Online Optimization of Power Networks

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March 2016
CISS Plenary Princeton

Key message

Large network of DERs

- Real-time optimization at scale
- Computational challenge: power flow solution

Online optimization [feedback control]

- Network computes power flow solutions in real time at scale for free
- Exploit it for our optimization/control
- Naturally adapts to evolving network conditions

Examples

- Slow timescale: OPF
- Fast timescale: frequency control

Outline

Background: smart grid

Online OPF

- Optimal power flow & relaxations
- Online algorithm
- Analysis and simulations

Gan & L, JSAC 2016

Load-side frequency control

- Dynamic model & design approach
- Distributed online algorithm
- Analysis and simulations

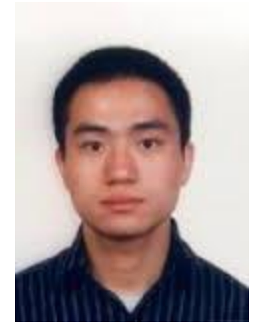
Zhao, Topcu, Li, L, TAC 2014
Mallada, Zhao, L, Allerton 2014
Zhao et al: CDC 2014, CISS 2015, PSCC 2016

Semidefinite relaxations of OPF

Bose (UIUC)

Chandy

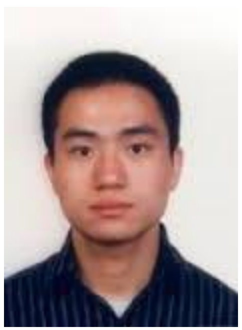
Farivar



Gan (FB)

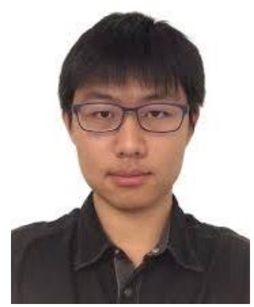
Lavaei (UCB)

Online OPF



Gan (FB)

Dvijotham



Tang

Dynamics

Mallada (JHU)

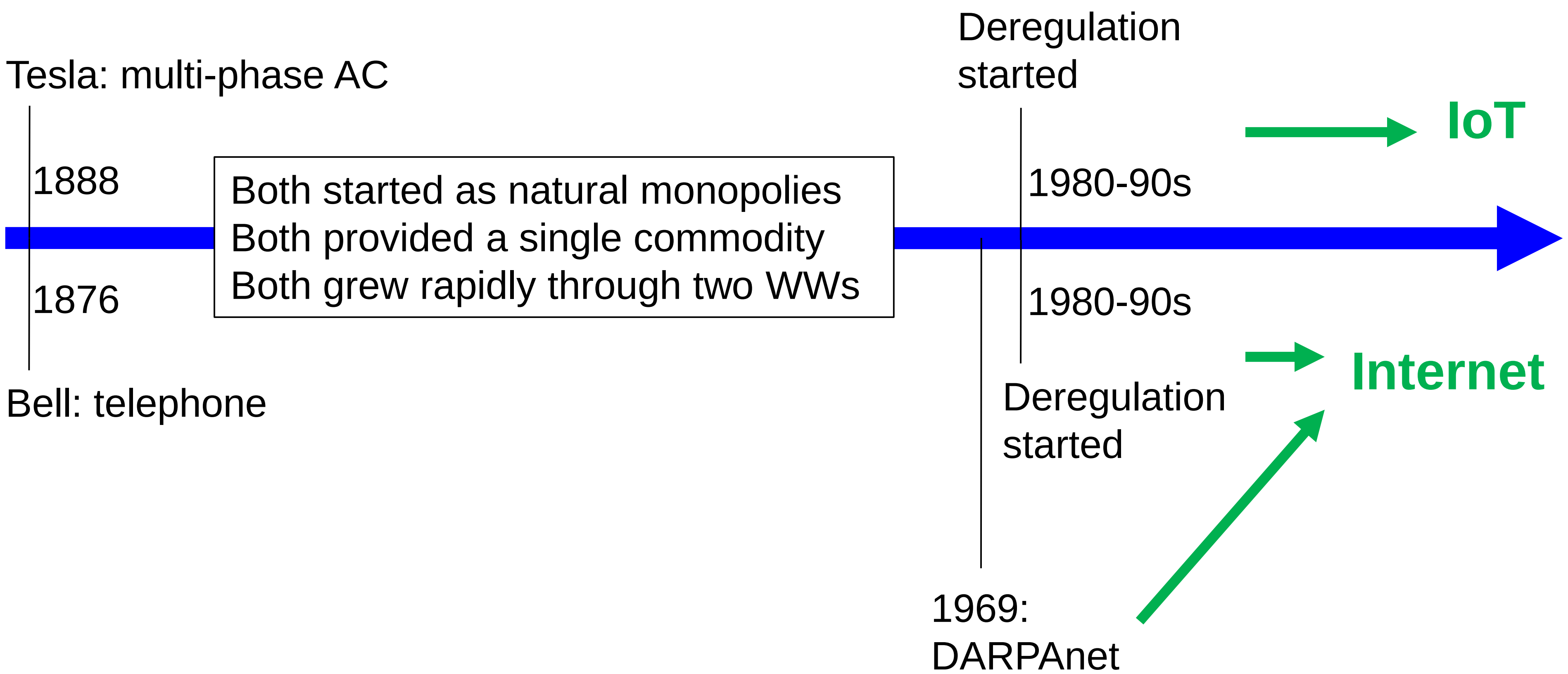
Li (Harvard)

Topcu (Austin)

Zhao

Watershed moment

Power network will undergo similar **architectural transformation** that phone network went through in the last two decades



Four drivers

Proliferation of renewables

Electrification of transportation

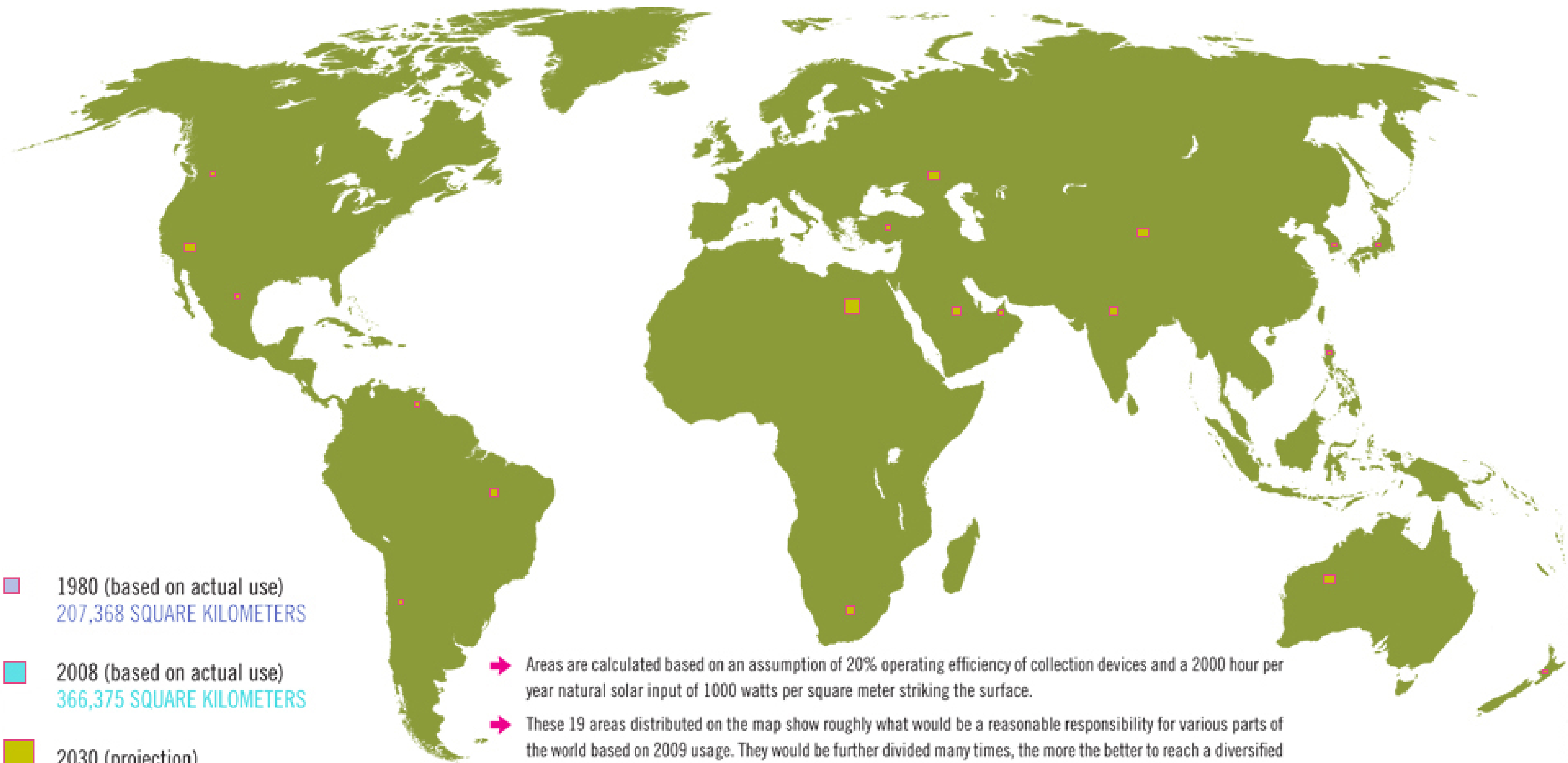
challenges

Advances in power electronics

Deployment of sensing, control, comm

enablers

Area to power the world by solar



1980 (based on actual use)
207,368 SQUARE KILOMETERS

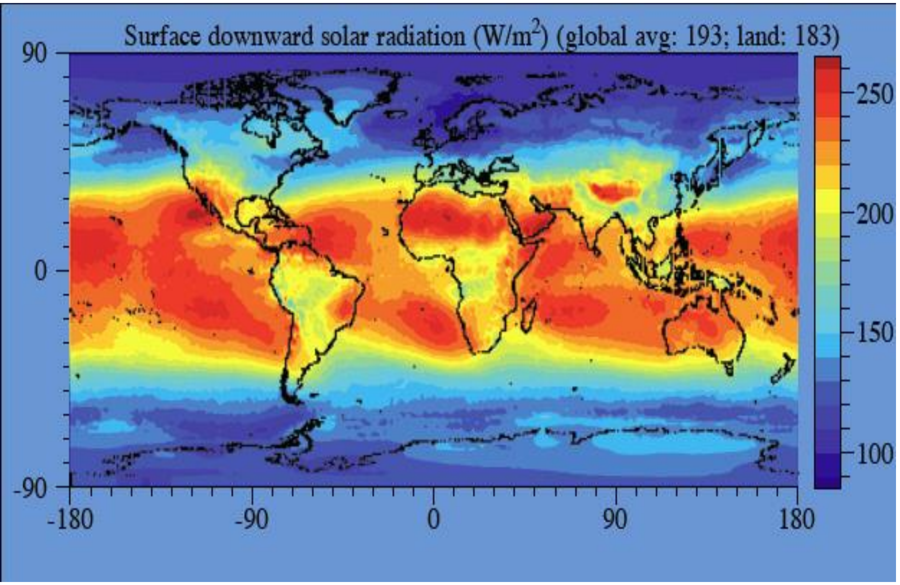
2008 (based on actual use)
366,375 SQUARE KILOMETERS

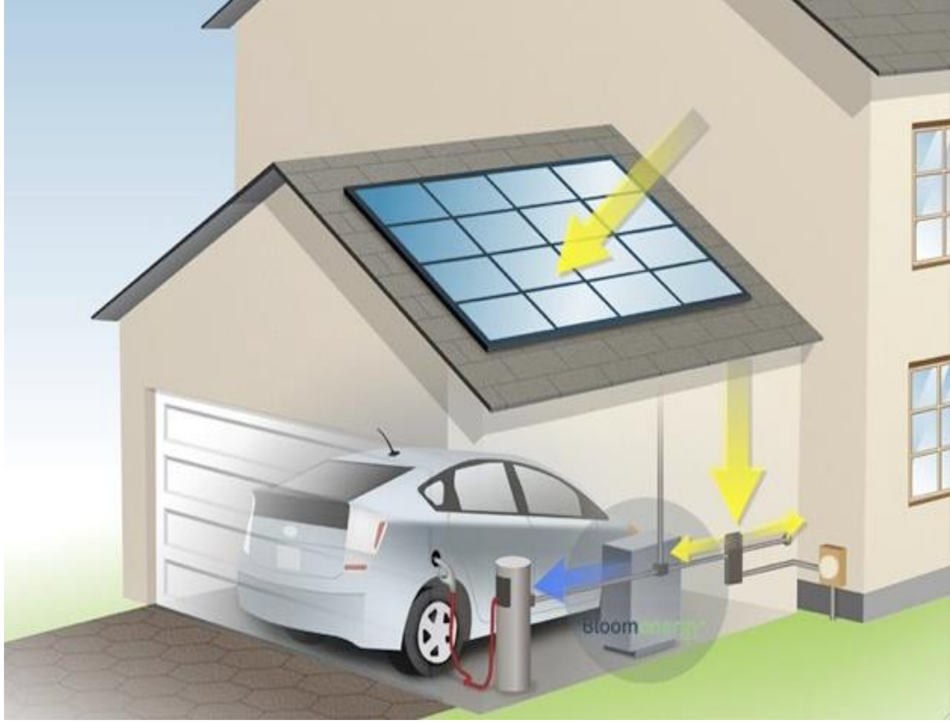
2030 (projection)
496,805 SQUARE KILOMETERS

- Areas are calculated based on an assumption of 20% operating efficiency of collection devices and a 2000 hour per year natural solar input of 1000 watts per square meter striking the surface.
- These 19 areas distributed on the map show roughly what would be a reasonable responsibility for various parts of the world based on 2009 usage. They would be further divided many times, the more the better to reach a diversified infrastructure that localizes use as much as possible.
- The large square in the Saharan Desert (1/4 of the overall 2030 required area) would power all of Europe and North Africa. Though very large, it is 18 times less than the total area of that desert.
- The definition of "power" covers the fuel required to run all electrical consumption, all machinery, and all forms of transportation. It is based on the US Department of Energy statistics of worldwide Btu consumption and estimates the 2030 usage (678 quadrillion Btu) to be 44% greater than that of 2008.
- Area calculations do not include magenta border lines.

LAND ART GENERATOR INITIATIVE

Solar power over land:
> 20x world energy demand





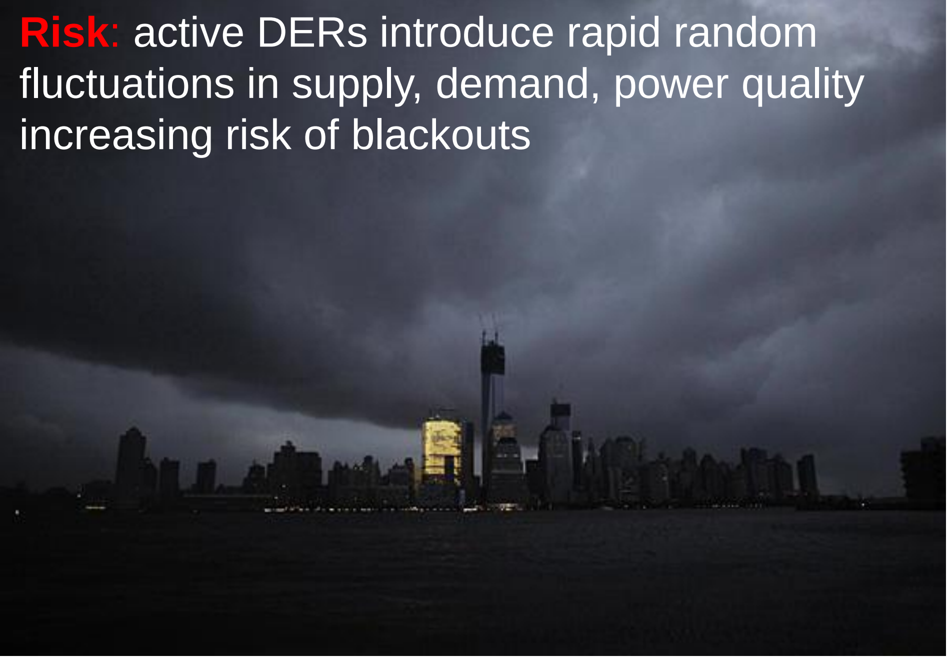
network of billions of active distributed energy resources (DERs)

DER: PV, wind tb, EV, storage, smart bldg / appl intelligence

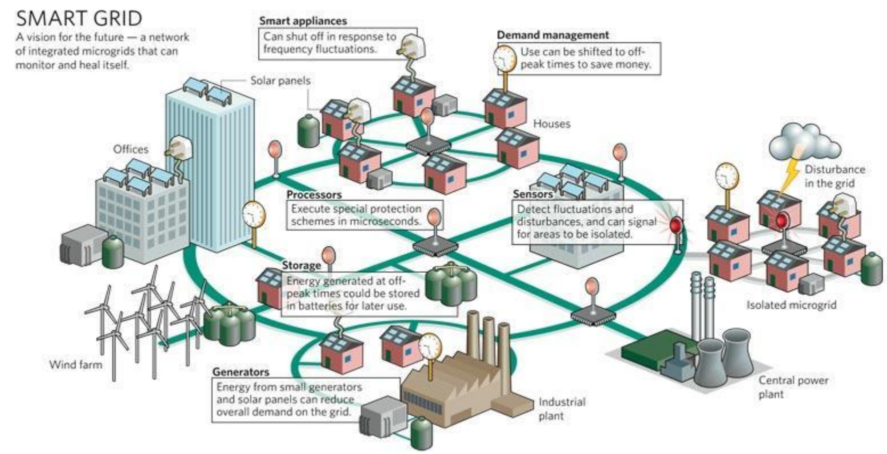
everywhere

connected

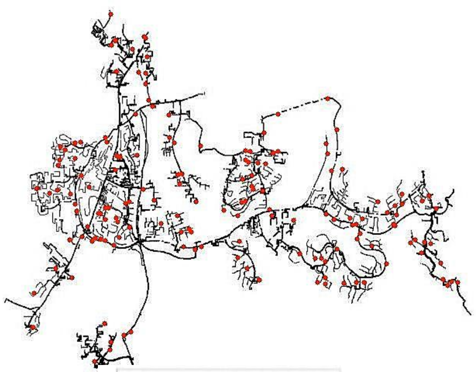
Risk: active DERs introduce rapid random fluctuations in supply, demand, power quality increasing risk of blackouts

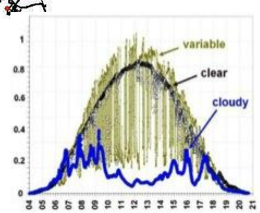


Opportunity: active DERs enables realtime dynamic network-wide feedback control, improving robustness, security, efficiency



Caltech research: distributed control of networked DERs





- Foundational theory, practical algorithms, concrete applications
- Integrate engineering and economics
- Active collaboration with industry

Recap

Global energy demand will continue to grow

Traditional supply is unsustainable

There is more renewable energy than the world ever needs

- Someone will figure out how to capture and store it

There will be connected intelligence everywhere

- Cost of computing, storage, communication and manufacturing will continue to drop

➔ Power system will transform into the largest and most complex Internet of Things

- Generation, transmission, distribution, consumption, storage

Recap

To develop technologies that will enable and guide the historic transformation of our power system

- Generation, transmission, distribution, consumption, storage
- Devices, systems, theory, algorithms
- Control, optimization, stochastics, data, economics

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Gan & L, JSAC 2016

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Optimal power flow (OPF)

OPF is solved routinely for

- network control & optimization decisions
- market operations & pricing
- at timescales of mins, hours, days, …

Non-convex and hard to solve

- Huge literature since 1962
- Common practice: DC power flow (LP)
- Also: Newton-Raphson, interior point, …

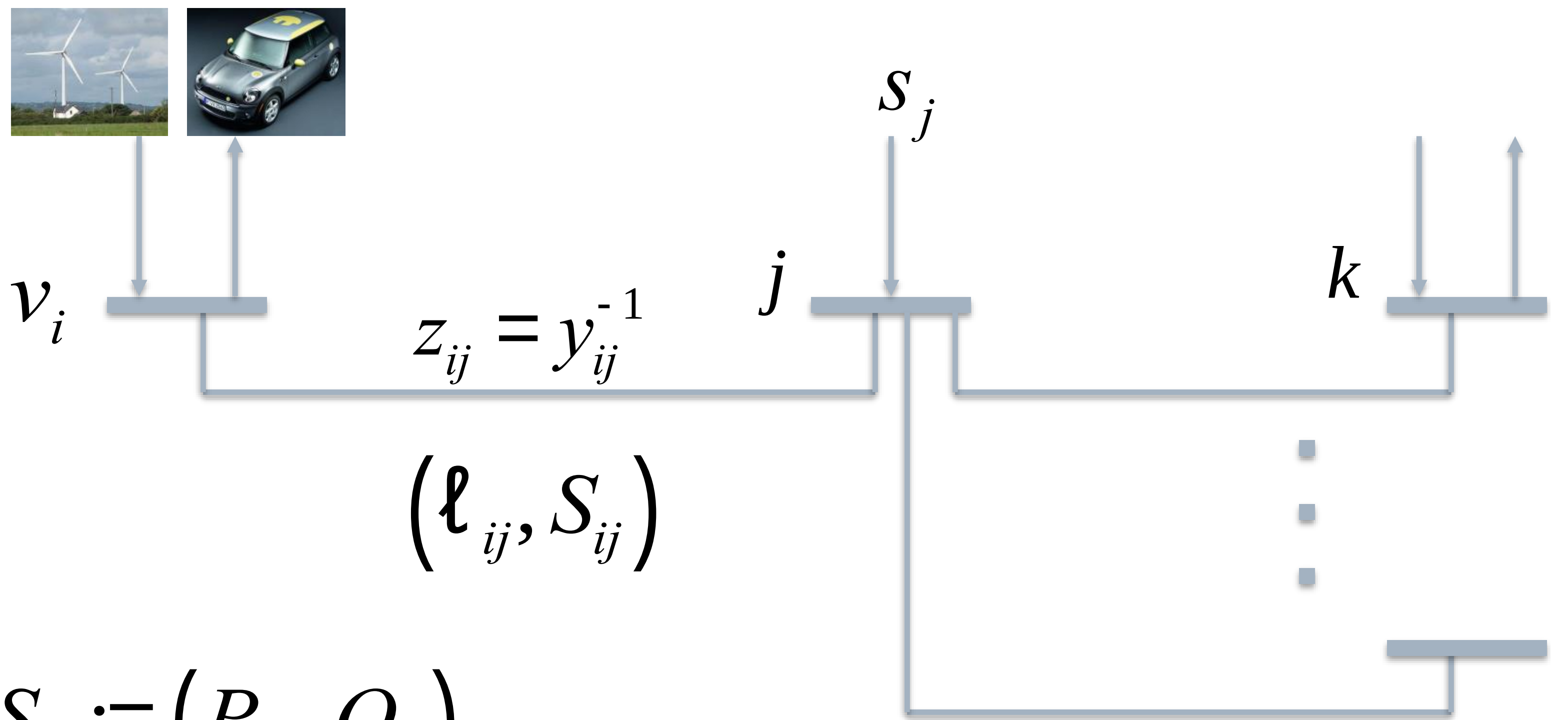
$$\min \ c(x) \quad \text{s. t.} \quad F(x) = 0, \ x \le \bar{x}$$

Optimal power flow (OPF)

OPF underlies many applications

- Unit commitment, economic dispatch
- State estimation
- Contingency analysis
- Feeder reconfiguration, topology control
- Placement and sizing of capacitors, storage
- Volt/var control in distribution systems
- Demand response, load control
- Electric vehicle charging
- Market power analysis
- …

Branch flow model



directed graph G

$$S_{ij} := (P_{ij},\ Q_{ij})$$

$$s_i := (p_i,\ q_i)$$

$$v_i := |V_i|^2, \quad \ell_{ij} := |I_{ij}|^2$$

Branch flow model

Branch flow model

power balance

$$\sum_{j \to k} S_{jk} = \sum_{i \to j} \left(S_{ij} - z_{ij} \ell_{ij} \right) + s_j$$

"Ohm's law"

$$v_i - v_j = 2\ \mathrm{Re}\left(z_{ij}^H S_{ij} \right) - \left| z_{ij} \right|^2 \ell_{ij}$$

def of power

$$v_i \ell_{ij} = \left| S_{ij} \right|^2$$

$$x := (s, v, S, \ell) \in \mathbf{R}^{3(m+n+1)}$$

$$= (p, q, v, P, Q, \ell)$$

Branch flow model

Branch flow model

linear

$$\sum_{j \to k} S_{jk} = \sum_{i \to j} \left(S_{ij} - z_{ij} \ell_{ij} \right) + s_j$$

$$v_i - v_j = 2 \operatorname{Re}\left(z_{ij}^H S_{ij} \right) - \left| z_{ij} \right|^2 \ell_{ij}$$

quadratic

$$v_i \ell_{ij} = \left| S_{ij} \right|^2$$

$$x := (s, v, S, \ell) \in \mathbf{R}^{3(m+n+1)}$$

$$= (p, q, v, P, Q, \ell)$$

DistFlow equations (radial nk)

Baran & Wu, 1989

Branch flow model

Branch flow model

$$\sum_{j \to k} S_{jk} = \sum_{i \to j} \left(S_{ij} - z_{ij} \ell_{ij} \right) + s_j$$

$$v_i - v_j = 2 \, \mathrm{Re} \left(z_{ij}^H S_{ij} \right) - \left| z_{ij} \right|^2 \ell_{ij}$$

$$v_i \ell_{ij} = \left| S_{ij} \right|^2$$

\+ cycle condition on

$$x := (s, v, S, \ell) \in \mathbf{R}^{3(m+n+1)}$$

SOCP relaxation

Branch flow model

$$\sum_{j \to k} S_{jk} = \sum_{i \to j} \left(S_{ij} - z_{ij} \ell_{ij} \right) + s_j$$

$$v_i - v_j = 2 \,\mathrm{Re} \left(z_{ij}^H S_{ij} \right) - \left| z_{ij} \right|^2 \ell_{ij}$$

$$v_i \ell_{ij} \geq \left| S_{ij} \right|^2$$

$$x := (s, v, S, \ell) \in \mathbf{R}^{3(m+n+1)}$$

Optimal power flow (OPF)

$$\min \quad f(x)$$

$$\text{over} \quad x := (s, v, \ell, S)$$

$$\text{s. t.} \quad \underline{s}_j \le s_j \le \overline{s}_j \qquad \underline{v}_j \le v_j \le \overline{v}_j$$

power flow equations nonconvex

Optimal power flow (OPF)

$$
\begin{aligned}
&\min && f(x) \\
&\text{over} && x := (s, v, \ell, S) \\
&\text{s. t.} && \underline{s}_j \text{ £ } s_j \text{ £ } \overline{s}_j \qquad \underline{v}_j \text{ £ } v_j \text{ £ } \overline{v}_j
\end{aligned}
$$

~~power flow equations~~ nonconvex

SOCP relaxation

SOCP relaxation of OPF

OPF: $$\min_{x \in \mathbf{X}} f(x)$$

SOCP: $$\min_{x \in \mathbf{X}^{+}} f(x)$$

SOCP relaxation of OPF

OPF: $$\min_{x \in \mathbf{X}} f(x)$$

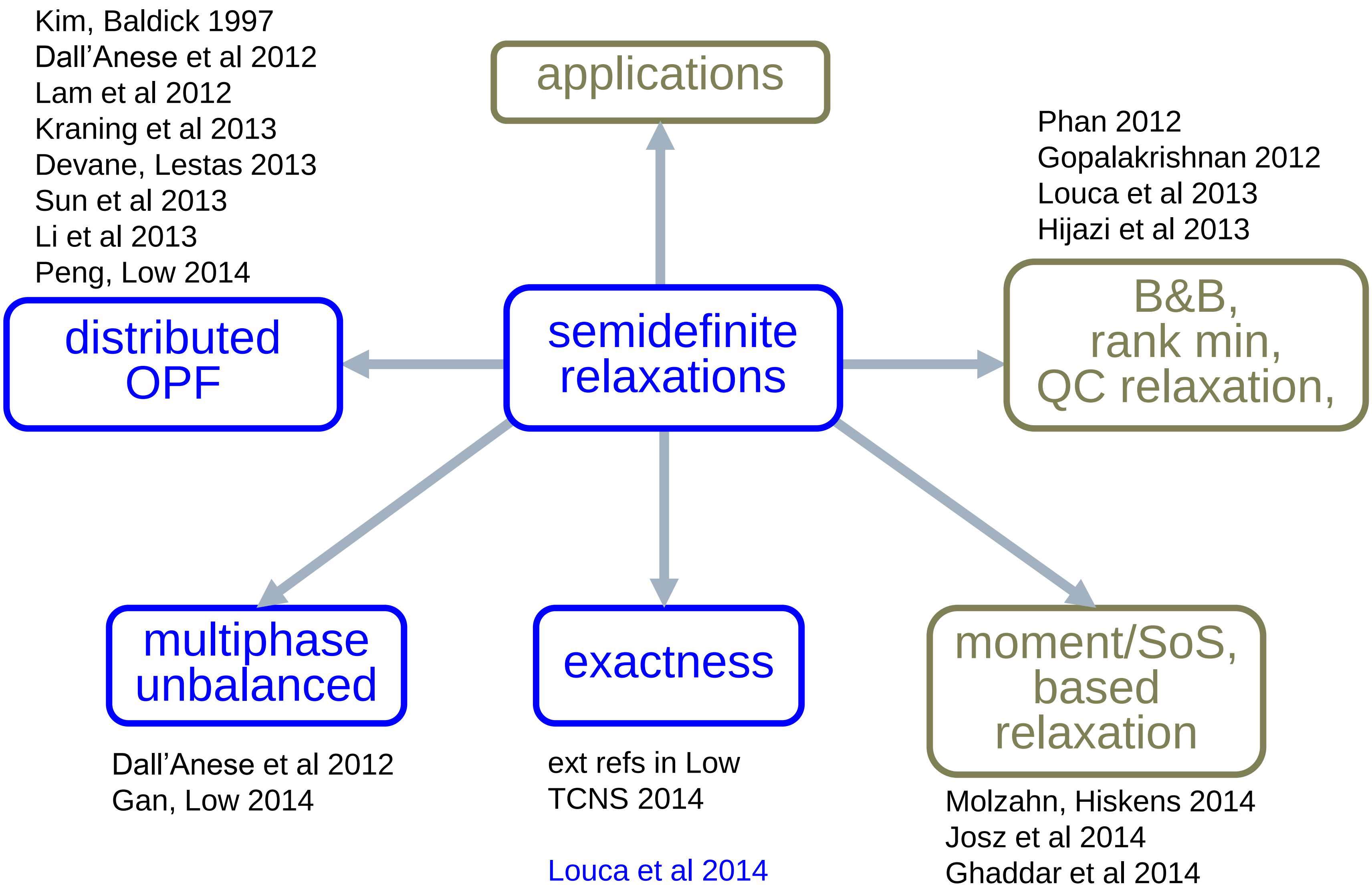
SOCP: $$\min_{x \in \mathbf{X}^+} f(x)$$

<u>Theorem</u>: SOCO is exact for tree networks if

1. Not both real & reactive power are both lower & upper bounded at each end of a line; *or*
2. Voltage upper bounds are not tight

[Farivar et al 2013, Gan et al 2014, Bose et al 2015]

Convex relaxations of OPF



SOCP relaxation of OPF

OPF: $$\min_{x \in \mathbf{X}} f(x)$$

relaxation: $$\min_{x \in \mathbf{X}^{+}} f(x)$$

But all these algorithms are offline …

… unsuitable for real-time optimization of
network of distributed energy resources

SOCP relaxation of OPF

OPF: $$\min_{x \in \mathbf{X}} f(x)$$

relaxation: $$\min_{x \in \mathbf{X}^{+}} f(x)$$

We will compare our online algorithm to
SOCP relaxation wrt optimality and speed

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Gan & L, JSAC 2016

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OPF

$$\min \quad \sum_{i=0}^{n} a_i p_i^2 + b_i p_i$$

$$\text{over} \quad x := (p_i, q_i, i \in N)$$

controllable devices

$$y := (p_0, q_0, v_i, i \in N; P_{ij}, Q_{ij}, \ell_{ij}, (i,j) \in E)$$

uncontrollable state

OPF

$$\min \quad c_0(y) + c(x)$$

$$\text{over} \quad x,\ y$$

s. t.

OPF

$$\min \quad c_0(y) + c(x)$$

$$\text{over} \quad x,\ y$$

$$\text{s. t.} \quad F(x, y) = 0$$

power flow equations

OPF

$$\min \quad c_0(y) + c(x)$$

$$\text{over} \quad x,\ y$$

$$\text{s. t.} \quad F(x, y) = 0$$ power flow equations

$$y \le \overline{y}$$ operational constraints

$$x \in X := \{ \underline{x} \le x \le \overline{x} \}$$ capacity limits

$$\text{Assume: } \frac{\partial F}{\partial y} \ne 0 \quad \Rightarrow \quad y(x) \ \text{ over } X$$

OPF: eliminate y

$$\min_{x} \quad c_0(y(x)) + c(x)$$

$$\text{s. t.} \quad y(x) \le \overline{y}$$

$$x \in X := \{\underline{x} \le x \le \overline{x}\}$$

OPF: add barrier

$$\min_{x} \quad c_0(y(x)) + c(x)$$

$$\text{s. t.} \quad y(x) \le \overline{y}$$

$$x \in X := \{ \underline{x} \le x \le \overline{x} \}$$

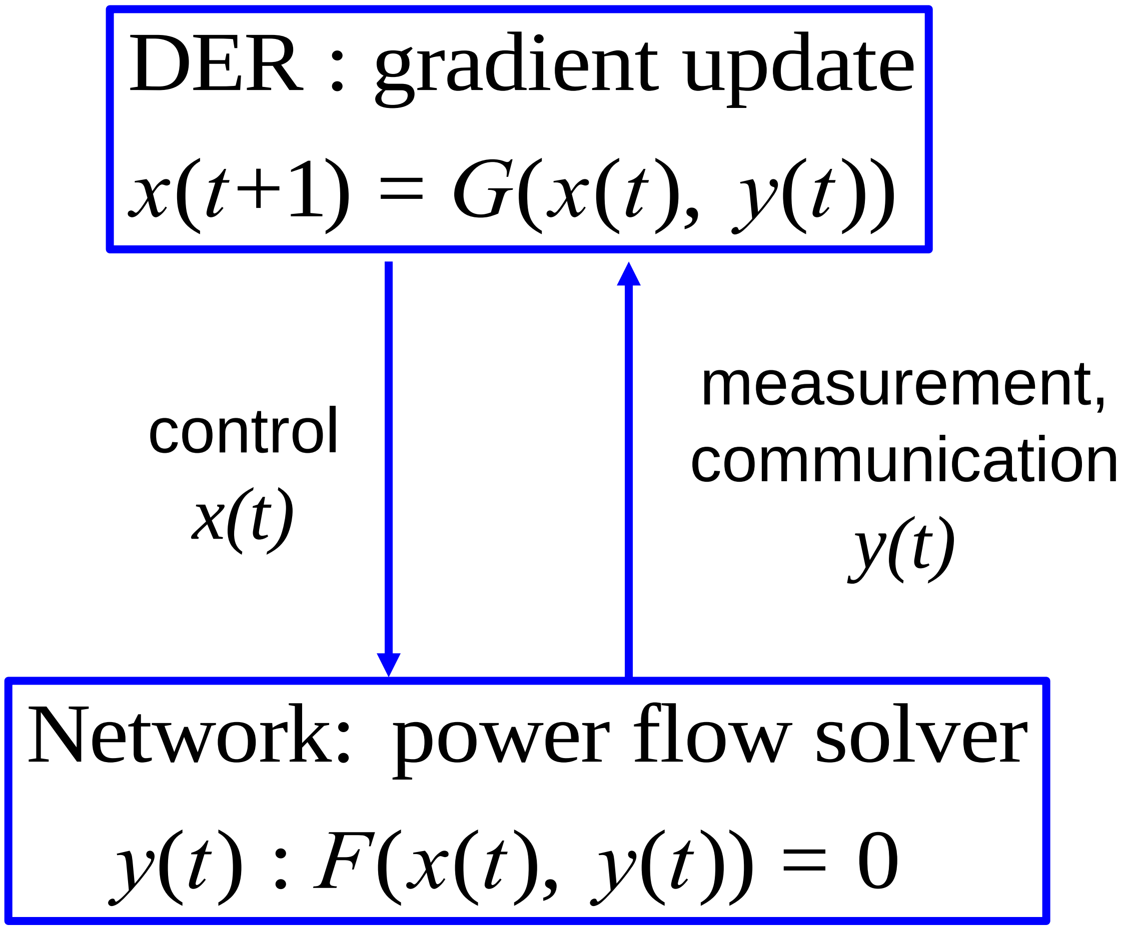
add barrier function
to remove
operational constraints

$$\min \quad L(x, y(x);\ \mu)$$

$$\text{over} \quad x \in X$$

L: nonconvex

Online (feedback) perspective



Gan & Low JSAC 2016
Dall'Anese & Simonetto 2016

Online gradient algorithm

$$\begin{array}{ll} \min & L(x, y(x);\ \mu) \\ \text{over} & x \in X \end{array}$$

gradient projection algorithm:

$$x(t+1) = \left[x(t) - \eta \frac{\partial L}{\partial x}(t) \right]_X \qquad \text{active control}$$

$$y(t) = y(x(t)) \qquad \text{law of physics}$$

- Explicitly exploits network as power flow solver
- Naturally tracks changing network conditions

Online gradient algorithm

$$
\begin{aligned}
\min \quad & L(x, y(x);\ \mu) \\
\text{over} \quad & x \in X
\end{aligned}
$$

gradient projection algorithm:

$$x(t+1) = \left[x(t) - \eta \frac{\partial L}{\partial x}(t) \right]_X$$

active control

$$y(t) = y(x(t))$$

law of physics

<u>Results</u>

1. Local optimality
2. Global optimality
3. Suboptimality bound
4. Tracking performance

Gan & L, JSAC 2016
Dvijotham, Tang & L, 2016

Local optimality

- $x(t)$ converges to set of local optima
- if #local optima is finite, $x(t)$ converges

Global optimality

Assume: $p_0(x)$ convex over X

$v_k(x)$ concave over X

$$A := \{ x \in X \; : \; v(x) \le \alpha\bar{v} + (1-\alpha)\underline{v} \}$$

Theorem

If co{local optima} are in A then

- $x(t)$ converges to the set of global optima
- $x(t)$ itself converges a global optimum if #local optima is finite

Global optimality

Assume: $p_0(x)$ convex over X

$v_k(x)$ concave over X

$$A := \{ x \in X \ : \ v(x) \leq \alpha \overline{v} + (1-\alpha)\underline{v} \}$$

Theorem

- Can choose α s.t.

  $A \rightarrow$ original feasible set

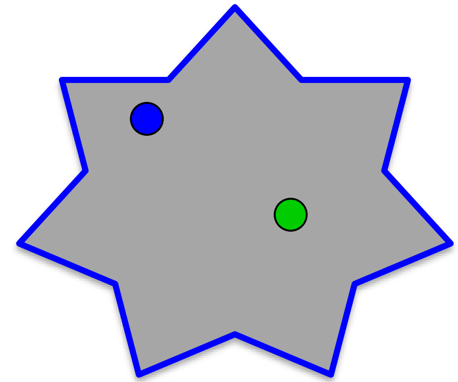
- If SOCP is exact over X, then assumption holds

Suboptimality gap

any local optimum

any original feasible pt slightly away from X boundary

$$L(x^*) - L(\hat{x}) \le r \approx 0$$



- Informally, a local minimum is almost as good as any strictly interior feasible point

Tracking performance

$$\min_{x} \quad c_0(y(x)) + c(x)$$
$$\text{s. t.} \quad y(x) \le \overline{y}$$
$$x \in X$$

$$\min_{x} \quad c_0(y(x), \gamma_t) + c(x, \gamma_t)$$
$$\text{s. t.} \quad y(x, \gamma_t) \le \overline{y}$$
$$x \in X$$

drifting OPF

Tracking performance

$$R(x, x^*) \; := \; \sum_{t=1}^{T} c_0(y(x), \gamma_t) + c(x, \gamma_t)$$

cost of Alg

dynamic regret

$$- \; \sum_{t=1}^{T} c_0(y(x^*), \gamma_t) + c(x^*, \gamma_t)$$

optimal cost

Tracking performance

$$R(x, x^*) \;:=\; \sum_{t=1}^{T} c_0(y(x), g_t) + c(x, g_t)$$ cost of Alg

dynamic regret

$$-\;\sum_{t=1}^{T} c_0(y(x^*), g_t) + c(x^*, g_t)$$ optimal cost

Theorem

$$R(x, x^*) = O\left(\sqrt{T}\left(1 + \sum_{t=1}^{T} \left\| x_{t+1}^* - x_t^* \right\|\right)\right) + \sum_{t=1}^{T} d_t$$

rate of drifting

subopt of local min

Tracking performance

$$R(x, x^*) := \sum_{t=1}^{T} c_0(y(x), g_t) + c(x, g_t)$$ cost of Alg

dynamic regret

$$- \sum_{t=1}^{T} c_0(y(x^*), g_t) + c(x^*, g_t)$$ optimal cost

<u>**Theorem**</u>

- If rate of drifting is $o\left(\sqrt{T}\right)$ then per-step $R(x, x^*)$ is asymptotically bounded by $\bar{\delta}$ (local min)
- Can made $\bar{\delta}$ arbitrarily small at cost of computation

Simulations

# bus	CVX		IPM		error	speedup
	obj	time(s)	obj	time(s)		
42	10.4585	6.5267	10.4585	0.2679	-0.0e-7	24.36
56	34.8989	7.1077	34.8989	0.3924	+0.2e-7	18.11
111	0.0751	11.3793	0.0751	0.8529	+5.4e-6	13.34
190	0.1394	20.2745	0.1394	1.9968	+3.3e-6	10.15
290	0.2817	23.8817	0.2817	4.3564	+1.1e-7	5.48
390	0.4292	29.8620	0.4292	2.9405	+5.4e-7	10.16
490	0.5526	36.3591	0.5526	3.0072	+2.9e-7	12.09
590	0.7035	43.6932	0.7035	4.4655	+2.4e-7	9.78
690	0.8546	51.9830	0.8546	3.2247	+0.7e-7	16.12
790	0.9975	62.3654	0.9975	2.6228	+0.7e-7	23.78
890	1.1685	67.7256	1.1685	2.0507	+0.8e-7	33.03
990	1.3930	74.8522	1.3930	2.7747	+1.0e-7	26.98
1091	1.5869	83.2236	1.5869	1.0869	+1.2e-7	76.57
1190	1.8123	92.4484	1.8123	1.2121	+1.4e-7	76.27
1290	2.0134	101.0380	2.0134	1.3525	+1.6e-7	74.70
1390	2.2007	111.0839	2.2007	1.4883	+1.7e-7	74.64
1490	2.4523	122.1819	2.4523	1.6372	+1.9e-7	74.83
1590	2.6477	157.8238	2.6477	1.8021	+2.0e-7	87.58
1690	2.8441	147.6862	2.8441	1.9166	+2.1e-7	77.06
1790	3.0495	152.6081	3.0495	2.0603	+2.1e-7	74.07
1890	3.8555	160.4689	3.8555	2.1963	+1.9e-7	73.06
1990	4.1424	171.8137	4.1424	2.3586	+1.9e-7	72.84

Key message

Large network of DERs

- Real-time optimization at scale
- Computational challenge: power flow solution

Online optimization

- Network computes power flow solutions in real time at scale for free
- Exploit it for our optimization/control
- Naturally adapts to evolving network conditions

Examples

- Slow timescale: OPF
- Fast timescale: frequency control

Outline

Optimal power flow

- Background: semidefinite relaxations
- Online algorithm
- Analysis and simulations

Load-side frequency control

- Dynamic model & design approach
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Zhao, Topcu, Li, L, TAC 2014
Mallada, Zhao, L, Allerton 2014
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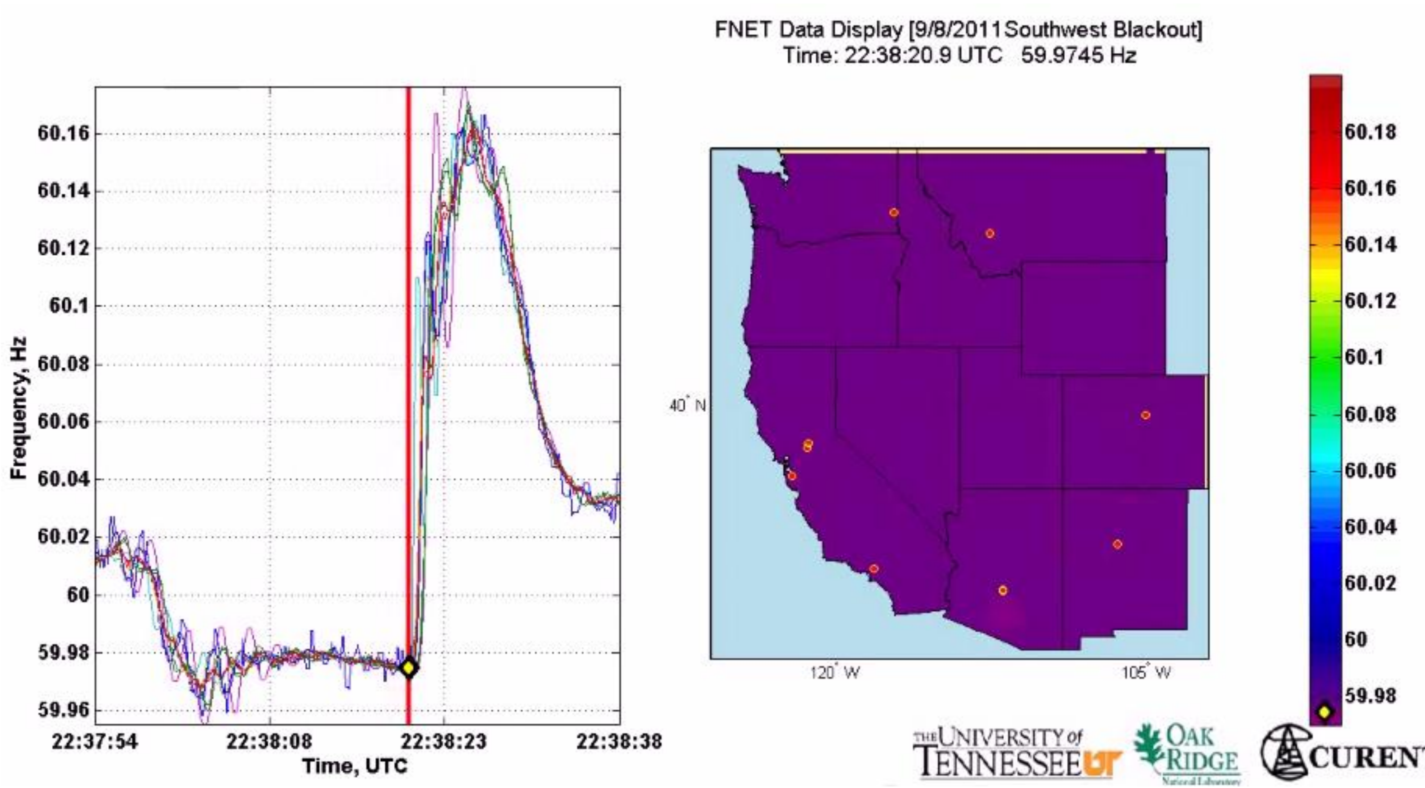
Motivation

- All buses synchronized to same nominal frequency (US: 60 Hz; Europe/China: 50 Hz)
- Supply-demand imbalance ➔ frequency fluctuation



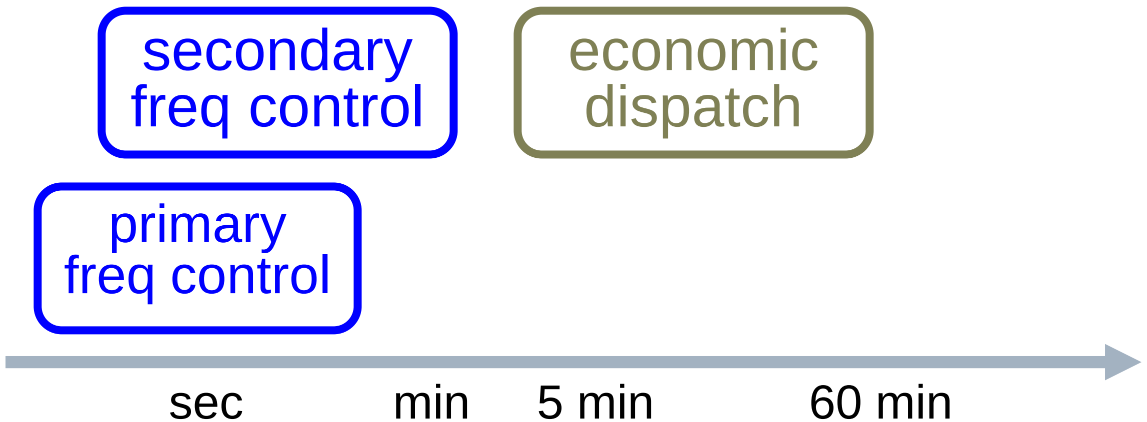
in)

2011 Southwest blackout

Why load-side participation

Ubiquitous continuous load-side control can supplement generator-side control

- faster (no/low inertia)
- more reliable (large #)
- better localize disturbances
- reducing generator-side control capacity



How

How to design load-side frequency control ?

How does it interact with generator-side control ?

Literature: load-side control

Original idea & early analytical work

- Schweppe et al 1980; Bergin, Hill, Qu, Dorsey, Wang, Varaiya …

Small scale trials around the world

- D.Hammerstrom et al 2007, UK Market Transform Programme 2008

Early simulation studies

- Trudnowski et al 2006, Lu and Hammerstrom 2006, Short et al 2007, Donnelly et al 2010, Brooks et al 2010, Callaway and I. A. Hiskens, 2011, Molina-Garcia et al 2011

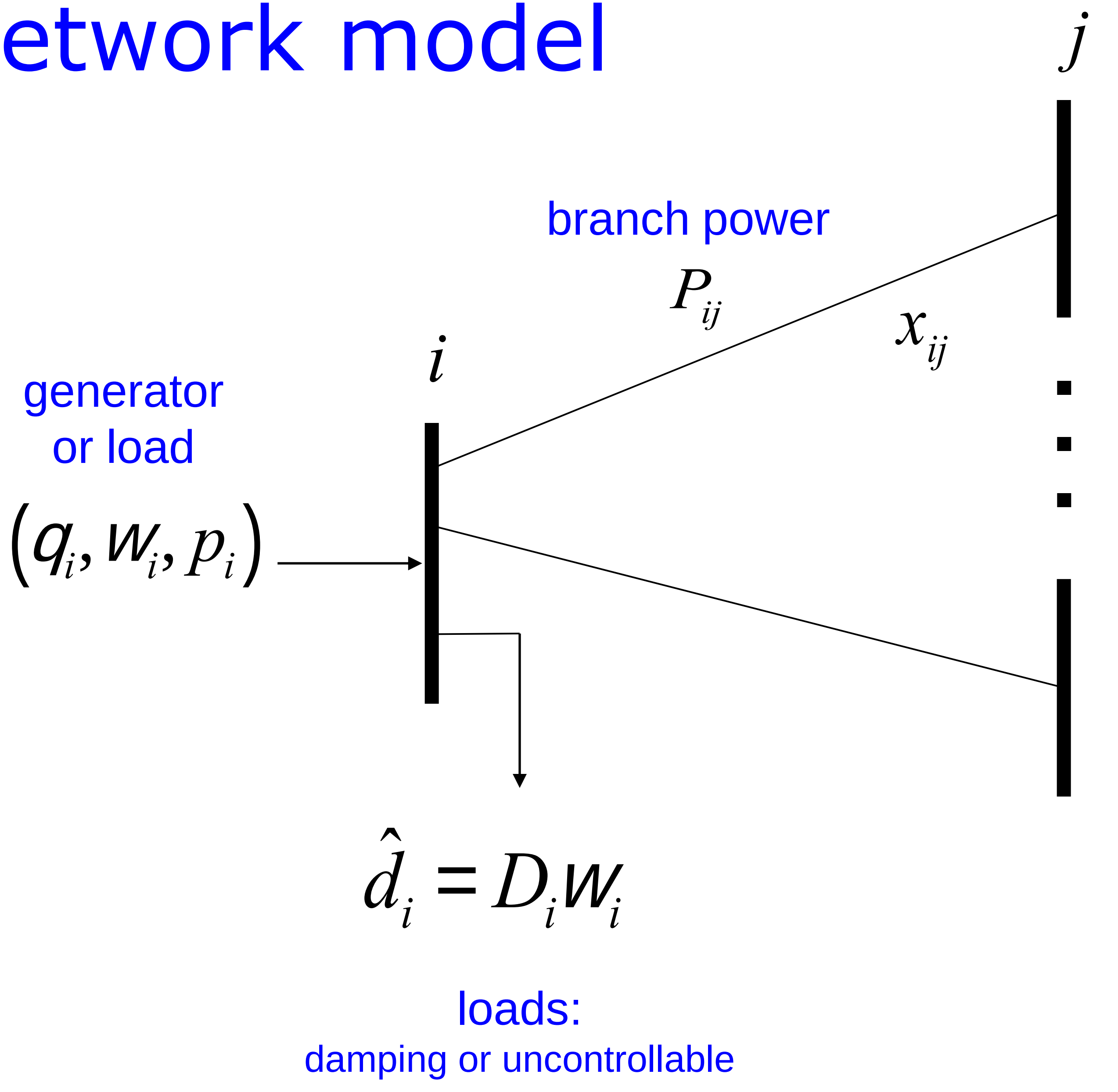
Analytical work – load-side control

- Zhao et al (2012/2014), Mallada and Low (2014), Mallada et al (2014), Zhao and Low (2014), Zhao et al (2015)
- Simpson-Porco et al 2013, You and Chen 2014, Zhang and Papachristodoulou (2014), Ma et al (2014), Zhao, et al (2014),

Recent analysis – generator-side/microgrid control:

- Andreasson et al (2013), Zhang and Papachristodoulou (2013), Li et al (2014), Burger et al (2014), You and Chen (2014), Simpson-Porco et al (2013), Hill et al (2014), Dorfler et al (2014)

Network model



i : region/control area/balancing authority

Network model

$$\dot{\theta}_i = \omega_i$$

$$M_i \dot{\omega}_i = -D_i \omega_i + p_i - \sum_e C_{ie} P_e$$

$$P_{ij} = b_{ij} \sin\left(\theta_i - \theta_j\right) \qquad \forall\ i \to j$$

Generator bus: $M_i > 0$

Load bus: $M_i = 0$

Network model

$$\dot{\theta}_i = \omega_i$$

$$M_i \dot{\omega}_i = -D_i \omega_i + \boxed{p_i} - \sum_e C_{ie} P_e$$

$$P_{ij} = b_{ij} \sin\left(\theta_i - \theta_j\right) \qquad \forall\, i \to j$$

Generator bus: p_i is real power injection

Load bus: p_i is controllable load

Generator-side control

$$\dot{\theta}_i = \omega_i$$

$$M_i \dot{\omega}_i = -D_i \omega_i + \boxed{p_i} - \sum_e C_{ie} P_e$$

$$P_{ij} = b_{ij} \sin\left(\theta_i - \theta_j\right) \qquad \forall\, i \rightarrow j$$

$$\dot{p}_i = -\frac{1}{\tau_{bi}}\left(p_i + a_i\right)$$

$$\dot{a}_i = -\frac{1}{\tau_{gi}}\left(a_i + p_i^c\right)$$

generator bus:

primary control $p_i^c(t) = p_i^c\left(w_i(t)\right)$

e.g. freq droop $p_i^c\left(w_i\right) = -b_i w_i$

Load-side control

$$\dot{\theta}_i = \omega_i$$

$$M_i \dot{\omega}_i = -D_i \omega_i + \boxed{p_i} - \sum_e C_{ie} P_e$$

$$P_{ij} = b_{ij} \sin\left(\theta_i - \theta_j\right) \qquad \forall\ i \to j$$

Load bus:

how to design feedback control ?

Network model

$$\dot{\theta}_i = \omega_i$$

$$M_i \dot{\omega}_i = -D_i \omega_i + p_i - \sum_e C_{ie} P_e$$

$$P_{ij} = b_{ij} \sin\left(\theta_i - \theta_j\right) \qquad \forall\ i \to j$$

Suppose the system is in steady state

Then: disturbance in gen/load …

Load-side controller design

$$\dot{\theta}_i = \omega_i$$

$$M_i \dot{\omega}_i = -D_i \omega_i + p_i - \sum_e C_{ie} P_e$$

$$P_{ij} = b_{ij} \sin\left(\theta_i - \theta_j\right) \qquad \forall\ i \to j$$

Control goals

Zhao, Topcu, Li, Low
TAC 2014
Mallada, Zhao, Low
Allerton, 2014

- Rebalance power & stabilize frequency
- Restore nominal frequency
- Restore scheduled inter-area flows
- Respect line limits

Load-side controller design

$$\dot{\theta}_i = \omega_i$$

$$M_i \dot{\omega}_i = -D_i \omega_i + p_i - \sum_e C_{ie} P_e$$

$$P_{ij} = b_{ij} \sin\left(\theta_i - \theta_j\right) \qquad \forall\ i \rightarrow j$$

Control goals (while min disutility)

Zhao, Topcu, Li, Low
TAC 2014
Mallada, Zhao, Low
Allerton, 2014

- Rebalance power & stabilize frequency
- Restore nominal frequency
- Restore scheduled inter-area flows
- Respect line limits

Load-side controller design

Design control law whose equilibrium solves:

$$\min_{d,P} \quad \sum_i c_i(d_i) \qquad \text{load disutility}$$

$$\text{s. t.} \quad P_i^m - d_i = \sum_e C_{ie} P_e \qquad \text{node } i \qquad \text{power balance}$$

$$\sum_{i \in N_k} \sum_e C_{ie} P_e = \hat{P}_k \qquad \text{area } k \qquad \text{inter-area flows}$$

$$\underline{P}_e \le P_e \le \overline{P}_e \qquad \text{line } e \qquad \text{line limits}$$

Control goals (while min disutility)

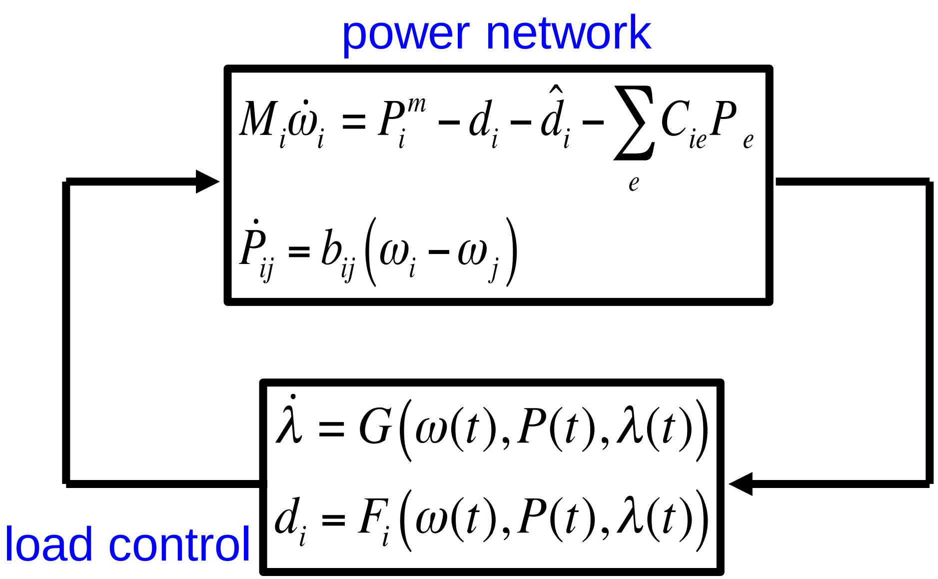
freq will emerge as Lagrange multiplier for power imbalance

- Rebalance power & stabilize frequency
- Restore nominal frequency
- Restore scheduled inter-area flows
- Respect line limits

Load-side controller design

Design control (G, F) s.t. closed-loop system

- is stable
- has equilibrium that is optimal



$$\min_{d,P} \quad \sum_i c_i(d_i)$$

$$\text{s. t.} \quad P_i^m - d_i = \sum_e C_{ie} P_e \qquad \text{node } i$$

$$\sum_{i \in N_k} \sum_e C_{ie} P_e = \hat{P}_k \qquad \text{area } k$$

$$\underline{P}_e \le P_e \le \overline{P}_e \qquad \text{line } e$$

Load-side controller design

Idea: exploit system dynamic as part of primal-dual algorithm for modified opt

- Distributed algorithm
- Stability analysis
- Control goals in equilibrium

power network

$$M_i \dot{\omega}_i = P_i^m - d_i - \hat{d}_i - \sum_e C_{ie} P_e$$

$$\dot{P}_{ij} = b_{ij} \left(\omega_i - \omega_j \right)$$

load control

$$\dot{\lambda} = G\left(\omega(t), P(t), \lambda(t) \right)$$

$$d_i = F_i\left(\omega(t), P(t), \lambda(t) \right)$$

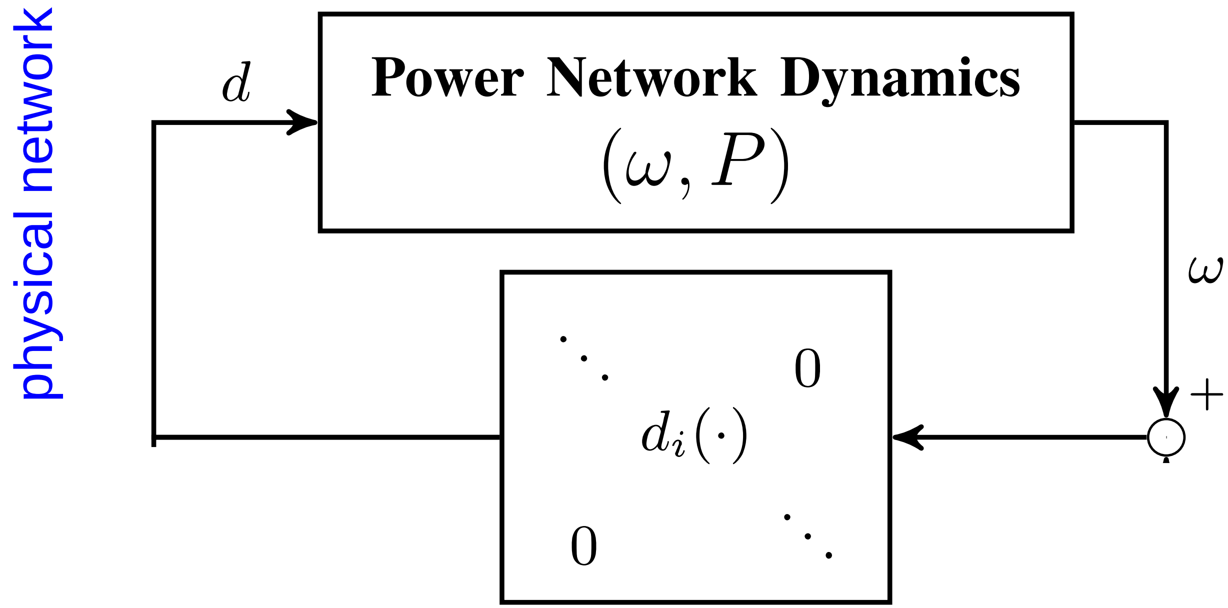
$$\min_{d,P} \quad \sum_i c_i(d_i)$$

$$\text{s. t.} \quad P_i^m - d_i = \sum_e C_{ie} P_e \qquad \text{node } i$$

$$\sum_{i \in N_k} \sum_e C_{ie} P_e = \hat{P}_k \qquad \text{area } k$$

$$\underline{P}_e \le P_e \le \overline{P}_e \qquad \text{line } e$$

Summary: control architecture

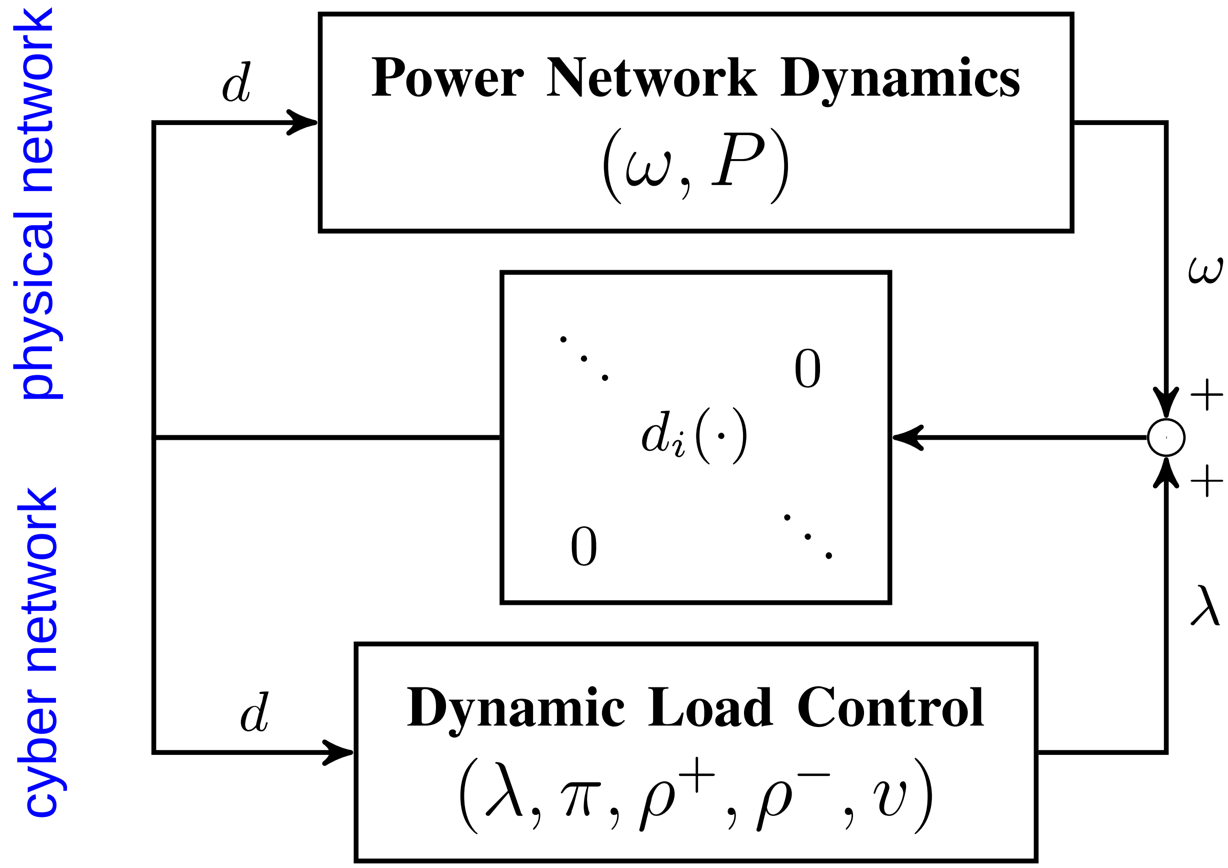


Primary load-side frequency control

- completely decentralized
- <u>Theorem</u>: stable dynamic, optimal equilibrium

Zhao, Topcu, Li, Low. TAC 2014

Summary: control architecture

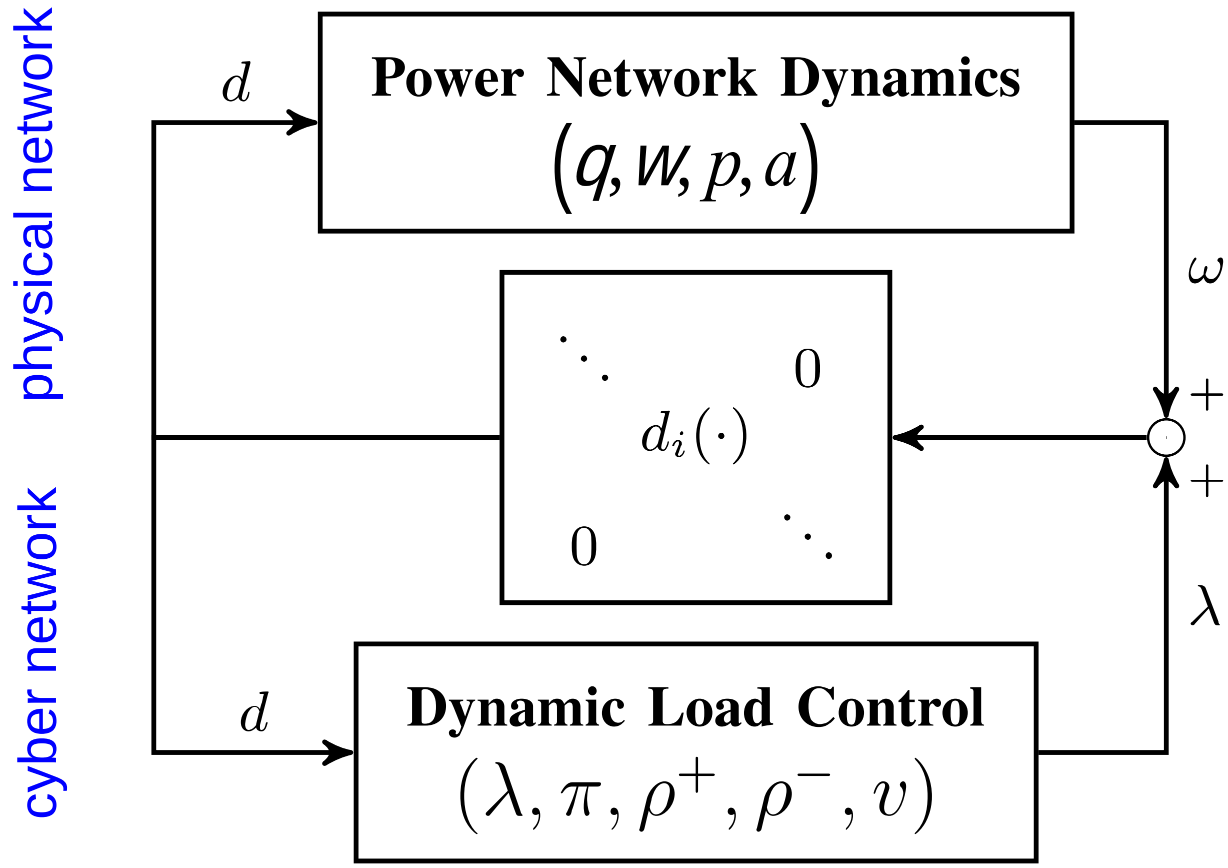


Secondary load-side frequency control

- communication with neighbors
- Theorem: stable dynamic, optimal equilibrium

Mallada, Zhao, Low. Allerton 2014

Summary: control architecture



With **generator-side** control, **nonlinear** power flow

- load-side improves both transient & eq
- <u>Theorem</u>: stable dynamic, optimal equilibrium

Zhao, Mallada, Low. CISS 2015

Simulations

Dynamic simulation of IEEE 39-bus system

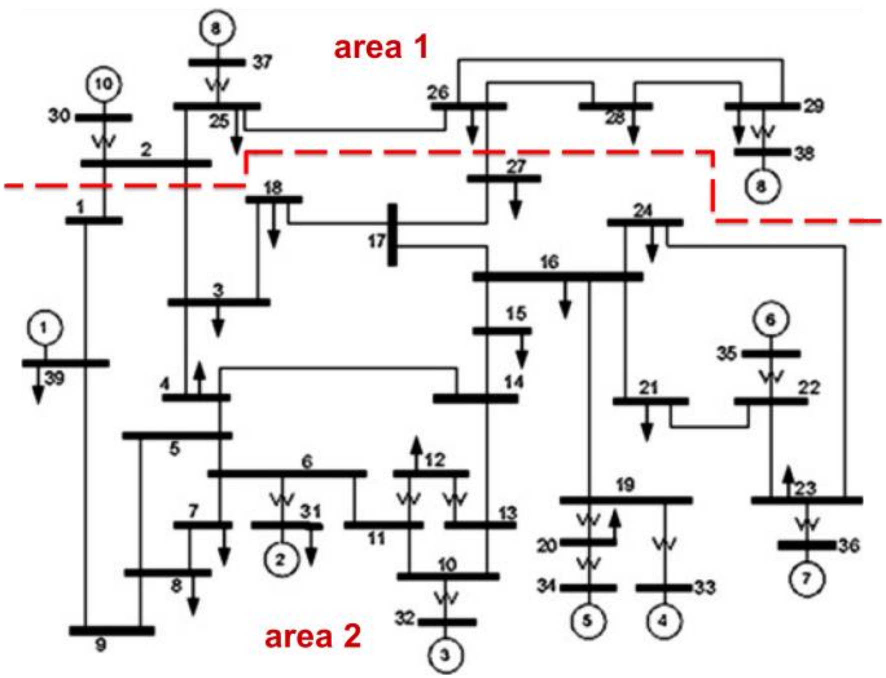
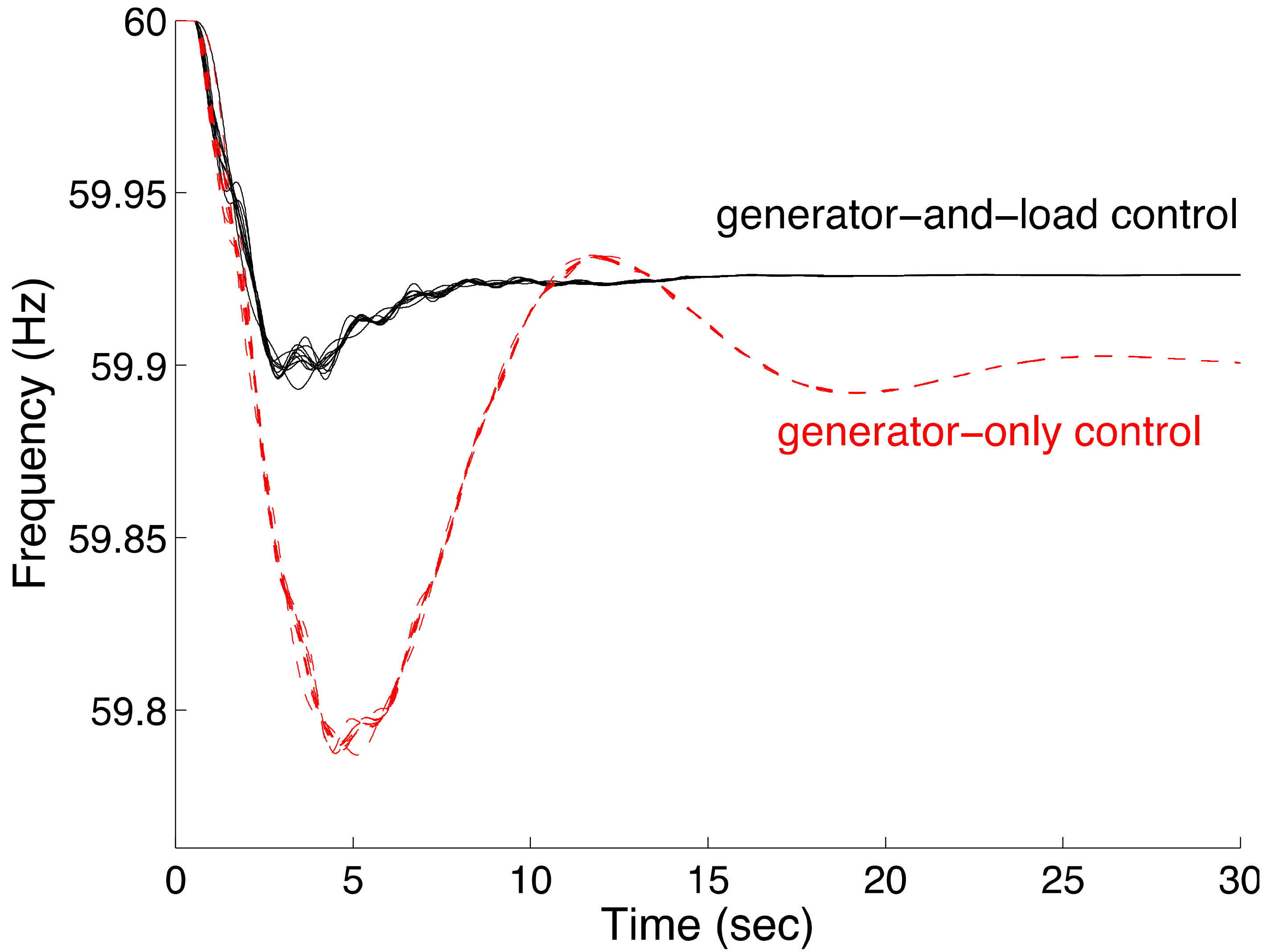


Fig. 2: IEEE 39 bus system : New England

- Power System Toolbox (RPI)
- Detailed generation model
- Exciter model, power system stabilizer model
- Nonzero resistance lines

Primary control



Secondary control

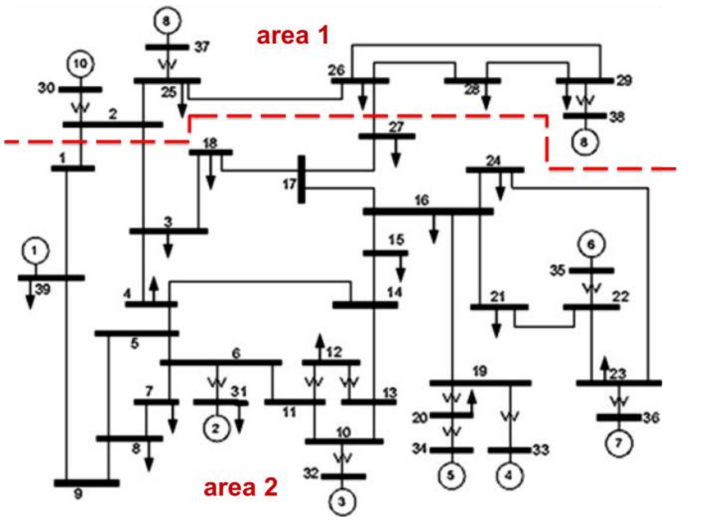
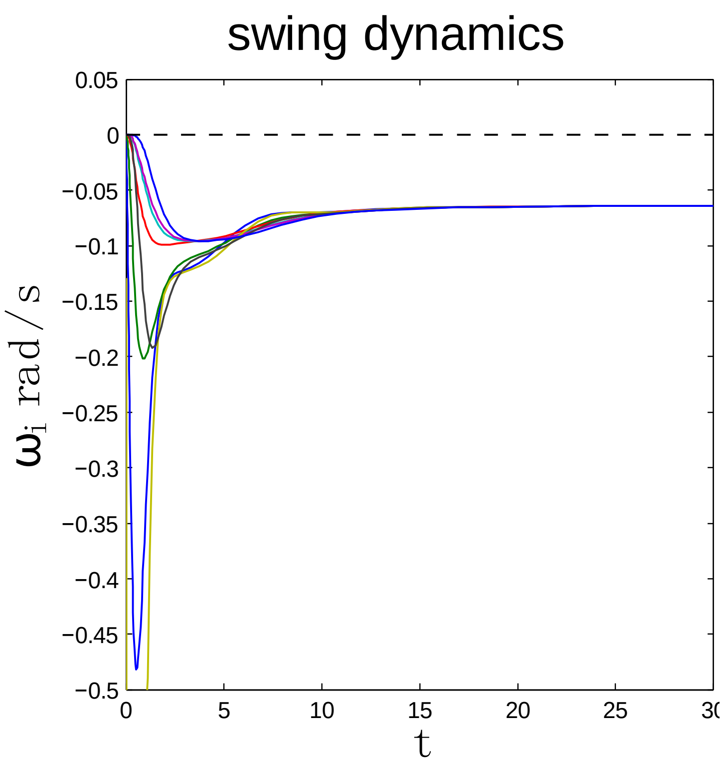
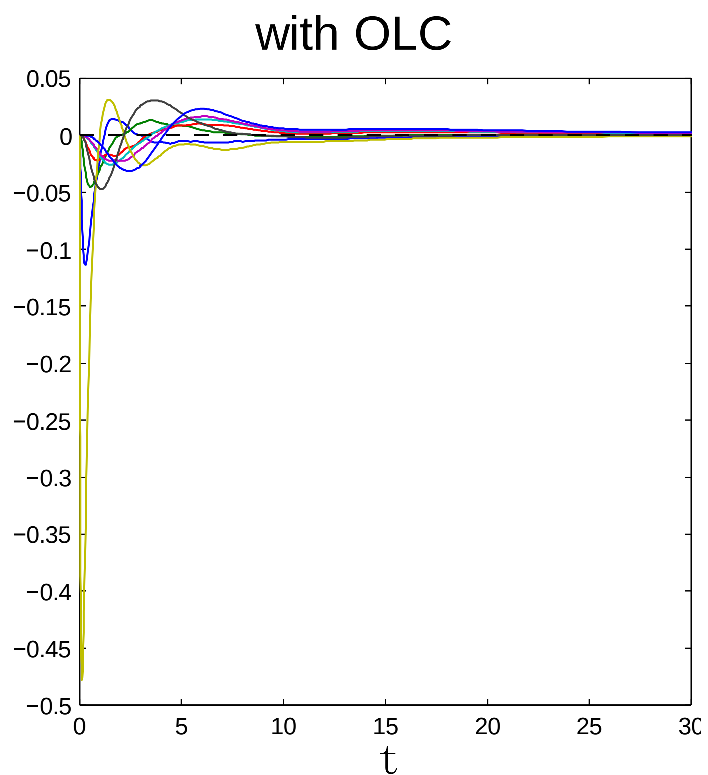


Fig. 2: IEEE 39 bus system : New England





area 1

Secondary control

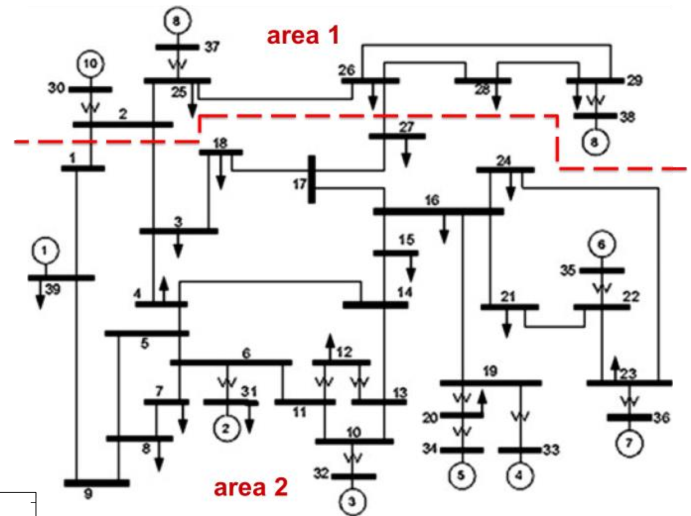
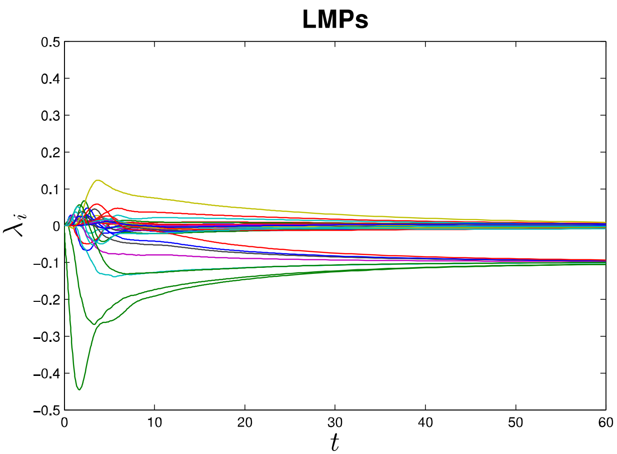
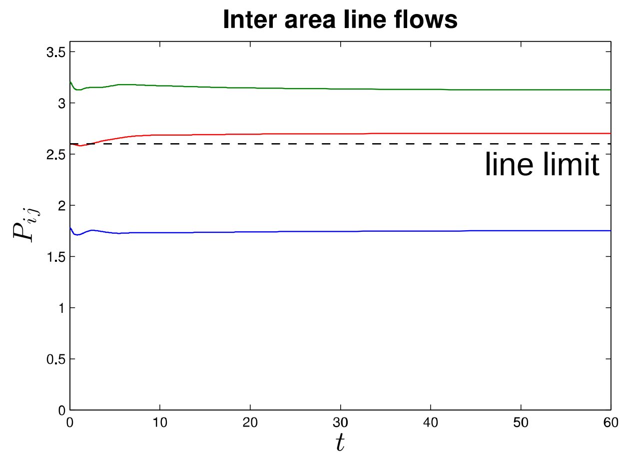
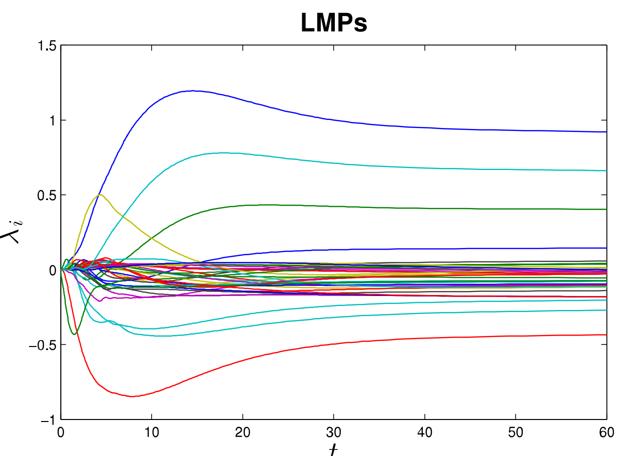


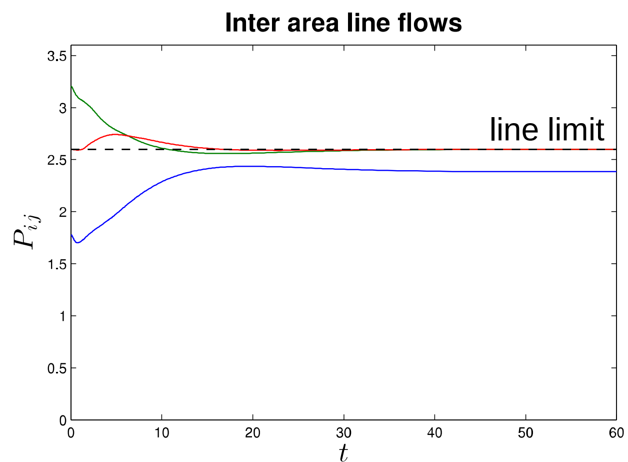
Fig. 2: IEEE 39 bus system : New England





no line limits





Total inter-area flow is the same in both cases

with line limits

Key message

Large network of DERs

- Real-time optimization at scale
- Computational challenge: power flow solution

Online optimization

- Network computes power flow solutions in real time at scale for free
- Exploit it for our optimization/control
- Naturally adapts to evolving network conditions

Examples

- Slow timescale: OPF
- Fast timescale: frequency control

Application: EV charging

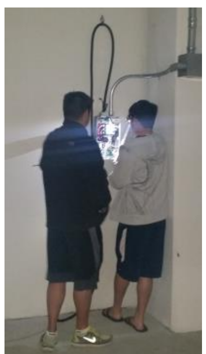
Online optimization of electric vehicle charging

- Enables mass deployment at much lower infrastructure costs
- First pilot @Caltech: 54 adaptive programmable chargers
- 2x 150kVA transformers, breakers, grid sensors, etc

Energized
Feb 13 Sat
2016

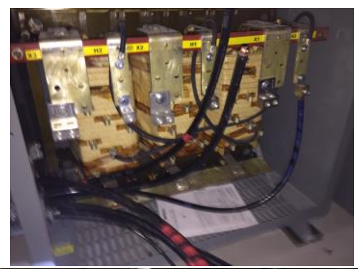
G. Lee

debugging

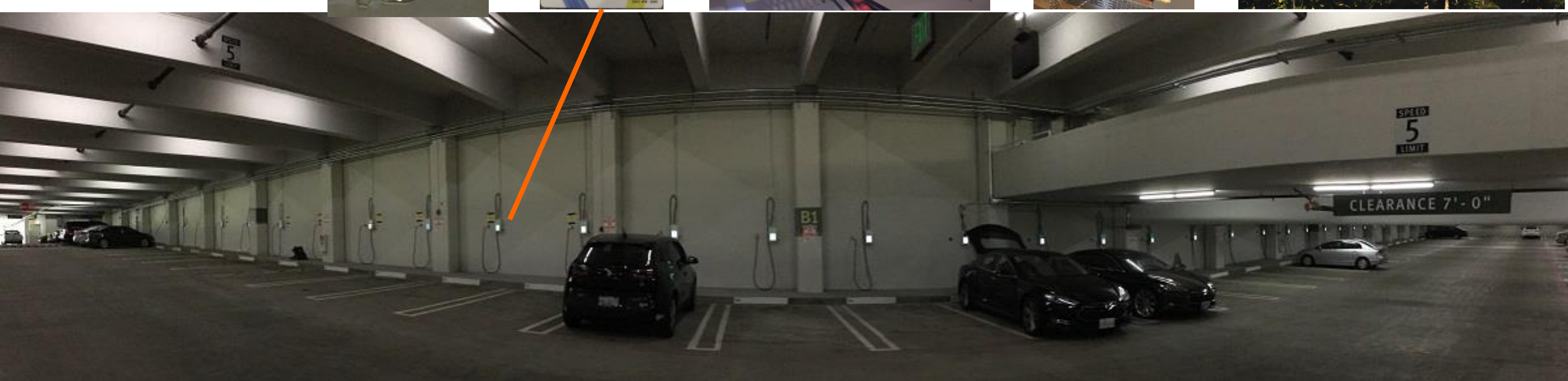


charger

150kVA transformer



main panel



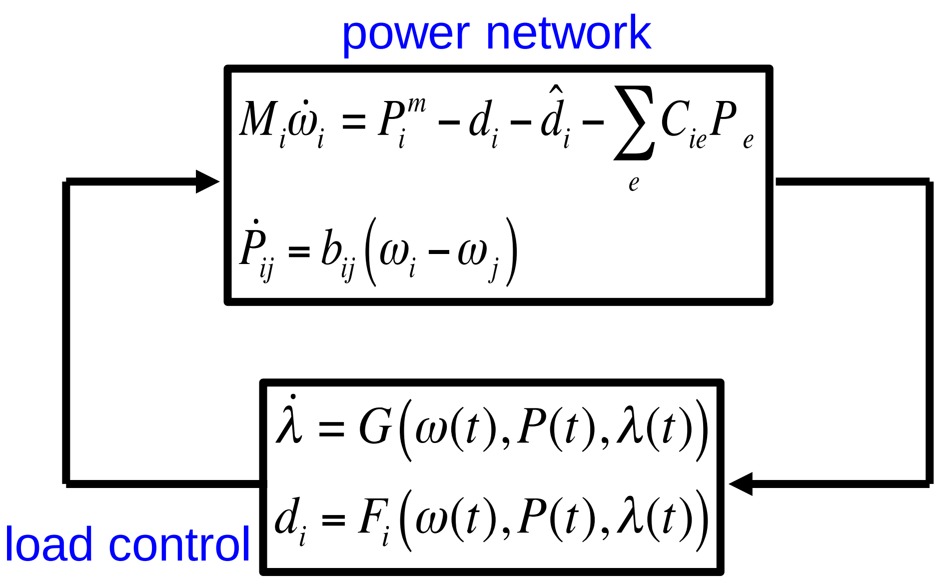
more details

(backup)

Recall: design approach

Idea: exploit system dynamic as part of primal-dual algorithm for modified opt

- closed-loop system is stable
- its equilibria are optimal



$$\min_{d,P} \quad \sum_i c_i(d_i)$$

$$\text{s. t.} \quad P_i^m - d_i = \sum_e C_{ie}P_e \qquad \text{node } i$$

$$\sum_{i \in N_k} \sum_e C_{ie}P_e = \hat{P}_k \qquad \text{area } k$$

$$\underline{P}_e \le P_e \le \overline{P}_e \qquad \text{line } e$$

Outline

Load-side frequency control

- Primary control   Zhao et al SGC2012, Zhao et al TAC2014
- Secondary control
- Interaction with generator-side control

Optimal load control (OLC)

$$\min_{d,\hat{d},P} \quad \sum_i \left(c_i(d_i) + \frac{\hat{d}_i^2}{2D_i} \right)$$

$$\text{s. t.} \quad P_i^m - \left(d_i + \hat{d}_i\right) = \sum_e C_{ie} P_{ie} \quad \forall\, i$$

demand = supply

disturbances

controllable loads

$$\min_{d,P} \quad \sum_i c_i(d_i)$$

$$\text{s. t.} \quad P_i^m - d_i = \sum_e C_{ie} P_e \qquad \text{node } i$$

$$\sum_{i \in N_k} \sum_e C_{ie} P_e = \hat{P}_k \qquad \text{area } k$$

$$\underline{P}_e \le P_e \le \overline{P}_e \qquad \text{line } e$$

system dynamics + load control = primal dual alg

swing dynamics

$$\dot{\omega}_i = -\frac{1}{M_i}\left(d_i(t) + D_i\omega_i(t) - P_i^m + \sum_{i \to j} P_{ij}(t) - \sum_{j \to i} P_{ji}(t) \right)$$

$$\dot{P}_{ij} = b_{ij}\left(\omega_i(t) - \omega_j(t)\right)$$

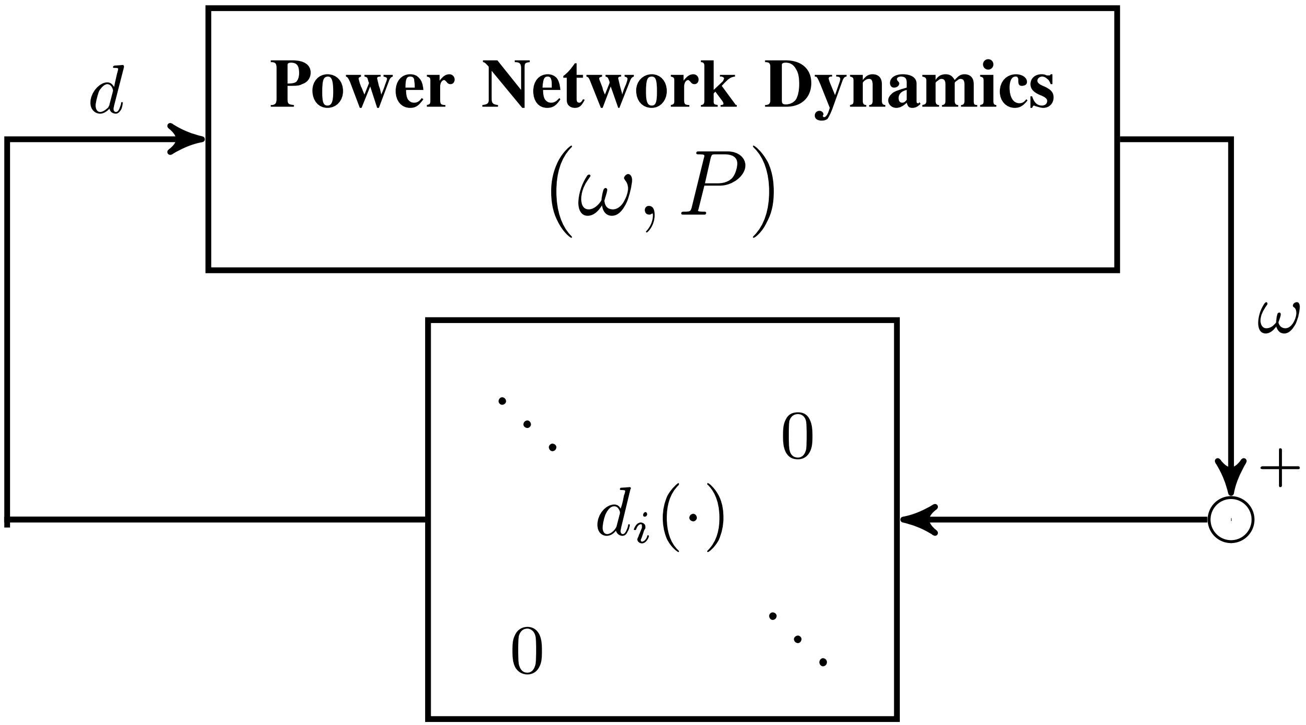
implicit

load control

$$d_i(t) := \left[c_i'^{-1}\left(\omega_i(t)\right) \right]_{\underline{d}_i}^{\overline{d}_i}$$

active control

Control architecture



Load-side primary control works

Theorem

Starting from any $\left(d(0),\ \hat{d}(0),\ w(0),\ P(0)\right)$

system trajectory $\left(d(t),\ \hat{d}(t),\ w(t),\ P(t)\right)$

converges to $\left(d^*,\ \hat{d}^*,\ w^*,\ P^*\right)$ as $t \to \infty$

- $\left(d^*,\ \hat{d}^*\right)$ is unique optimal of OLC
- w^* is unique optimal for dual

- completely decentralized
- frequency deviations contain right info for local decisions that are globally optimal

Recap: control goals

- Yes — Rebalance power
- Yes — Stabilize frequencies

- No — Restore nominal frequency $\left(w^{*}\ 1\ 0\right)$
- No — Restore scheduled inter-area flows
- No — Respect line limits

Outline

Load-side frequency control

- Primary control
- Secondary control
- Interaction with generator-side control

Mallada, Low, IFAC 2014
Mallada et al, Allerton 2014

OLC for secondary control

$$\min_{d,\hat{d},P,v} \quad \sum_i \left(c_i(d_i) + \frac{1}{2D_i}\hat{d}_i^2 \right)$$

s. t. $\quad P^m - (d + \hat{d}) = CP \qquad$ demand = supply

$\quad P^m - d = CBC^T v \qquad$ restore nominal freq

$$\min_{d,P} \quad \sum_i c_i(d_i)$$

s. t. $\quad P_i^m - d_i = \sum_e C_{ie} P_e \qquad$ node i

$\quad \sum_{i \in N_k} \sum_e C_{ie} P_e = \hat{P}_k \qquad$ area k

$\quad \underline{P}_e \le P_e \le \overline{P}_e \qquad$ line e

OLC for secondary control

$$\min_{d,\hat{d},P,v} \quad \sum_i \left(c_i(d_i) + \frac{1}{2D_i}\hat{d}_i^2 \right)$$

s. t. $\quad P^m - (d + \hat{d}) = CP \quad$ demand = supply

$\quad P^m - d = CBC^T v \quad$ restore nominal freq

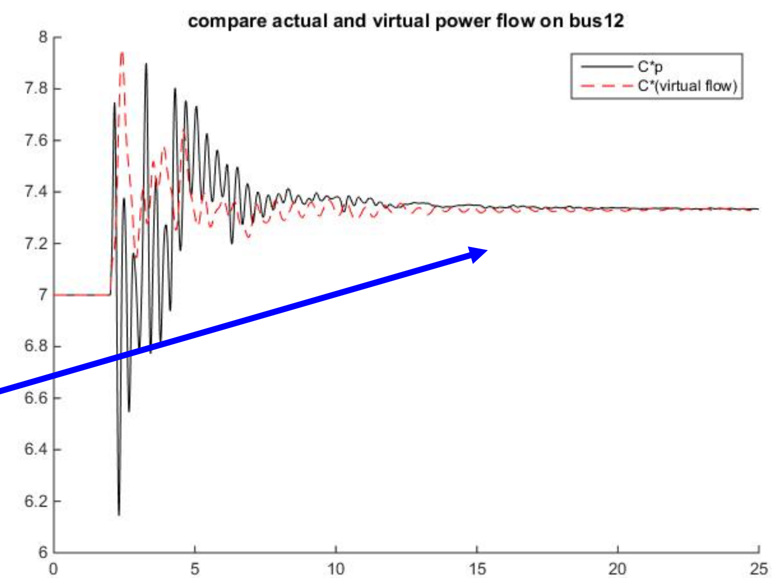
key idea: "virtual flows"

$$BC^T v$$

in steady state:

virtual flow = real flows

$$BC^T v = P$$



OLC for secondary control

$$\min_{d,\hat{d},P,v} \sum_i \left(c_i(d_i) + \frac{1}{2D_i}\hat{d}_i^2 \right)$$

s. t.

$$P^m - (d + \hat{d}) = CP$$ demand = supply

$$P^m - d = CBC^T v$$ restore nominal freq

$$\hat{C}BC^T v = \hat{P}$$ restore inter-area flow

$$\underline{P} \le BC^T v \le \overline{P}$$ respect line limit

in steady state:

virtual flow = real flows

$$BC^T v = P$$

Recall: primary control

swing dynamics:

$$\dot{\omega}_i = -\frac{1}{M_i}\left(d_i(t) + D_i\omega_i(t) - P_i^m + \sum_{e \in E} C_{ie}P_e(t)\right)$$

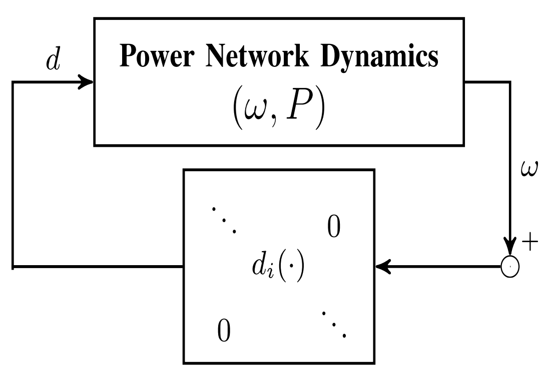
$$\dot{P}_{ij} = b_{ij}\left(\omega_i(t) - \omega_j(t)\right)$$

implicit

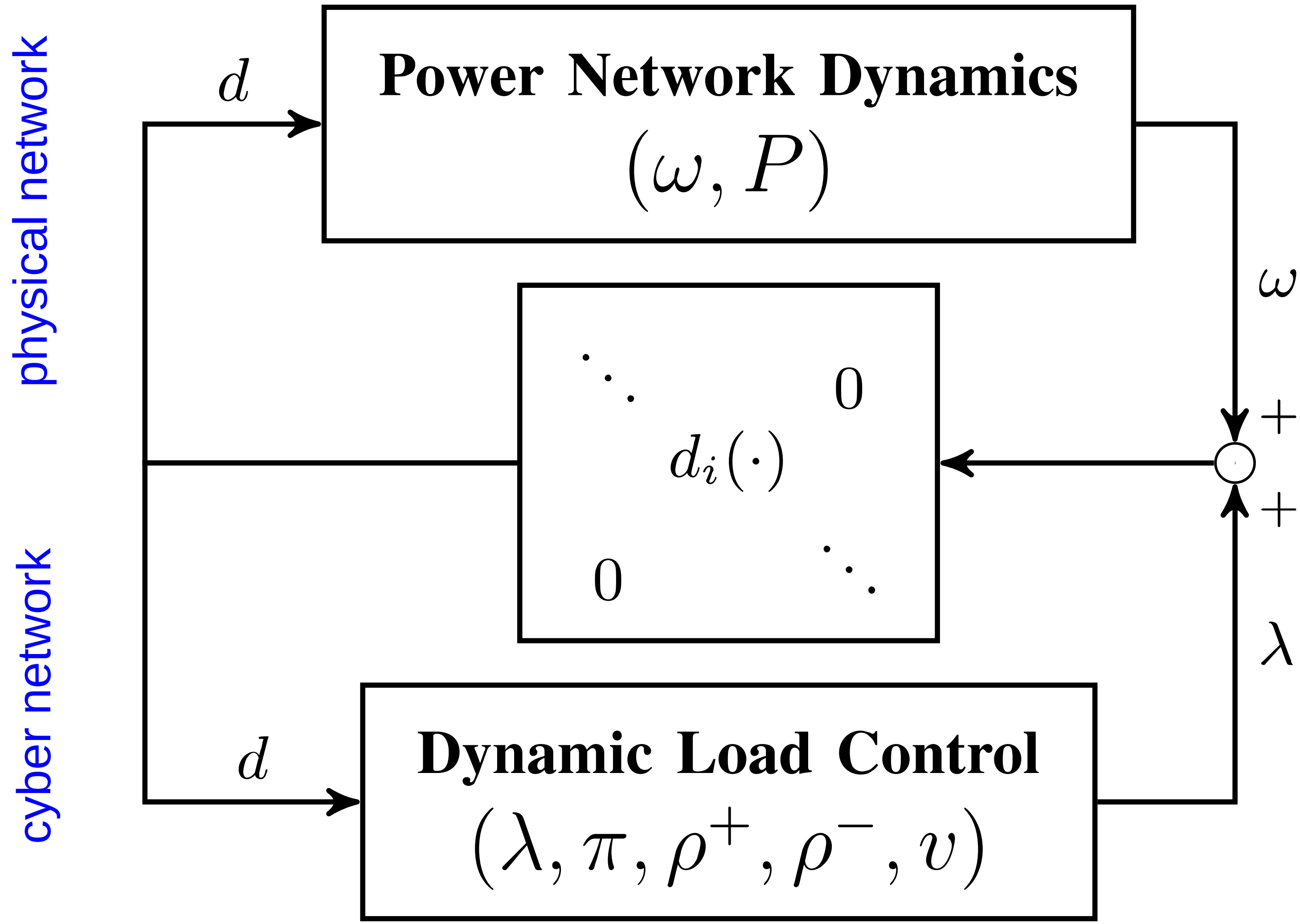
load control:

$$d_i(t) := \left[c_i'^{-1}\left(\omega_i(t)\right)\right]_{\underline{d}_i}^{\overline{d}_i}$$

active control



Control architecture



Secondary frequency control

load control: $d_i(t) := \left[c_i'^{-1}\left(\omega_i(t) + \lambda_i(t)\right) \right]_{\underline{d}_i}^{\bar{d}_i}$

computation & communication:

primal var: $$\dot{v} = \chi^v \left(L_B \lambda - C D_B \hat{C}^T \pi - C D_B (\rho^+ - \rho^-) \right)$$

dual vars: $$\dot{\lambda} = \zeta^\lambda \left(P^m - d - L_B v \right)$$

$$\dot{\pi} = \zeta^\pi \left(\hat{C} D_B C^T v - \hat{P} \right)$$

$$\dot{\rho}^+ = \zeta^{\rho^+} \left[D_B C^T v - \bar{P} \right]_{\rho^+}^+$$

$$\dot{\rho}^- = \zeta^{\rho^-} \left[\underline{P} - D_B C^T v \right]_{\rho^-}^+$$

Secondary control works

<u>Theorem</u>

starting from any initial point, system trajectory converges s. t.

- $\left(d^*, \hat{d}^*, P^*, v^*\right)$ is unique optimal of OLC
- nominal frequency is restored $w^* = 0$
- inter-area flows are restored $\hat{C}P^* = \hat{P}$
- line limits are respected $\underline{P}$ £ P^* £ $\overline{P}$

Recap: key ideas

Design optimal load control (OLC) problem

- Objective function, constraints

Derive control law as primal-dual algorithms

- Lyapunov stability
- Achieve original control goals in equilibrium

Distributed algorithms

primary control: $d_i(t) := c_i'^{-1}\left(\omega_i(t)\right)$

secondary control: $d_i(t) := c_i'^{-1}\left(\omega_i(t) + \lambda_i(t)\right)$

Recap: key ideas

Design optimal load control (OLC) problem

- Objective function, constraints

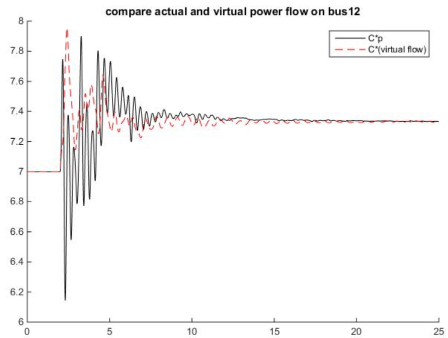
Derive control law as primal-dual algorithms

- Lyapunov stability
- Achieve original control goals in equilibrium

Distributed algorithms

Virtual flows

- Enforce desired properties on line flows



in steady state: virtual flow = real flows

$$BC^T v = P$$

Recap: control goals

- Yes ■ Rebalance power
- Yes ■ Resynchronize/stabilize frequency

Zhao, et al TAC2014

- Yes ■ Restore nominal frequency $\left(w^{*}\ {}^{1}\ 0\right)$
- Yes ■ Restore scheduled inter-area flows
- Yes ■ Respect line limits

Mallada, et al Allerton2014

Secondary control restores nominal frequency but requires local communication

Outline

Load-side frequency control

- Primary control
- Secondary control
- Interaction with generator-side control

Zhao and Low, CDC2014
Zhao, Mallada, Low, CISS 2015
Zhao, Mallada, Low, Bialek, PSCC 2016

Generator-side control

New model: nonlinear PF, with generator control

$$\dot{\theta}_i = \omega_i$$

$$M_i \dot{\omega}_i = -D_i \omega_i + \boxed{p_i} - \sum_e C_{ie} P_e$$

$$P_{ij} = b_{ij} \sin\left(\theta_i - \theta_j\right) \qquad \forall\ i \rightarrow j$$

Recall model: linearized PF, no generator control

$$M_i \dot{\omega}_i = -D_i \omega_i + \boxed{P_i^m - d_i} - \sum_e C_{ie} P_e$$

$$\dot{P}_{ij} = b_{ij}\left(\omega_i - \omega_j\right) \qquad \forall\ i \rightarrow j$$

Generator-side control

New model: nonlinear PF, with generator control

$$\dot{\theta}_i = \omega_i$$

$$M_i \dot{\omega}_i = -D_i \omega_i + p_i - \sum_e C_{ie} P_e$$

$$P_{ij} = b_{ij} \sin\left(\theta_i - \theta_j\right) \qquad \forall\ i \rightarrow j$$

generator bus: real power injection
load bus: controllable load

Generator-side control

New model: nonlinear PF, with generator control

$$\dot{\theta}_i = \omega_i$$

$$M_i\dot{\omega}_i = -D_i\omega_i + p_i - \sum_e C_{ie} P_e$$

$$P_{ij} = b_{ij} \sin\left(\theta_i - \theta_j\right) \qquad \forall\ i \rightarrow j$$

generator buses:

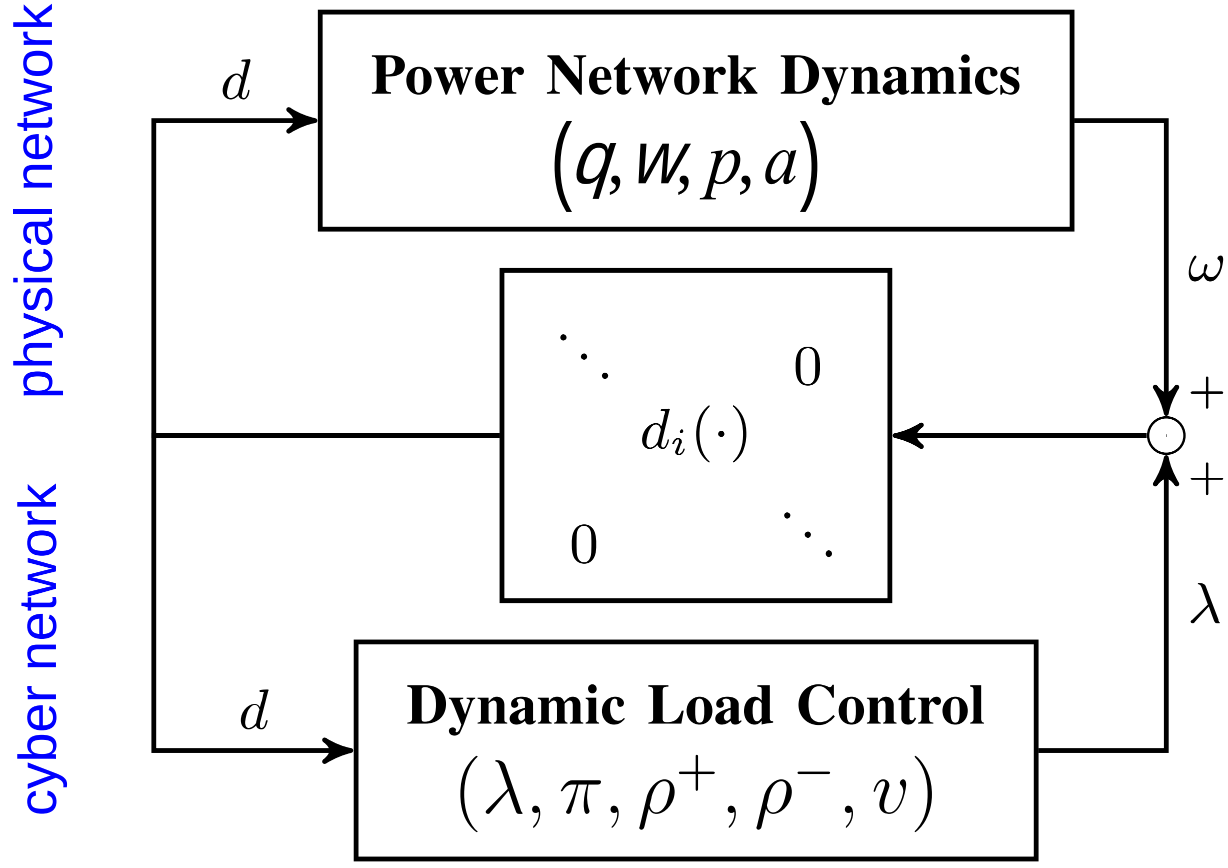
$$\dot{p}_i = -\frac{1}{\tau_{bi}}\left(p_i + a_i\right)$$

$$\dot{a}_i = -\frac{1}{\tau_{gi}}\left(a_i + p_i^c\right)$$

primary control $p_i^c(t) = p_i^c\left(w_i(t)\right)$

e.g. freq droop $p_i^c\left(w_i\right) = -b_i w_i$

Load-side control



Load-side primary control works

Theorem

- Every closed-loop equilibrium solves OLC and its dual

Suppose $\left|p_i^c(w) - p_i^c(w^*)\right| \pounds L_i \left|w - w^*\right|$

near w^* for some $L_i < D_i$

- Any closed-loop equilibrium is (locally) asymptotically stable provided

$$\left|q_i^* - q_j^*\right| < \frac{p}{2}$$

Conclusion

Forward-engineering design facilitates

- explicit control goals
- distributed algorithms
- stability analysis

Load-side frequency regulation

- primary & secondary control works
- helps generator-side control

Key message

Large network of DERs

- Real-time optimization at scale
- Computational challenge: power flow solution

Online optimization

- Network computes power flow solutions in real time at scale for free
- Exploit it for our optimization/control
- Naturally adapts to evolving network conditions

Examples

- Slow timescale: OPF
- Fast timescale: frequency control

Online optimal power flow

Large network of DERs

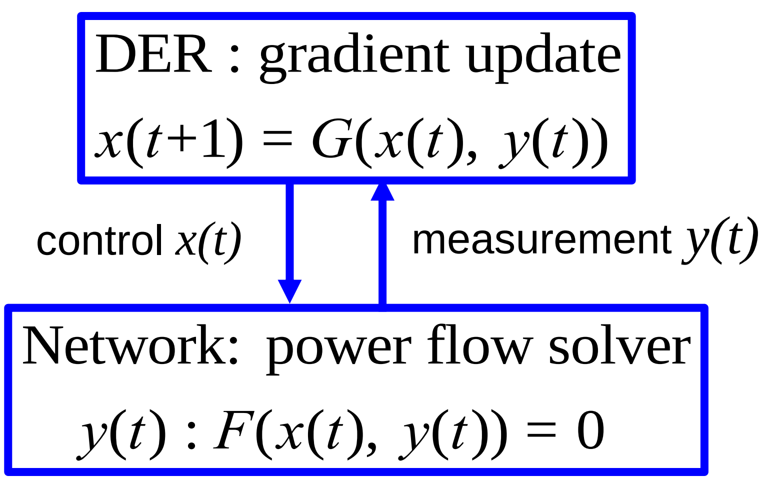
- Need real-time optimization at scale
- Computational challenge: power flow solution

Online optimization

- Network computes power flow solutions in real time at scale for free
- Exploit it for our optimization/control
- Naturally adapts to evolving network conditions

generic OPF problem

$$
\begin{aligned}
\min \quad & c(x, y) \\
\text{over} \quad & x \quad \text{(controllable devices)} \\
& y \quad \text{(uncontrollable states)} \\
\text{subj to} \quad & F(x, y) = 0 \quad \text{(power flow eqns)}
\end{aligned}
$$



Application: EV charging

Online optimization of electric vehicle charging

- Enables mass deployment at much lower infrastructure costs
- First pilot @Caltech: 54 adaptive programmable chargers
- 2x 150kVA transformers, breakers, grid sensors, etc

Energized Feb 13 Sat 2016

debugging

charger

150kVA transformer

main panel

1st customer