Design and Stability of Load-side Frequency Control

Changhong Zhao*
Enrique Mallada*
Steven Low
EE, CMS, Caltech

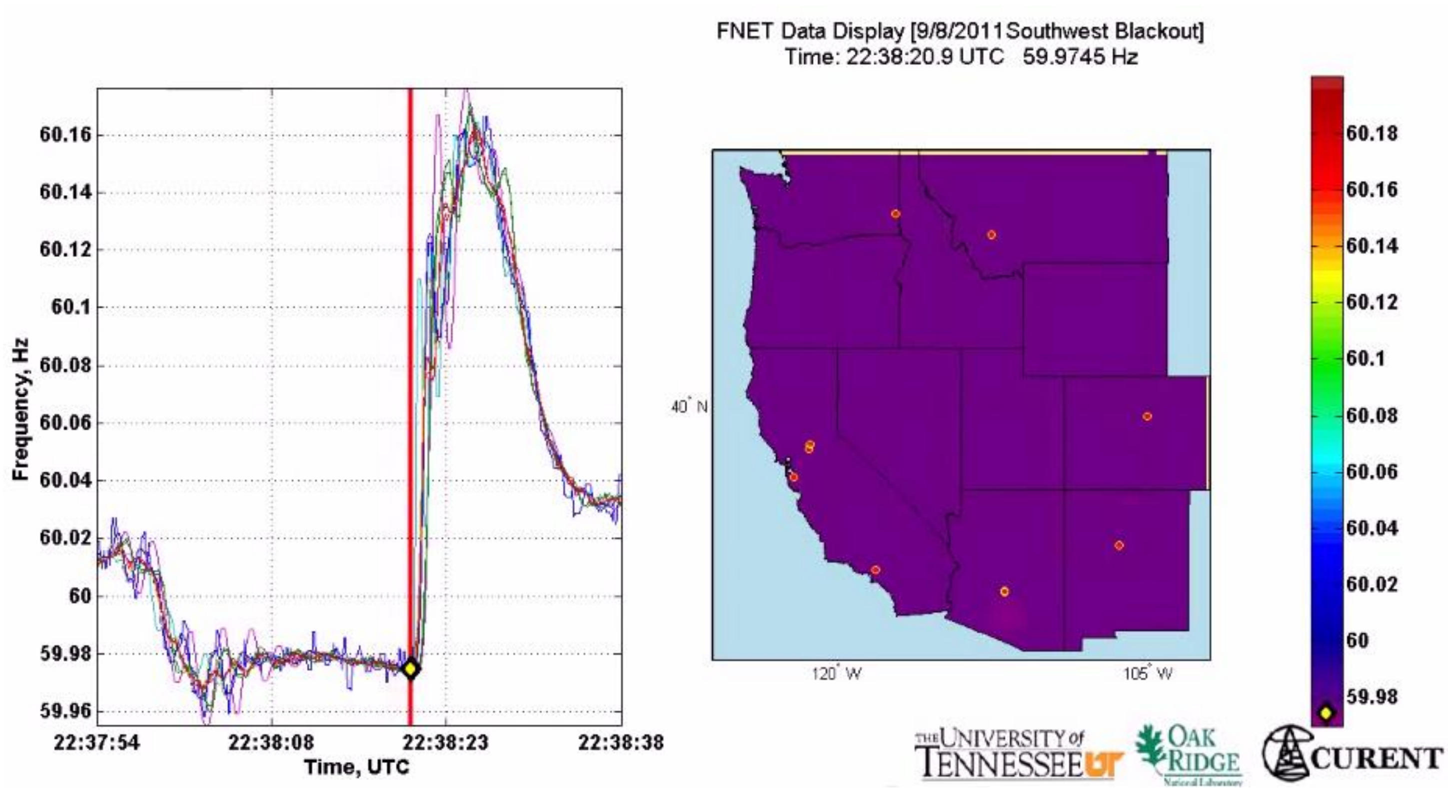
Ufuk Topcu
U Penn

Lina Li
Harvard

July 2015

Motivation

- All buses synchronized to same nominal frequency (US: 60 Hz; Europe/China: 50 Hz)
- Supply-demand imbalance ➔ frequency fluctuation



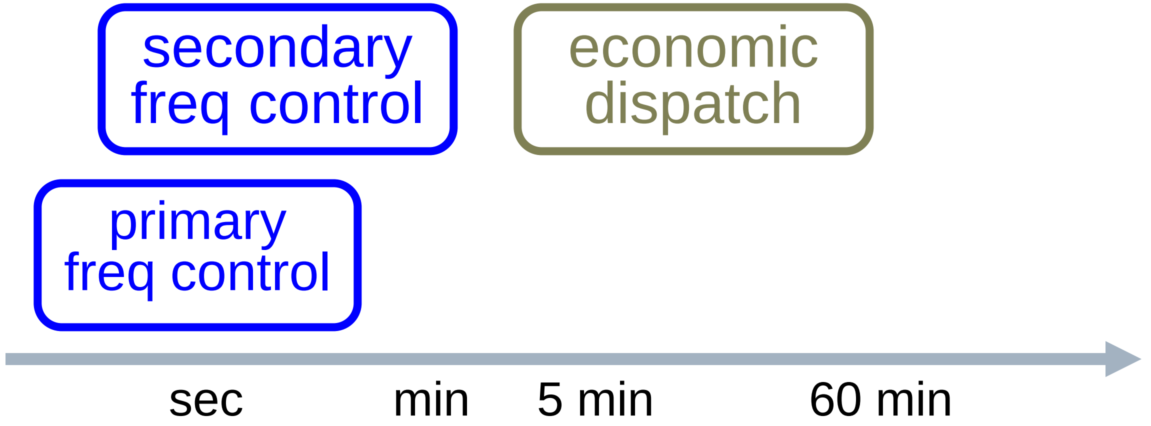
in)

2011 Southwest blackout

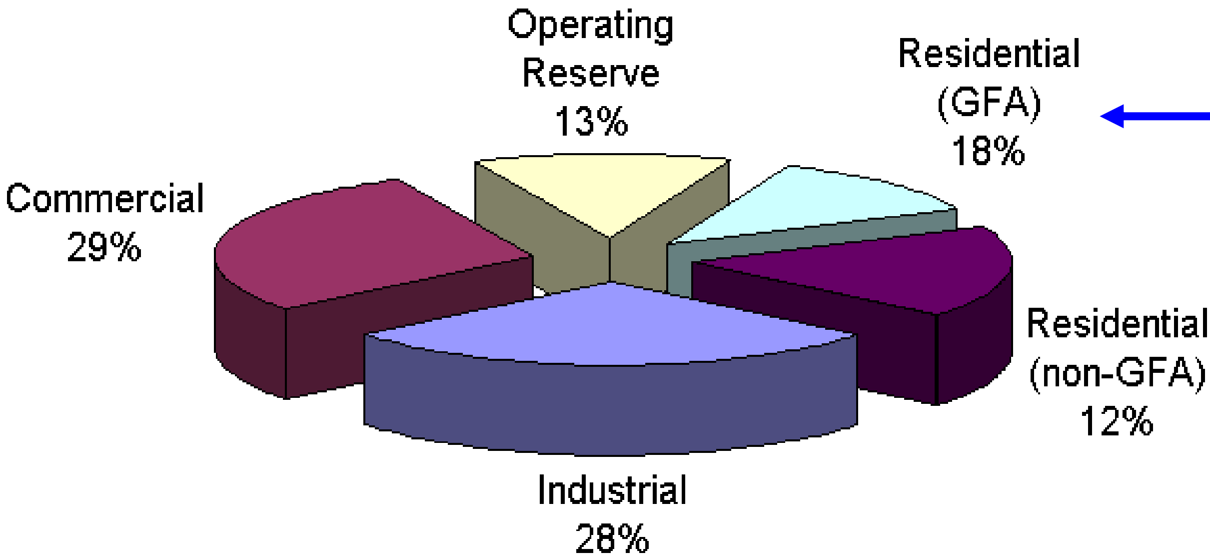
Why load-side participation

Ubiquitous continuous load-side control can supplement generator-side control

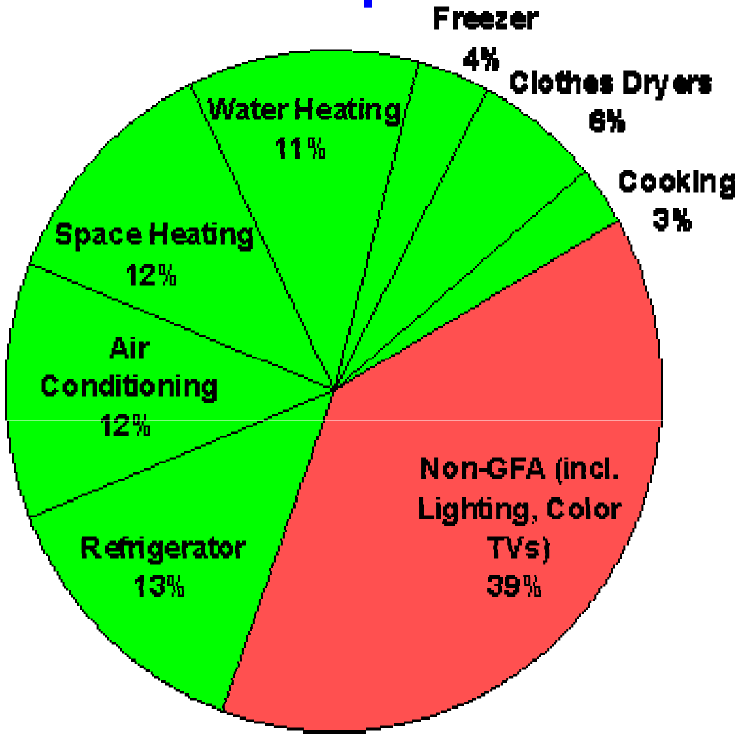
- faster (no/low inertia)
- no extra waste or emission
- more reliable (large #)
- better localize disturbances
- reducing generator-side control capacity



What is the potential



- Residential load accounts for ~1/3 of peak demand
- 61% residential appliances are Grid Friendly



US:
operating reserve: 13% of peak
total GFA capacity: 18%

Lu & Hammerstrom (2006), PNNL

How

How to design load-side frequency control ?

How does it interact with generator-side control ?

Literature: load-side control

Original idea & early analytical work

- Schweppe et al 1980; Bergin, Hill, Qu, Dorsey, Wang, Varaiya …

Small scale trials around the world

- D.Hammerstrom et al 2007, UK Market Transform Programme 2008

Early simulation studies

- Trudnowski et al 2006, Lu and Hammerstrom 2006, Short et al 2007, Donnelly et al 2010, Brooks et al 2010, Callaway and I. A. Hiskens, 2011, Molina-Garcia et al 2011

Analytical work – load-side control

- Zhao et al (2012/2014), Mallada and Low (2014), Mallada et al (2014), Zhao and Low (2014), Zhao et al (2015)
- Simpson-Porco et al 2013, You and Chen 2014, Zhang and Papachristodoulou (2014), Ma et al (2014), Zhao, et al (2014),

Recent analysis – generator-side/microgrid control:

- Andreasson et al (2013), Zhang and Papachristodoulou (2013), Li et al (2014), Burger et al (2014), You and Chen (2014), Simpson-Porco et al (2013), Hill et al (2014), Dorfler et al (2014)

Outline

Network model

Load-side frequency control

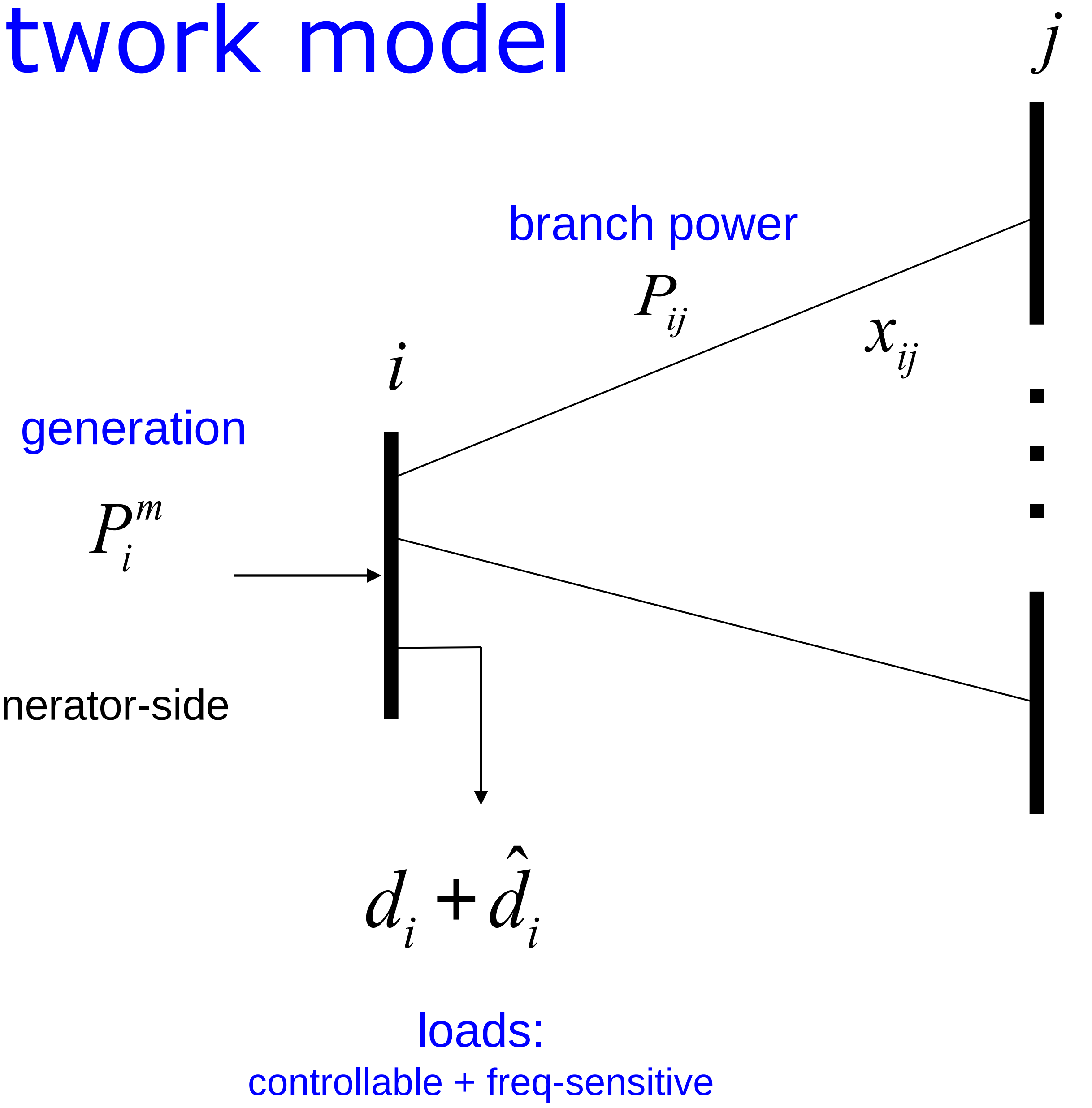
Simulations

Details

Main references:

Zhao, Topcu, Li, Low, TAC 2014
Mallada, Zhao, Low, Allerton 2014
Zhao, Low, CDC 2014, Zhao et al CISS 2015

Network model



Will include generator-side control later

i : region/control area/balancing authority

Network model

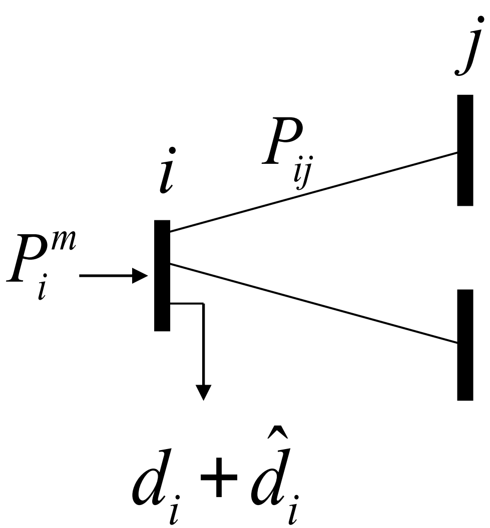
$$M_i \dot{w}_i = P_i^m - d_i - \hat{d}_i - \sum_e C_{ie} P_e$$

Generator bus: $M_i > 0$

Load bus: $M_i = 0$

Damping/uncontr loads: $\hat{d}_i = D_i w_i$

Controllable loads: d_i

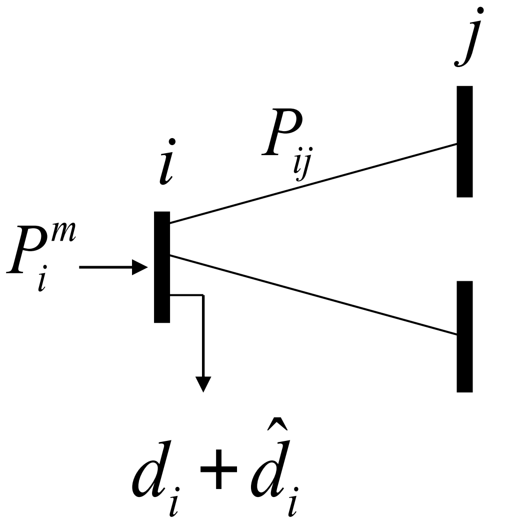


Network model

$$M_i \dot{w}_i = P_i^m - d_i - \hat{d}_i - \sum_e C_{ie} P_e$$

$$\dot{P}_{ij} = b_{ij}\left(w_i - w_j\right) \qquad \forall \; i \to j$$

- swing dynamics
- all variables are deviations from nominal
- extends to nonlinear power flow



Frequency control

$$M_i \dot{\omega}_i = P_i^m - d_i - \hat{d}_i - \sum_e C_{ie} P_e$$

$$\dot{P}_{ij} = b_{ij}\left(\omega_i - \omega_j\right) \qquad \forall \; i \to j$$

Suppose the system is in steady state

$$\dot{\omega}_i = 0 \qquad \dot{P}_{ij} = 0 \qquad \omega_i = 0$$

Then: disturbance in gen/load …

Frequency control

$$M_i \dot{\omega}_i = P_i^m - d_i - \hat{d}_i - \sum_e C_{ie} P_e$$

$$\dot{P}_{ij} = b_{ij}\left(\omega_i - \omega_j\right) \qquad \forall \; i \rightarrow j$$

current approach

load-side control

Outline

Network model

Load-side frequency control

Simulations

Details

Main references:

Zhao, Topcu, Li, Low, TAC 2014
Mallada, Zhao, Low, Allerton 2014
Zhao, Low, CDC 2014, Zhao et al CISS 2015

Load-side controller design

$$M_i \dot{w}_i = P_i^m - d_i - \hat{d}_i - \sum_e C_{ie} P_e$$

$$\dot{P}_{ij} = b_{ij}\left(w_i - w_j\right) \qquad " \quad i \to j$$

Control goals

Zhao, Topcu, Li, Low
TAC 2014

Mallada, Zhao, Low
Allerton, 2014

- Rebalance power & stabilize frequency
- Restore nominal frequency
- Restore scheduled inter-area flows
- Respect line limits

Load-side controller design

$$M_i \dot{\omega}_i = P_i^m - d_i - \hat{d}_i - \sum_e C_{ie} P_e$$

$$\dot{P}_{ij} = b_{ij}\left(\omega_i - \omega_j\right) \qquad \forall\ i \rightarrow j$$

Control goals (while min disutility)

- Rebalance power & stabilize frequency
- Restore nominal frequency
- Restore scheduled inter-area flows
- Respect line limits

Zhao, Topcu, Li, Low
TAC 2014

Mallada, Zhao, Low
Allerton, 2014

Load-side controller design

Design control law whose equilibrium solves:

$$\min_{d,P} \sum_i c_i(d_i) \qquad \text{load disutility}$$

$$\text{s. t.} \quad P_i^m - d_i = \sum_e C_{ie} P_e \quad \text{node } i \qquad \text{power balance}$$

$$\sum_{i \in N_k} \sum_e C_{ie} P_e = \hat{P}_k \quad \text{area } k \qquad \text{inter-area flows}$$

$$\underline{P}_e \le P_e \le \overline{P}_e \quad \text{line } e \qquad \text{line limits}$$

Control goals (while min disutility)

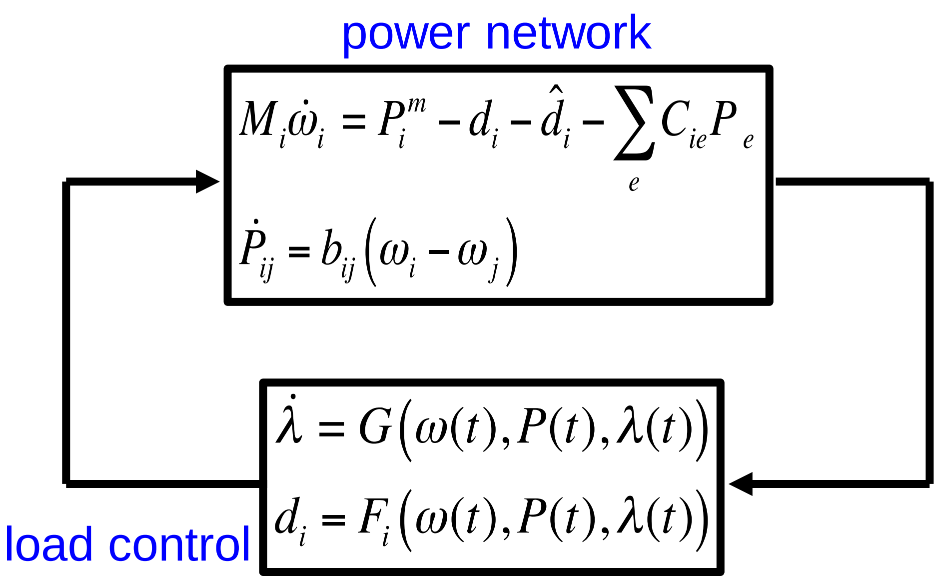
- Rebalance power & stabilize frequency
- Restore nominal frequency
- Restore scheduled inter-area flows
- Respect line limits

freq will emerge as Lagrange multiplier for power imbalance

Load-side controller design

Design control (G, F) s.t. closed-loop system

- is stable
- has equilibrium that is optimal



$$M_i \dot{\omega}_i = P_i^m - d_i - \hat{d}_i - \sum_e C_{ie} P_e$$

$$\dot{P}_{ij} = b_{ij} \left(\omega_i - \omega_j \right)$$

$$\dot{\lambda} = G\left(\omega(t), P(t), \lambda(t) \right)$$

$$d_i = F_i\left(\omega(t), P(t), \lambda(t) \right)$$

$$\begin{aligned}
\min_{d,P} \quad & \sum_i c_i(d_i) & \\
\text{s. t.} \quad & P_i^m - d_i = \sum_e C_{ie} P_e & \text{node } i \\
& \sum_{i \in N_k} \sum_e C_{ie} P_e = \hat{P}_k & \text{area } k \\
& \underline{P}_e \le P_e \le \overline{P}_e & \text{line } e
\end{aligned}$$

Load-side controller design

Idea: exploit system dynamic as part of primal-dual algorithm for modified opt

- Distributed algorithm
- Stability analysis
- Control goals in equilibrium

power network

$$M_i \dot{\omega}_i = P_i^m - d_i - \hat{d}_i - \sum_e C_{ie} P_e$$

$$\dot{P}_{ij} = b_{ij}\left(\omega_i - \omega_j\right)$$

load control

$$\dot{\lambda} = G\left(\omega(t), P(t), \lambda(t)\right)$$

$$d_i = F_i\left(\omega(t), P(t), \lambda(t)\right)$$

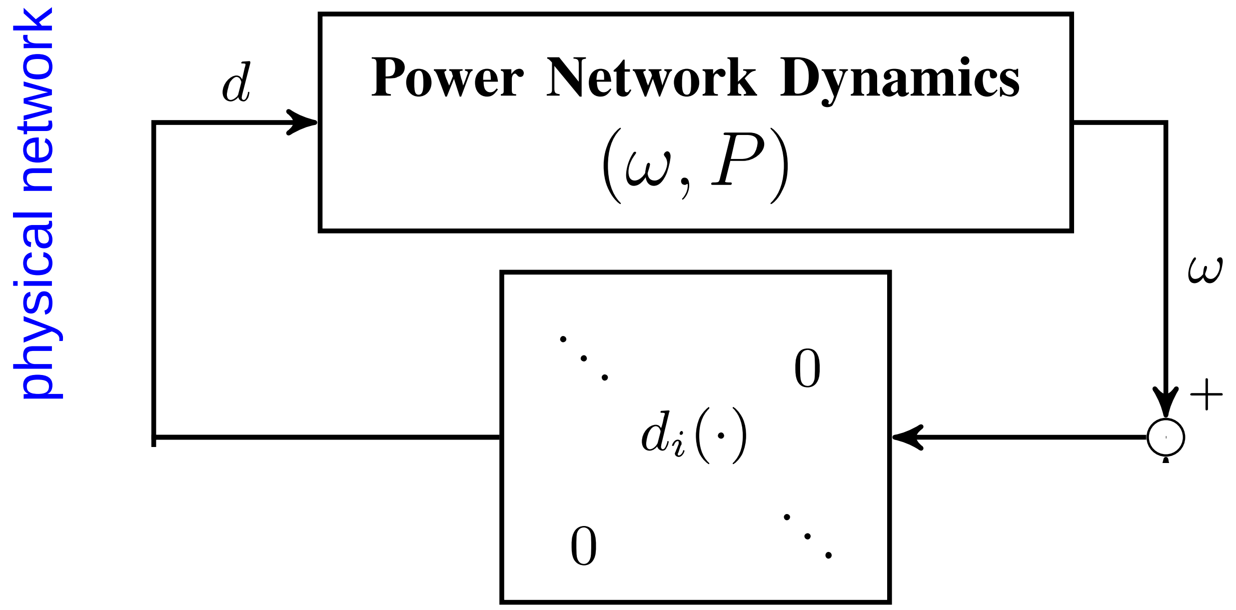
$$\min_{d,P} \quad \sum_i c_i(d_i)$$

$$\text{s. t.} \quad P_i^m - d_i = \sum_e C_{ie} P_e \qquad \text{node } i$$

$$\sum_{i \in N_k} \sum_e C_{ie} P_e = \hat{P}_k \qquad \text{area } k$$

$$\underline{P}_e \le P_e \le \overline{P}_e \qquad \text{line } e$$

Summary: control architecture

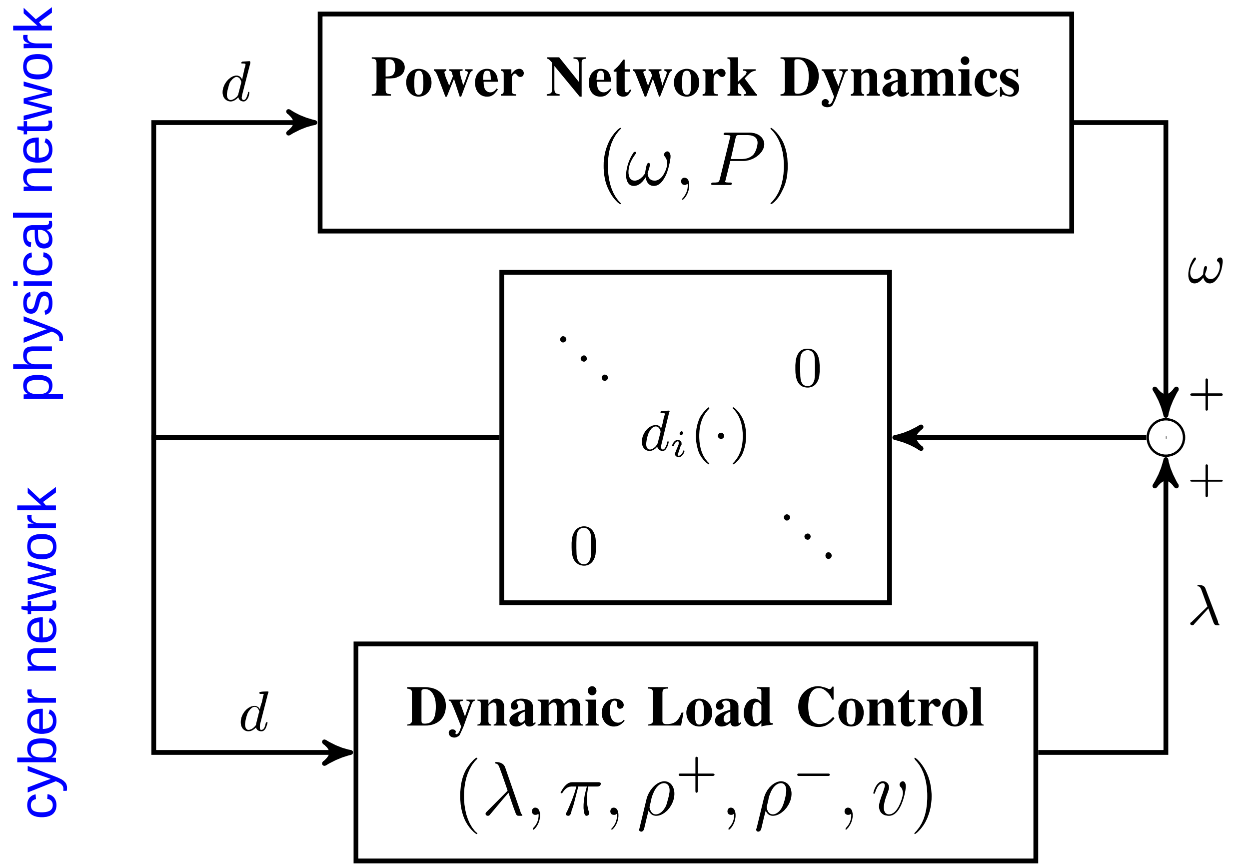


Primary load-side frequency control

- completely decentralized
- Theorem: stable dynamic, optimal equilibrium

Zhao, Topcu, Li, Low. TAC 2014

Summary: control architecture

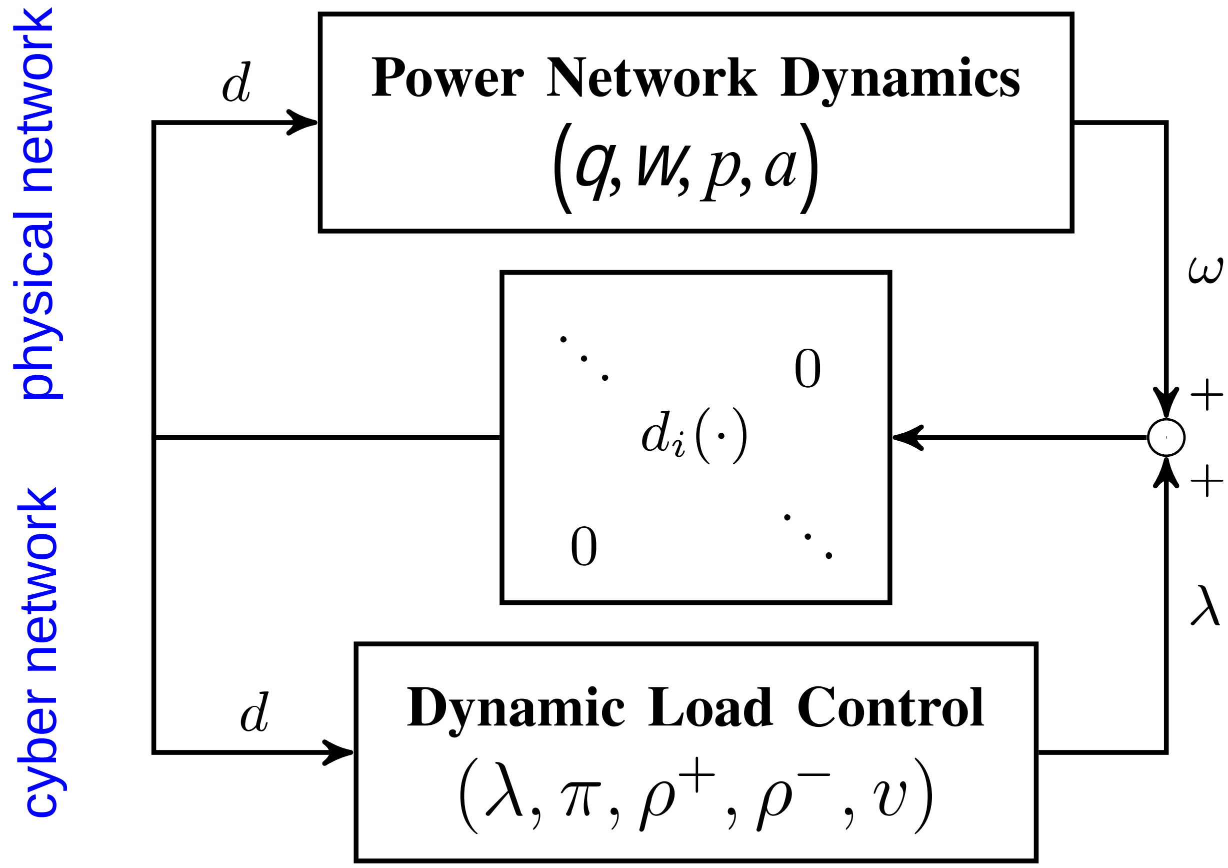


Secondary load-side frequency control

- communication with neighbors
- Theorem: stable dynamic, optimal equilibrium

Mallada, Zhao, Low. Allerton 2014

Summary: control architecture



With **generator-side** control, **nonlinear** power flow

- load-side improves both transient & eq
- <u>Theorem</u>: stable dynamic, optimal equilibrium

Zhao, Mallada, Low. CISS 2015

Outline

Network model

Load-side frequency control

Simulations

Details

Main references:

Zhao, Topcu, Li, Low, TAC 2014
Mallada, Zhao, Low, Allerton 2014
Zhao, Low, CDC 2014, Zhao et al CISS 2015

Simulations

Dynamic simulation of IEEE 39-bus system

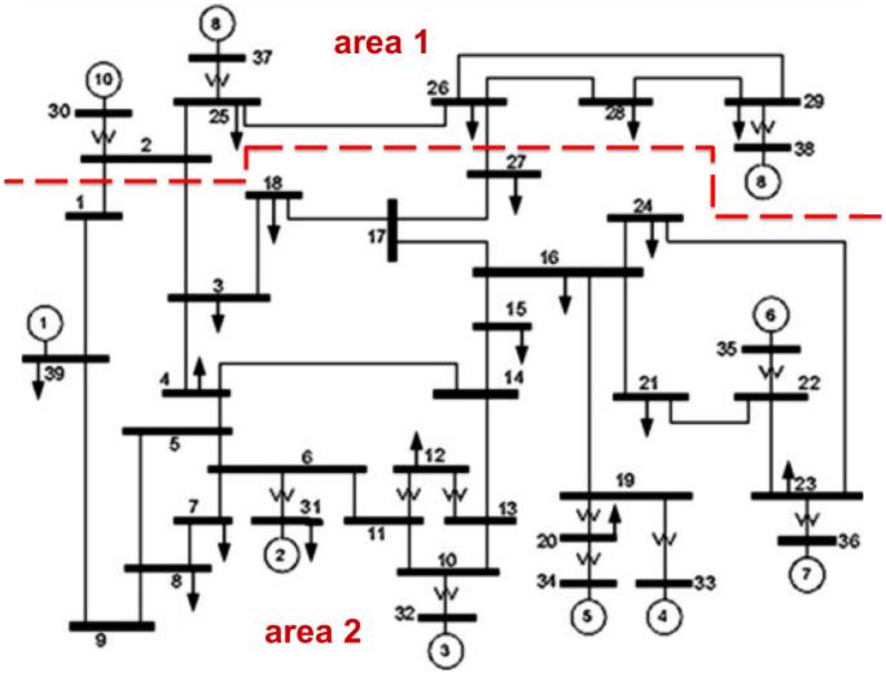
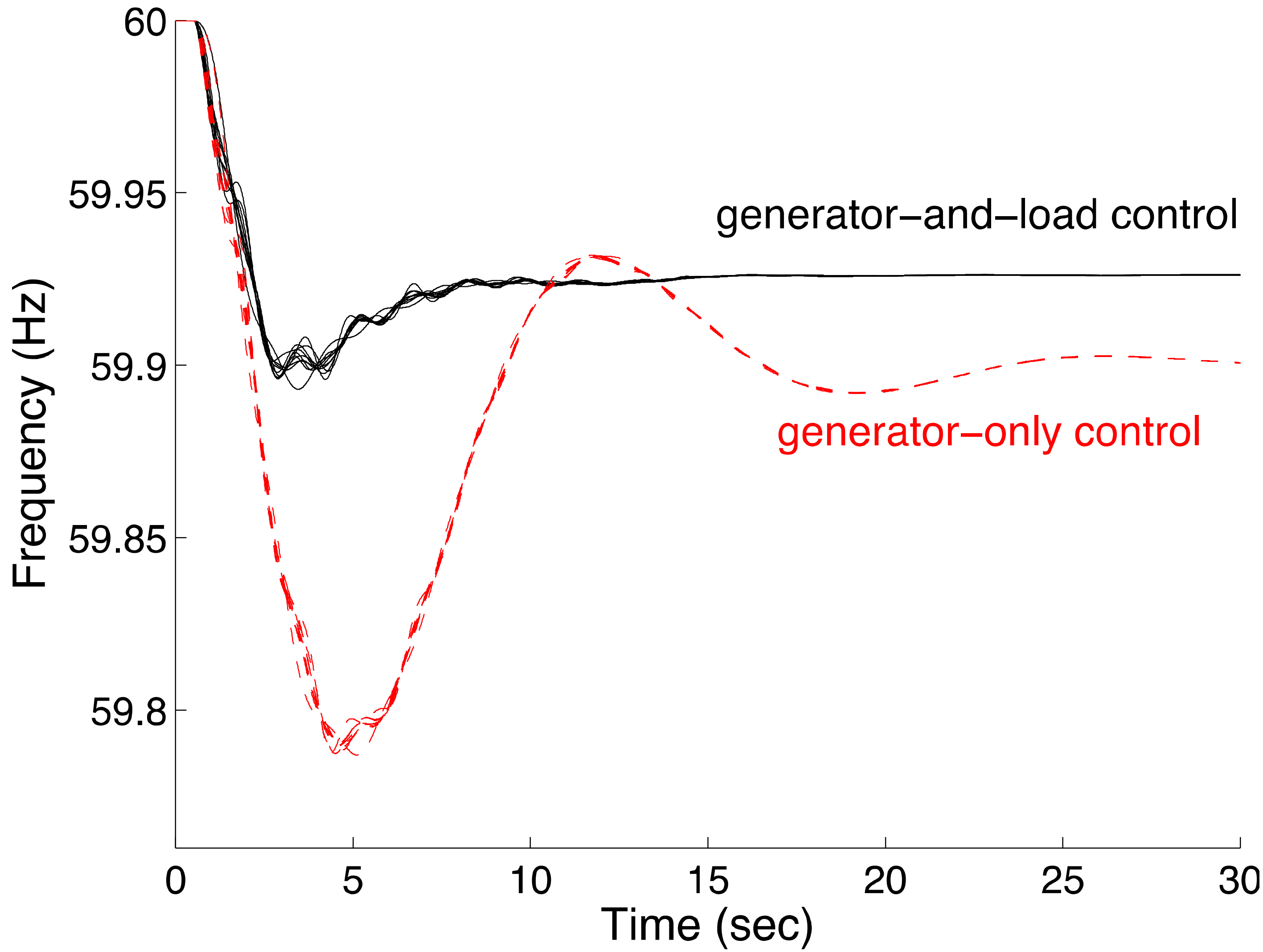


Fig. 2: IEEE 39 bus system : New England

- Power System Toolbox (RPI)
- Detailed generation model
- Exciter model, power system stabilizer model
- Nonzero resistance lines

Primary control



Secondary control

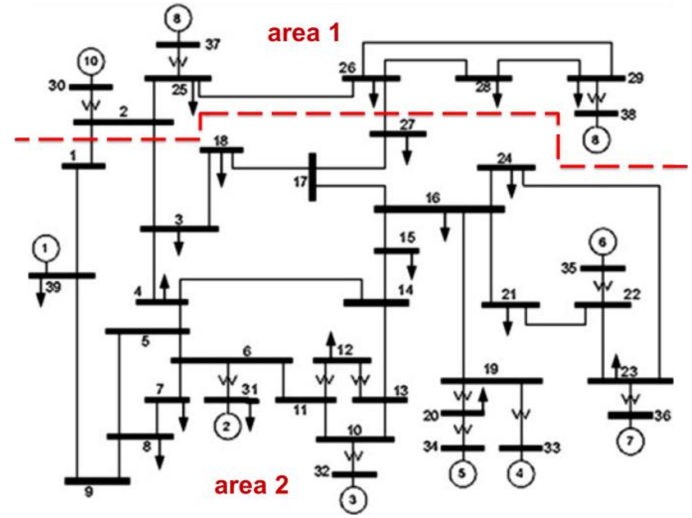
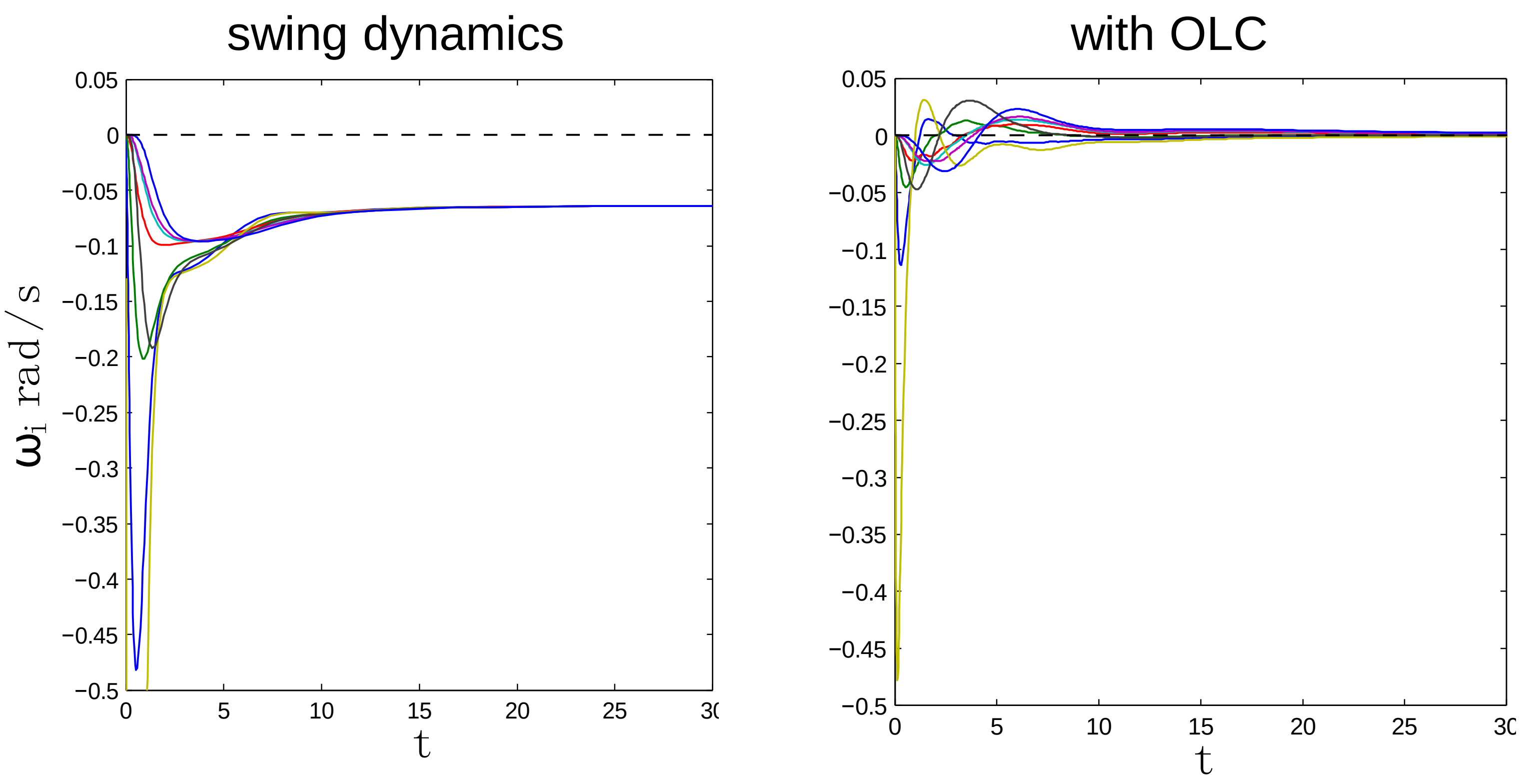


Fig. 2: IEEE 39 bus system: New England



area 1

Secondary control

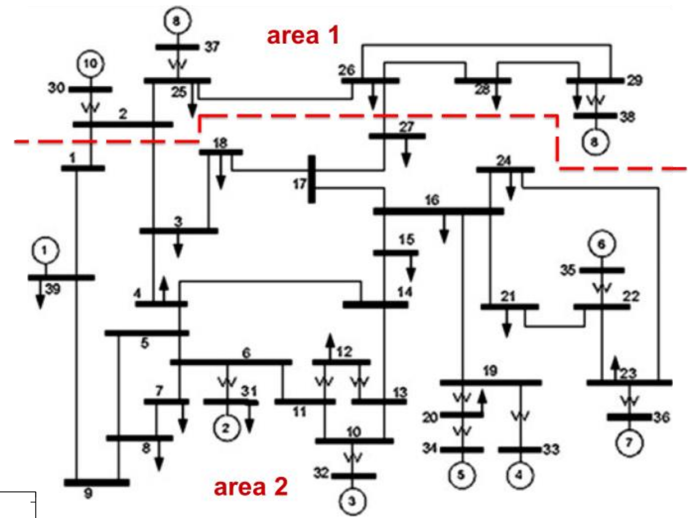
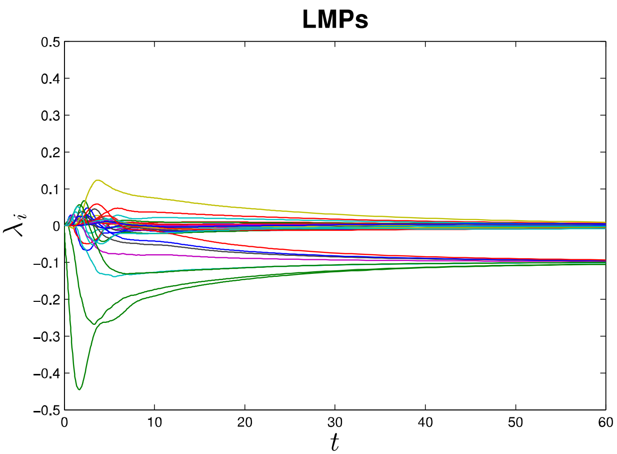
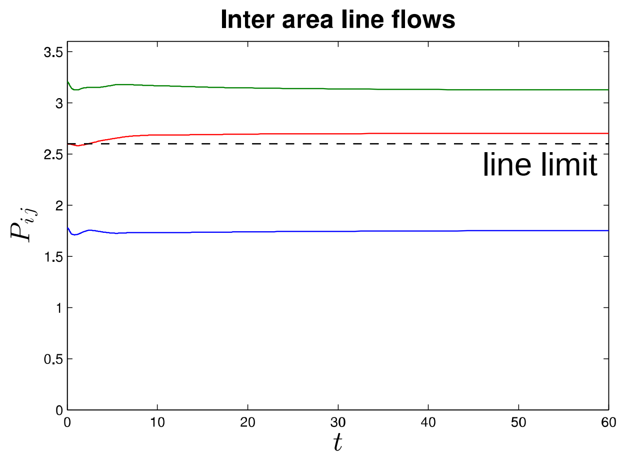
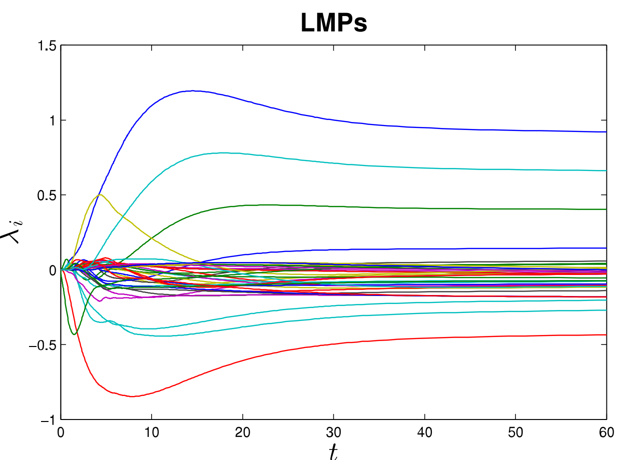


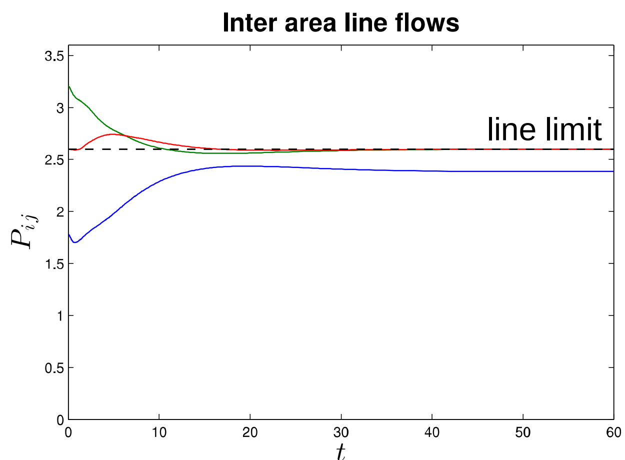
Fig. 2: IEEE 39 bus system : New England





no line limits





Total inter-area flow is the same in both cases

with line limits

Conclusion

Forward-engineering design facilitates

- explicit control goals
- distributed algorithms
- stability analysis

Load-side frequency regulation

- primary & secondary control works
- helps generator-side control

Outline

Network model

Load-side frequency control

Simulations

Details

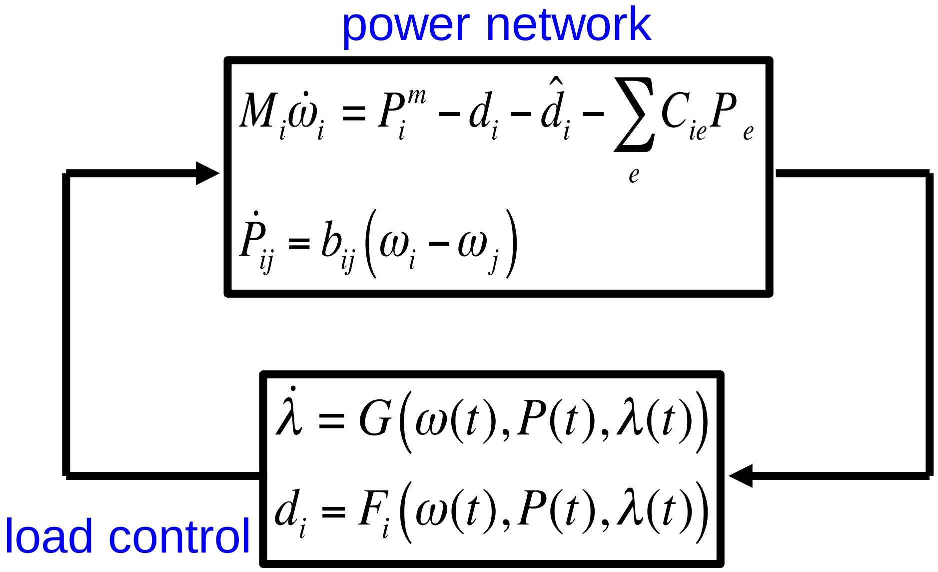
Main references:

Zhao, Topcu, Li, Low, TAC 2014
Mallada, Zhao, Low, Allerton 2014
Zhao, Low, CDC 2014, Zhao et al CISS 2015

Recall: design approach

Idea: exploit system dynamic as part of primal-dual algorithm for modified opt

- closed-loop system is stable
- its equilibria are optimal



$$
\begin{aligned}
\min_{d,P} \quad & \sum_i c_i(d_i) & \\
\text{s. t.} \quad & P_i^m - d_i = \sum_e C_{ie} P_e & \text{node } i \\
& \sum_{i \in N_k} \sum_e C_{ie} P_e = \hat{P}_k & \text{area } k \\
& \underline{P}_e \le P_e \le \overline{P}_e & \text{line } e
\end{aligned}
$$

Outline

Load-side frequency control

- Primary control — Zhao et al SGC2012, Zhao et al TAC2014
- Secondary control
- Interaction with generator-side control

Optimal load control (OLC)

$$\min_{d,\hat{d},P} \quad \sum_i \left(c_i(d_i) + \frac{\hat{d}_i^2}{2D_i} \right)$$

$$\text{s. t.} \quad P_i^m - \left(d_i + \hat{d}_i\right) = \sum_e C_{ie} P_{ie} \quad \forall i$$

demand = supply

disturbances

controllable loads

$$\min_{d,P} \quad \sum_i c_i(d_i)$$

$$\text{s. t.} \quad P_i^m - d_i = \sum_e C_{ie} P_e \qquad \text{node } i$$

$$\sum_{i \in N_k} \sum_e C_{ie} P_e = \hat{P}_k \qquad \text{area } k$$

$$\underline{P}_e \le P_e \le \overline{P}_e \qquad \text{line } e$$

system dynamics + load control = primal dual alg

swing dynamics

$$\dot{\omega}_i = -\frac{1}{M_i}\left(d_i(t) + D_i \omega_i(t) - P_i^m + \sum_{i \to j} P_{ij}(t) - \sum_{j \to i} P_{ji}(t) \right)$$

$$\dot{P}_{ij} = b_{ij}\left(\omega_i(t) - \omega_j(t) \right)$$

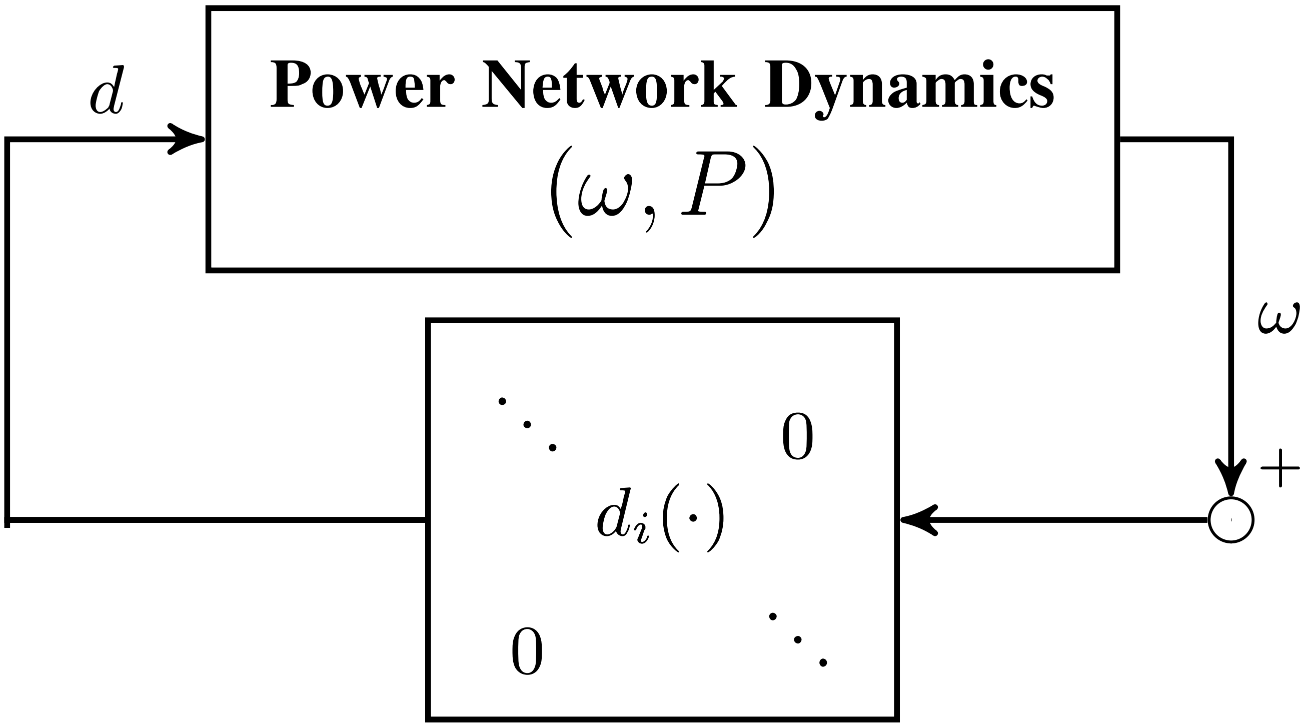
implicit

load control

$$d_i(t) := \left[c_i'^{-1}\left(\omega_i(t)\right) \right]_{\underline{d}_i}^{\overline{d}_i}$$

active control

Control architecture



Load-side primary control works

Theorem

Starting from any $\left(d(0),\ \hat{d}(0),\ w(0),\ P(0)\right)$

system trajectory $\left(d(t),\ \hat{d}(t),\ w(t),\ P(t)\right)$

converges to $\left(d^*,\ \hat{d}^*,\ w^*,\ P^*\right)$ as $t \to \infty$

- $\left(d^*,\ \hat{d}^*\right)$ is unique optimal of OLC
- w^* is unique optimal for dual

- completely decentralized
- frequency deviations contain right info for local decisions that are globally optimal

Recap: control goals

- Yes — Rebalance power
- Yes — Stabilize frequencies

- No — Restore nominal frequency $\left(W^{*}\ 1\ 0\right)$
- No — Restore scheduled inter-area flows
- No — Respect line limits

Outline

Load-side frequency control

- Primary control
- Secondary control
- Interaction with generator-side control

Mallada, Low, IFAC 2014
Mallada et al, Allerton 2014

OLC for secondary control

$$\min_{d,\hat{d},P,v} \quad \sum_i \left(c_i(d_i) + \frac{1}{2D_i}\hat{d}_i^2 \right)$$

$$\text{s. t.} \quad P^m - (d + \hat{d}) = CP \qquad \text{demand = supply}$$

$$P^m - d = CBC^T v \qquad \text{restore nominal freq}$$

$$\min_{d,P} \quad \sum_i c_i(d_i)$$

$$\text{s. t.} \quad P_i^m - d_i = \sum_e C_{ie} P_e \qquad \text{node } i$$

$$\sum_{i \in N_k} \sum_e C_{ie} P_e = \hat{P}_k \qquad \text{area } k$$

$$\underline{P}_e \le P_e \le \overline{P}_e \qquad \text{line } e$$

OLC for secondary control

$$\min_{d,\hat{d},P,v} \quad \sum_i \left(c_i(d_i) + \frac{1}{2D_i}\hat{d}_i^2 \right)$$

$$\text{s. t.} \quad P^m - (d + \hat{d}) = CP \qquad \text{demand = supply}$$

$$P^m - d = CBC^T v \qquad \text{restore nominal freq}$$

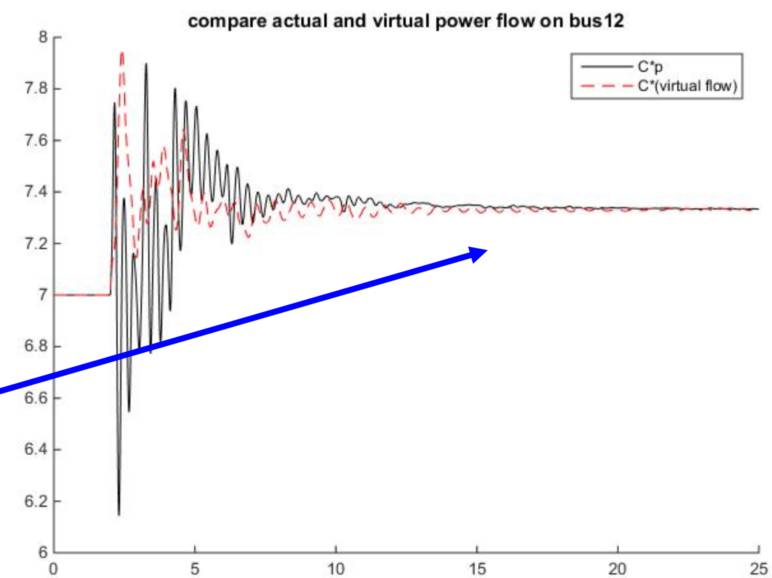
key idea: "virtual flows"

$$BC^T v$$

in steady state:

virtual flow = real flows

$$BC^T v = P$$



OLC for secondary control

$$\min_{d,\hat{d},P,v} \quad \sum_i \left(c_i(d_i) + \frac{1}{2D_i}\hat{d}_i^2 \right)$$

s. t.

$$P^m - (d + \hat{d}) = CP \qquad \text{demand = supply}$$

$$P^m - d = CBC^T v \qquad \text{restore nominal freq}$$

$$\hat{C}BC^T v = \hat{P} \qquad \text{restore inter-area flow}$$

$$\underline{P} \le BC^T v \le \overline{P} \qquad \text{respect line limit}$$

in steady state:

virtual flow = real flows

$$BC^T v = P$$

Recall: primary control

swing dynamics:

$$\dot{\omega}_i = -\frac{1}{M_i}\left(d_i(t) + D_i \omega_i(t) - P_i^m + \sum_{e \in E} C_{ie} P_e(t) \right)$$

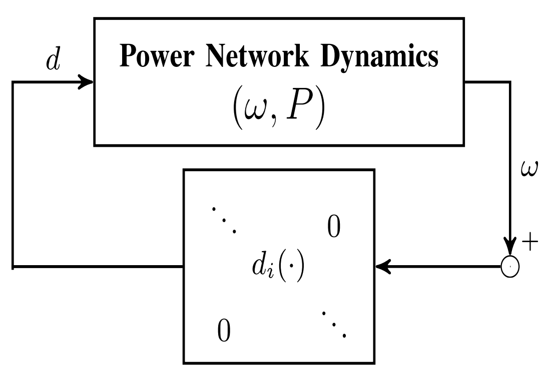
$$\dot{P}_{ij} = b_{ij}\left(\omega_i(t) - \omega_j(t)\right)$$

implicit

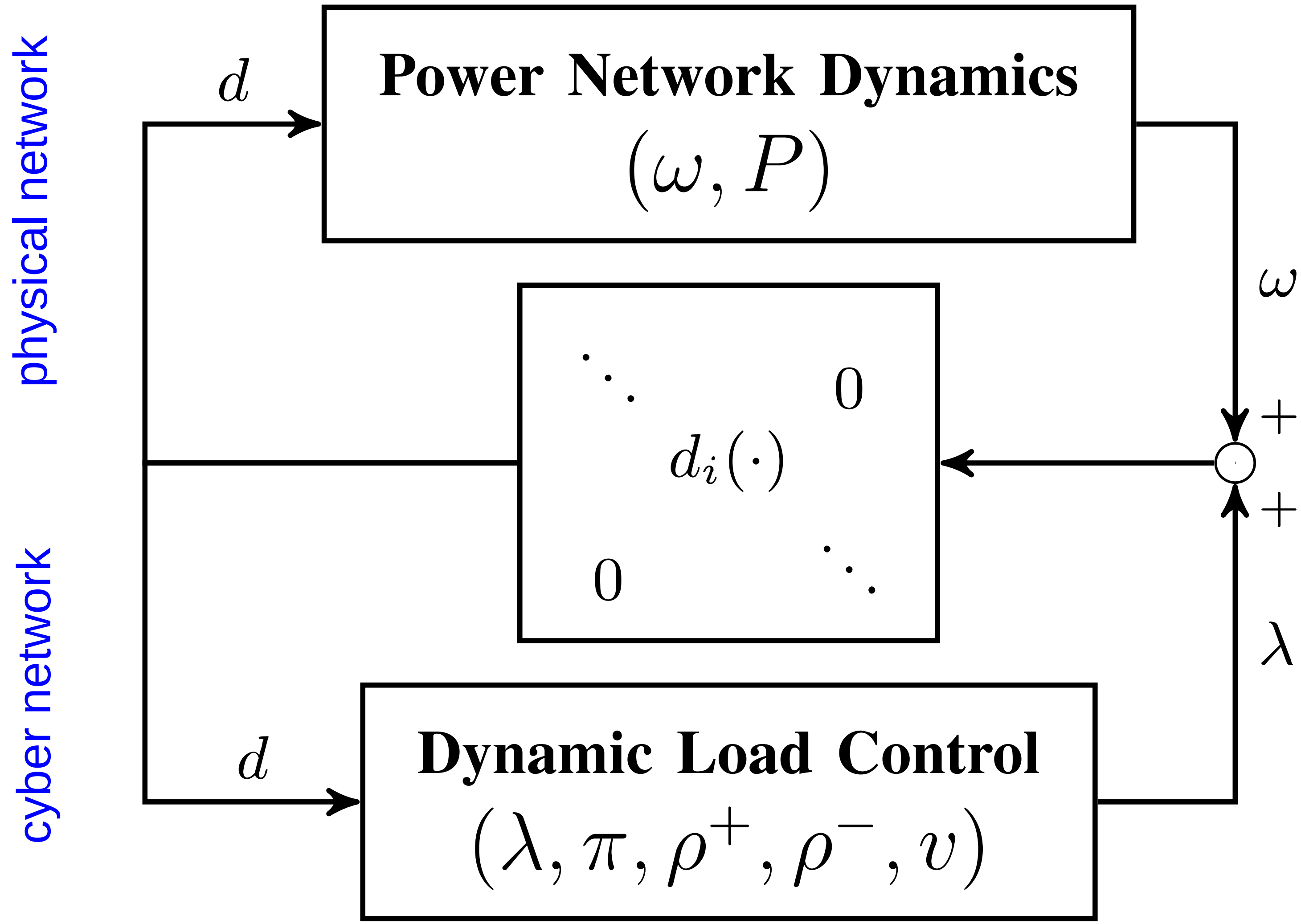
load control:

$$d_i(t) := \left[c_i'^{-1}\left(\omega_i(t)\right) \right]_{\underline{d}_i}^{\bar{d}_i}$$

active control



Control architecture



Secondary frequency control

load control: $$d_i(t) := \left[c_i'^{-1}\left(\omega_i(t) + \lambda_i(t)\right) \right]_{\underline{d}_i}^{\overline{d}_i}$$

computation & communication:

primal var: $$\dot{v} = \chi^v \left(L_B \lambda - C D_B \hat{C}^T \pi - C D_B (\rho^+ - \rho^-) \right)$$

dual vars: $$\dot{\lambda} = \zeta^\lambda \left(P^m - d - L_B v \right)$$

$$\dot{\pi} = \zeta^\pi \left(\hat{C} D_B C^T v - \hat{P} \right)$$

$$\dot{\rho}^+ = \zeta^{\rho^+} \left[D_B C^T v - \bar{P} \right]^+_{\rho^+}$$

$$\dot{\rho}^- = \zeta^{\rho^-} \left[\underline{P} - D_B C^T v \right]^+_{\rho^-}$$

Secondary control works

Theorem

starting from any initial point, system trajectory converges s. t.

- $\left(d^*, \hat{d}^*, P^*, v^*\right)$ is unique optimal of OLC
- nominal frequency is restored $w^* = 0$
- inter-area flows are restored $\hat{C}P^* = \hat{P}$
- line limits are respected $\underline{P}$ £ P^* £ $\overline{P}$

Recap: key ideas

Design optimal load control (OLC) problem

- Objective function, constraints

Derive control law as primal-dual algorithms

- Lyapunov stability
- Achieve original control goals in equilibrium

Distributed algorithms

primary control: $d_i(t) := c_i^{'-1}\left(w_i(t)\right)$

secondary control: $d_i(t) := c_i^{'-1}\left(W_i(t) + /_i(t)\right)$

Recap: key ideas

Design optimal load control (OLC) problem

- Objective function, constraints

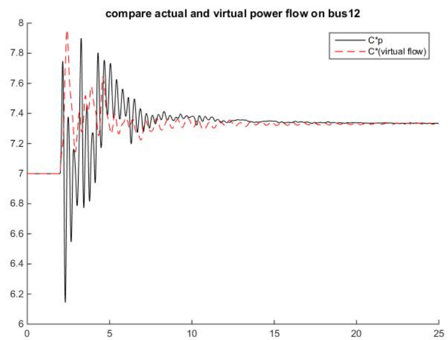
Derive control law as primal-dual algorithms

- Lyapunov stability
- Achieve original control goals in equilibrium

Distributed algorithms

Virtual flows

- Enforce desired properties on line flows



in steady state: virtual flow = real flows

$$BC^T v = P$$

Recap: control goals

Yes ■ Rebalance power

Yes ■ Resynchronize/stabilize frequency

Zhao, et al TAC2014

Yes ■ Restore nominal frequency $\left(w^{*}\ 1\ 0\right)$

Yes ■ Restore scheduled inter-area flows

Yes ■ Respect line limits

Mallada, et al Allerton2014

Secondary control restores nominal frequency but requires local communication

Outline

Load-side frequency control

- Primary control
- Secondary control
- Interaction with generator-side control

Zhao and Low, CDC2014
Zhao, Mallada, Low, CISS 2015

Generator-side control

New model: nonlinear PF, with generator control

$$\dot{\theta}_i = \omega_i$$

$$M_i \dot{\omega}_i = -D_i \omega_i + \boxed{p_i} - \sum_e C_{ie} P_e$$

$$P_{ij} = b_{ij} \sin\left(\theta_i - \theta_j\right) \qquad \forall\; i \rightarrow j$$

Recall model: linearized PF, no generator control

$$M_i \dot{\omega}_i = -D_i \omega_i + \boxed{P_i^m - d_i} - \sum_e C_{ie} P_e$$

$$\dot{P}_{ij} = b_{ij} \left(\omega_i - \omega_j\right) \qquad \forall\; i \rightarrow j$$

Generator-side control

New model: nonlinear PF, with generator control

$$\dot{\theta}_i = \omega_i$$

$$M_i \dot{\omega}_i = -D_i \omega_i + p_i - \sum_e C_{ie} P_e$$

$$P_{ij} = b_{ij} \sin\left(\theta_i - \theta_j\right) \qquad \forall\ i \rightarrow j$$

generator bus: real power injection
load bus: controllable load

Generator-side control

New model: nonlinear PF, with generator control

$$\dot{\theta}_i = \omega_i$$

$$M_i \dot{\omega}_i = -D_i \omega_i + p_i - \sum_e C_{ie} P_e$$

$$P_{ij} = b_{ij} \sin\left(\theta_i - \theta_j\right) \qquad \forall\, i \to j$$

generator buses:

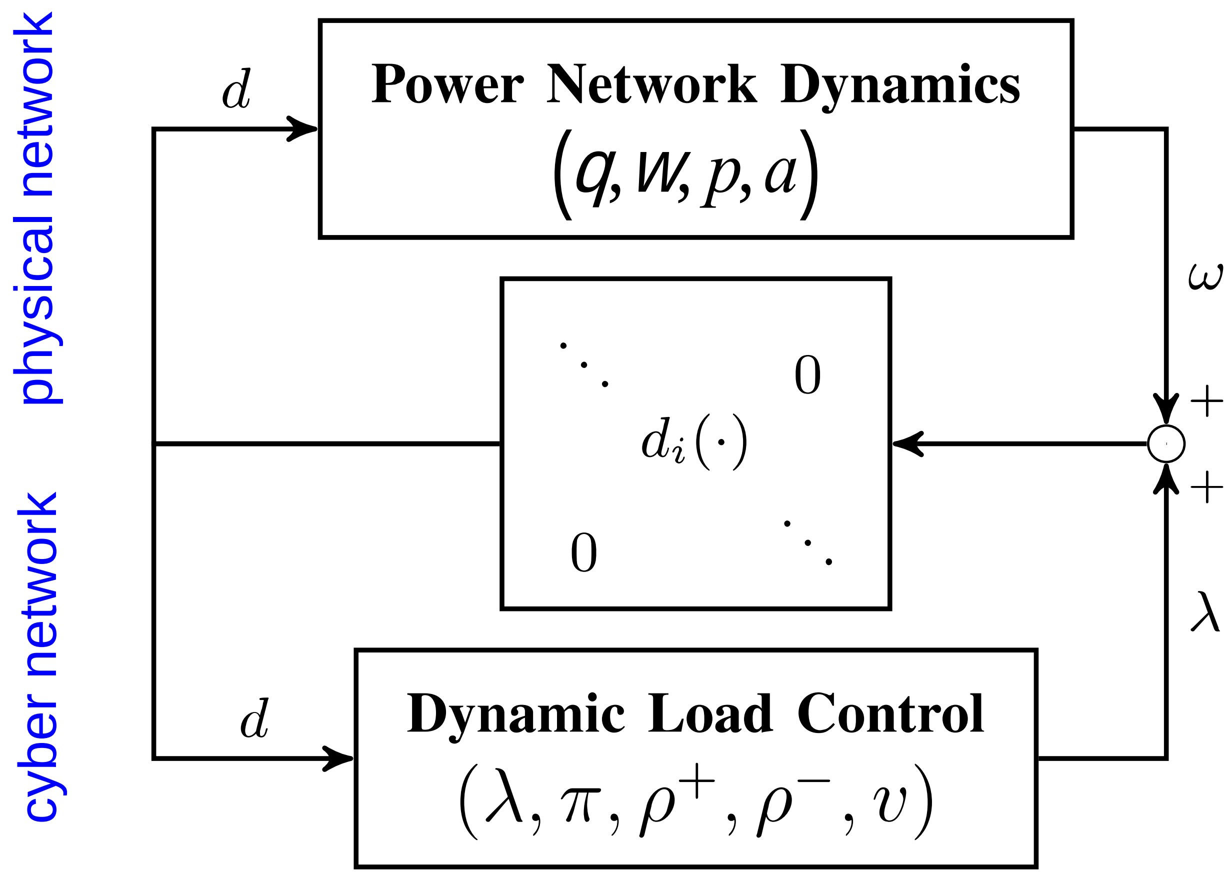
$$\dot{p}_i = -\frac{1}{\tau_{bi}}\left(p_i + a_i\right)$$

primary control $p_i^c(t) = p_i^c\left(w_i(t)\right)$

$$\dot{a}_i = -\frac{1}{\tau_{gi}}\left(a_i + p_i^c\right)$$

e.g. freq droop $p_i^c\left(w_i\right) = -\, b_i w_i$

Load-side control



Load-side primary control works

Theorem

- Every closed-loop equilibrium solves OLC and its dual

Suppose $\left| p_i^c(\omega) - p_i^c\left(\omega^*\right) \right| \le L_i \left| \omega - \omega^* \right|$

near ω^* for some $L_i < D_i$

- Any closed-loop equilibrium is (locally) asymptotically stable provided

$$\left| \theta_i^* - \theta_j^* \right| < \frac{\pi}{2}$$