Optimal Storage Placement and Power Flow Solution

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Outline

Optimal storage placement

- Continuous network model
- Structural properties

Tang

Power flow solution

- Monotone operators
- L_2 contraction
- $L_{¥}$ contraction

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Motivation

Storage on distribution network

- Useful for renewable/DG integration
- CA mandates
  - RPS: 33% by 2020, 50% by 2030
  - 3 IOUs to deploy 1.3GW battery by 2020
    - At least ½ owned by IOUs

Design question: ideal deployment ?

- Where to place and how to size
- ... to minimize line loss
- ... assuming
  - no other constraints such as real estate
  - optimal charging/discharging operation

Conclusion

Interested in design guidelines

- Structural properties of optimal solution

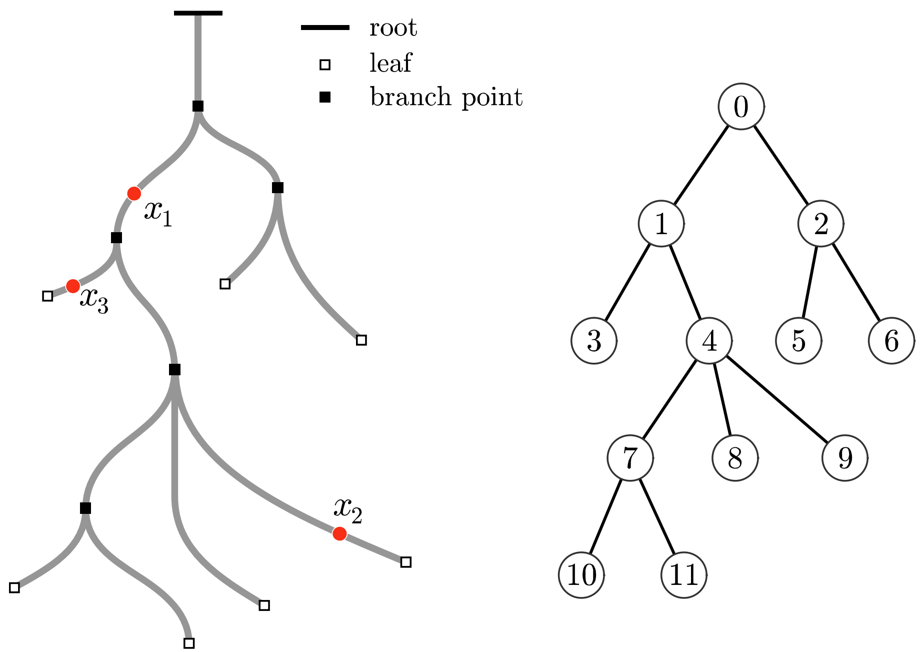
Placement and sizing

- (scaled) Monotone deployment is optimal

Charging/discharging schedule

- Always charge till full (when generation is highest)
- Always discharge till empty (when demand is highest)

Continuous tree

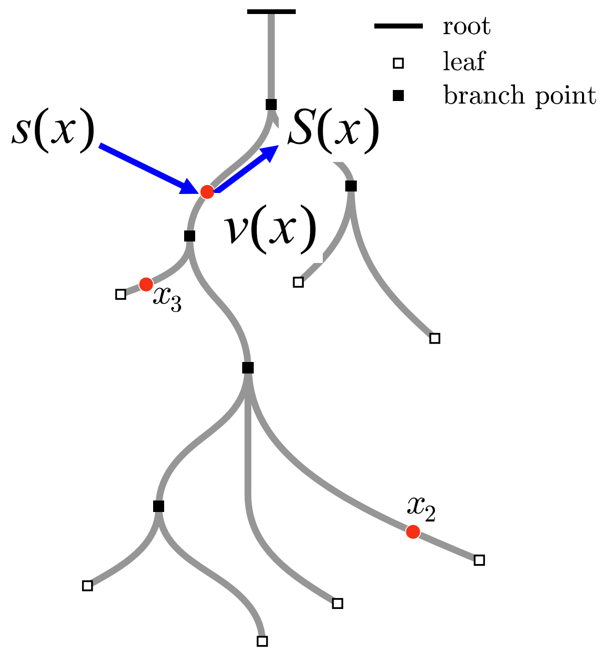


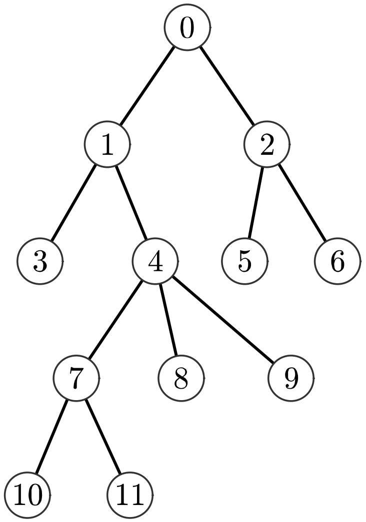
A continuous tree with underlying discrete tree

- Each segment (corresponding to a node in discrete tree) can be treated like a closed interval in $\mathbf{R}$
- Order and integration are well defined on each segment

[similar to Wang Turitsyn Chertkov (2012) ODE model of feeder line for power flow solution]

Continuous power flow equations



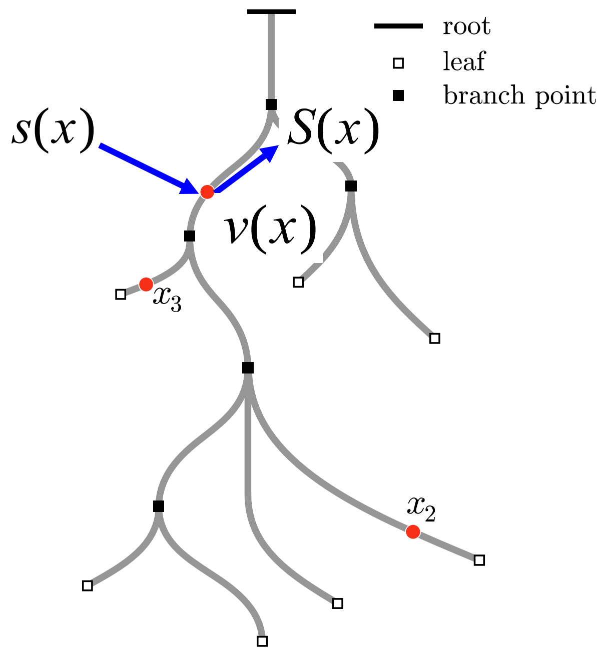


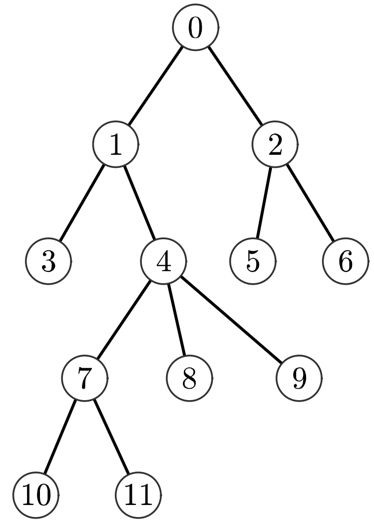
$s(x) = p(x) + jq(x)$ injection at x

$S(x) = P(x) + jQ(x)$ branch power flow at x

$v(x)$ squared voltage magnitude

Continuous power flow equations





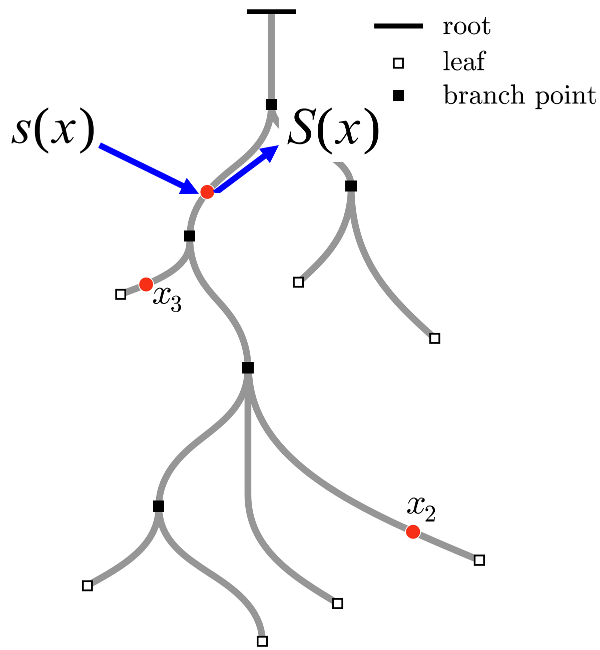
line loss

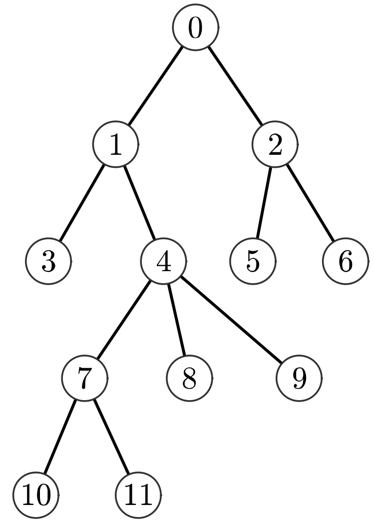
$$S(x) = \int_{y \geq x} \left(s(y) - z(y) \frac{|S(y)|^2}{v(y)} \right) dy$$

$$v(x) = 1 + \int_{[0,x]} 2\mathrm{Re}\left(z^*(y) S(y) \right) dy$$

[power flow equations: continuous version of Baran and Wu (1989)]

Linear approximation





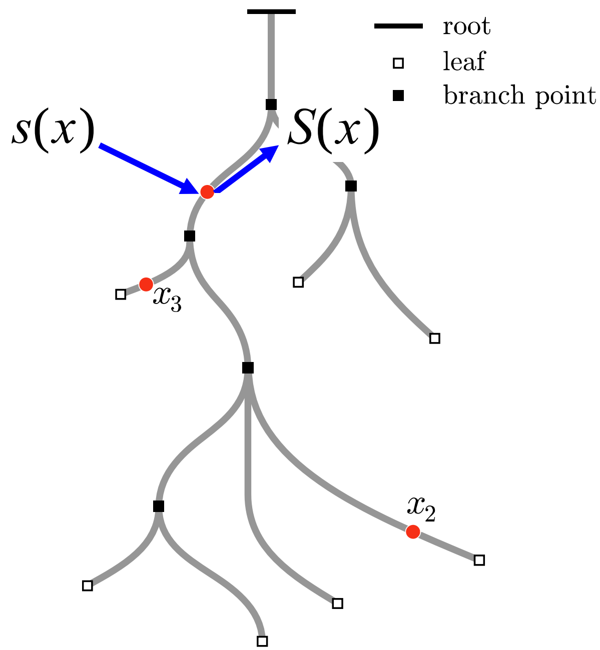
$$S(x,t) \approx \int_{y \geq x} s(y,t)\,dy$$

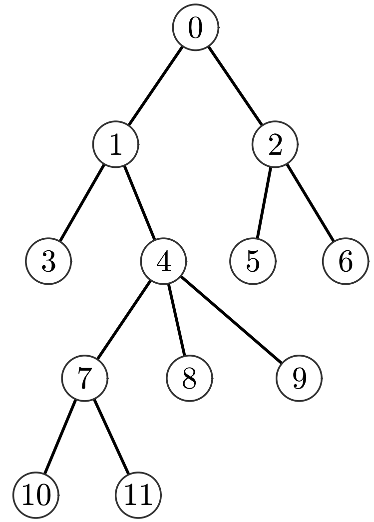
Assumptions

- loss is small relative to line flow
- voltage magnitude ~ 1 pu

[power flow equations: continuous version of Baran and Wu (1989)]

Linear approximation





$$S(x,t) \approx \int_{y \geq x} s(y,t)\,dy$$

<u>Assumptions</u>

- loss is small relative to line flow
- voltage magnitude ~ 1 pu

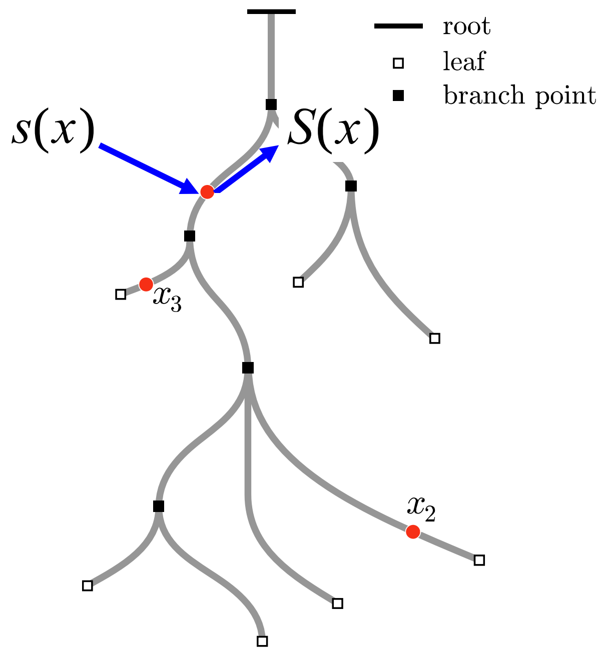
$$s(x,t) = \underline{a(x)p(t) + b(x)} - u(x,t)$$ **charging rate**

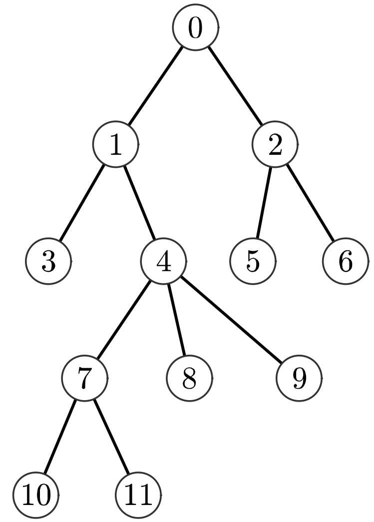
<u>background injection:</u>

- common load shape in time
- location-dependent scale & offset

[Smith, Wong, Rajagopal 2012]

Storage model





$$s(x,t) = a(x)p(t) + b(x) - u(x,t)$$

charging rate

$$\frac{\partial b(x,t)}{\partial t} = u(x,t), \quad 0 \leq b(x,t) \leq B(x)$$

SoC

Optimal placement & sizing

line flow

$$\min_{\substack{b(x,t)\\ B(x)}} \int_{\text{time}} \int_{\text{tree}} r(x)\, P^2(x,t)\, dx\, dt$$

energy loss

s. t.

linear power flow equation

state of charge

budget constraint

Optimal placement & sizing

$$\min_{\substack{b(x,t)\\ B(x)}} \int_{\text{time}} \int_{\text{tree}} r(x)\, P^2(x,t)\, dx\, dt$$

energy loss

$$\text{s. t.}\quad P(x,t) = \int_{y \geq x} \left(a(y)\, p(t) + b(y) - \frac{\partial b}{\partial t}(y,t) \right) dy$$

linear power flow equation

background injection

charging rate

state of charge

budget constraint

Optimal placement & sizing

$$\min_{\substack{b(x,t)\\ B(x)}} \int_{\text{time}} \int_{\text{tree}} r(x)\, P^2(x,t)\, dx\, dt$$

energy loss

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linear power flow equation

state of charge

$$0 \leq b(x,t) \leq B(x)$$

budget constraint

Optimal placement & sizing

$$\min_{\substack{b(x,t)\\ B(x)}} \int_{\text{time}} \int_{\text{tree}} r(x)\, P^2(x,t)\,dx\,dt$$

energy loss

$$\text{s. t.}\quad P(x,t) = \int_{y \geq x} \left(\alpha(y)p(t) + \beta(y) - \frac{\partial b}{\partial t}(y,t) \right) dy$$

linear power flow equation

$$0 \leq b(x,t) \leq B(x)$$

state of charge

$$\int_{\text{tree}} B(x)\,dx \leq B_{\text{total}}$$

budget constraint

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Power flow solution

- Monotone operators
- L_2 contraction
- $L_{¥}$ contraction

Linear network

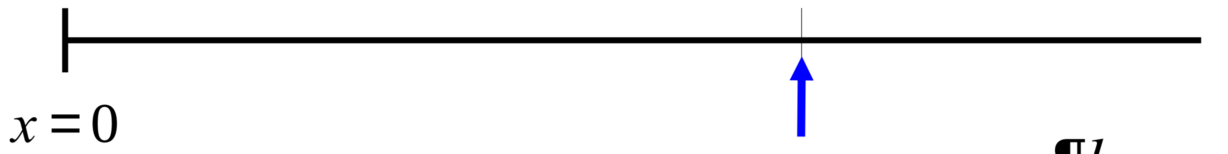
compute: $b(x,t),\ B(x)$

$x = 0$

given: $\alpha(x)p(t) + \beta(x) - \dfrac{\partial b}{\partial t}(x,t)$

Linear network

compute: $b(x,t),\ B(x)$

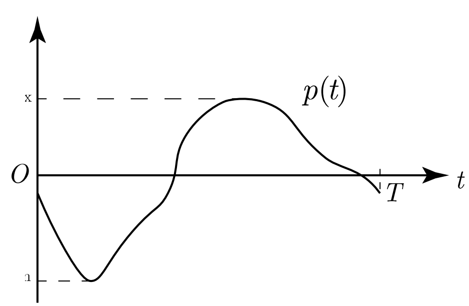


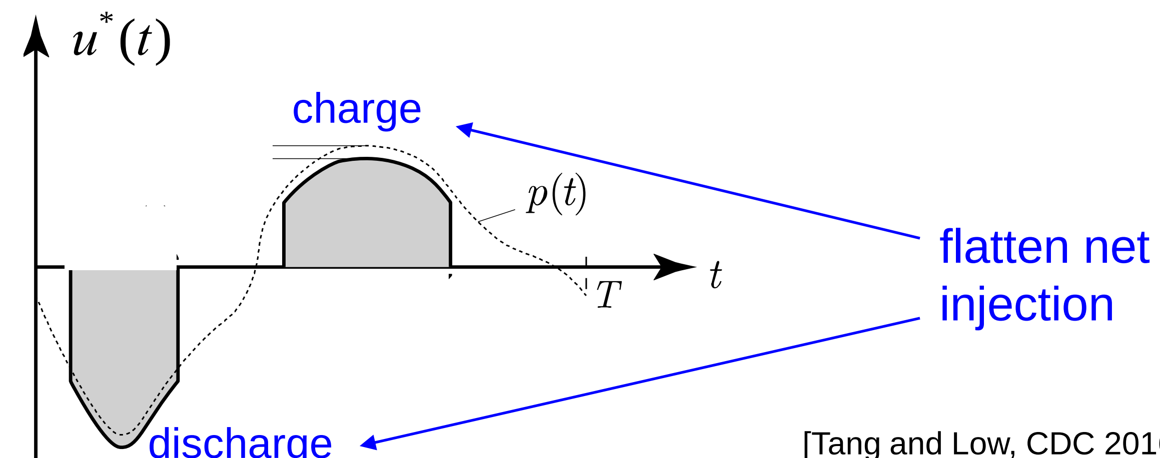
given: $a(x)p(t) + b(x) - \frac{\partial b}{\partial t}(x,t)$

Theorem Optimal charging schedule

Follow load shape *p(t)* with an offset, or OFF

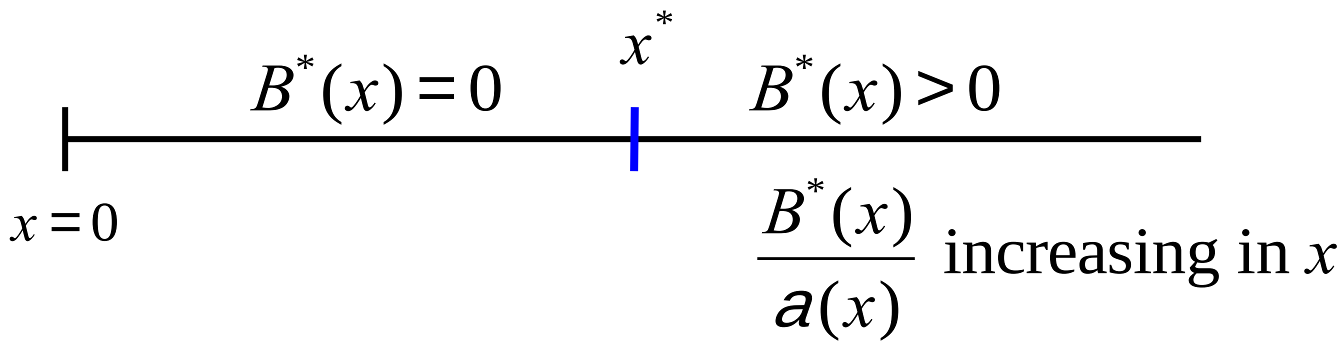
- Charge when injection is highest till full
- Discharge when injection is lowest till empty





[Tang and Low, CDC 2016]

Linear network



Theorem Optimal placement & sizing

- Deploy most storage far away from substation
- Decrease (scaled) storage capacity moving towards substation
- ... until running out of storage budget

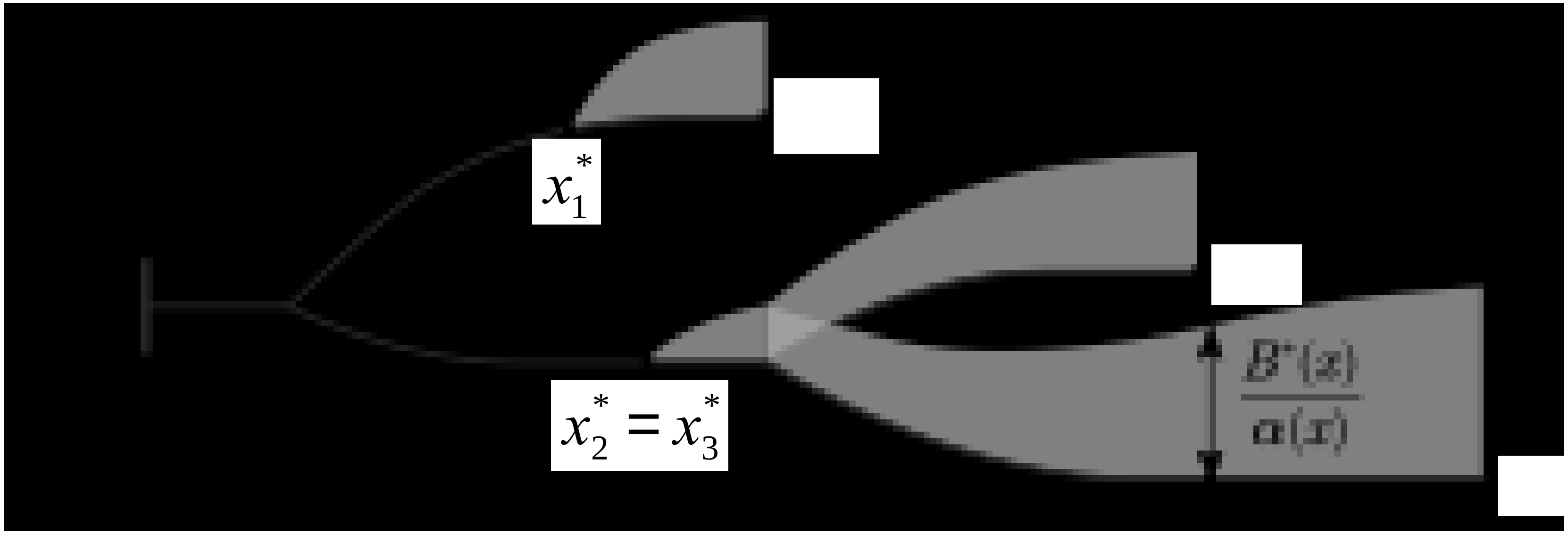
Every node can have distributed generation (2-way power flow)
... as long as all nodes have same load shape (with different scales & offsets)

[Tang and Low, CDC 2016]

Tree network

Theorem

Structural properties (roughly) extend to tree

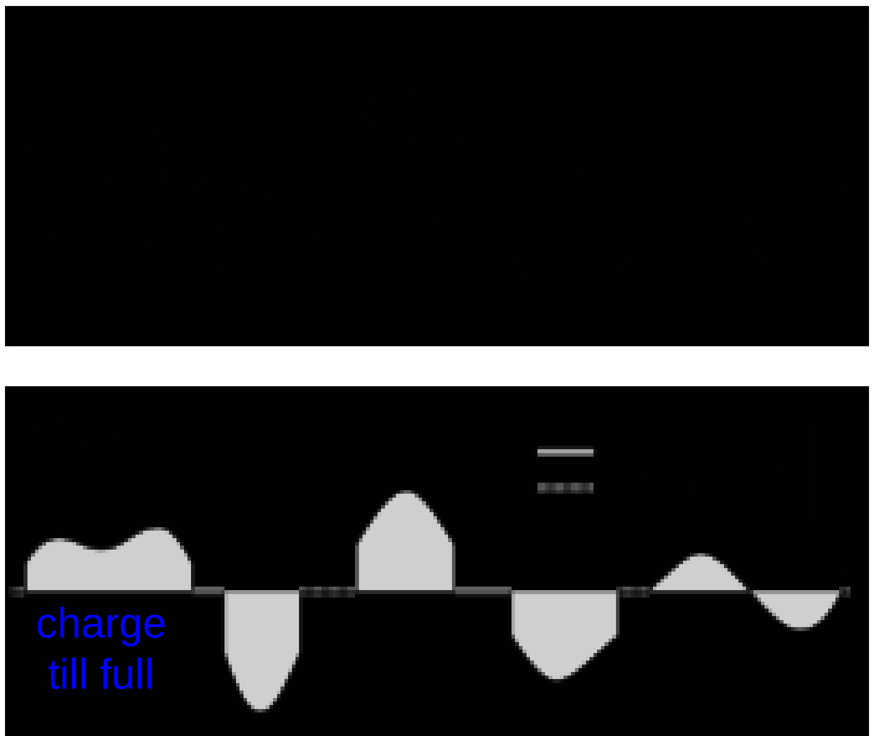


optimal placement & sizing

Tree network

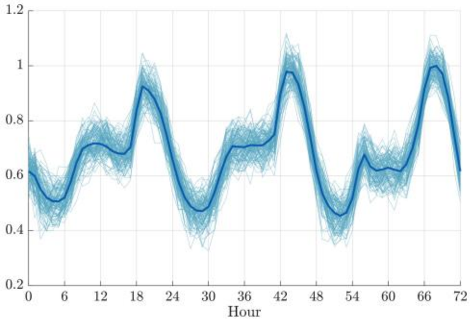
Theorem

Structural properties (roughly) extend to tree



Simulations

IEEE 123-bus test system



More realistic model

- Nonlinear DistFlow model
- Different nodes have slightly different load shapes (from SCE)

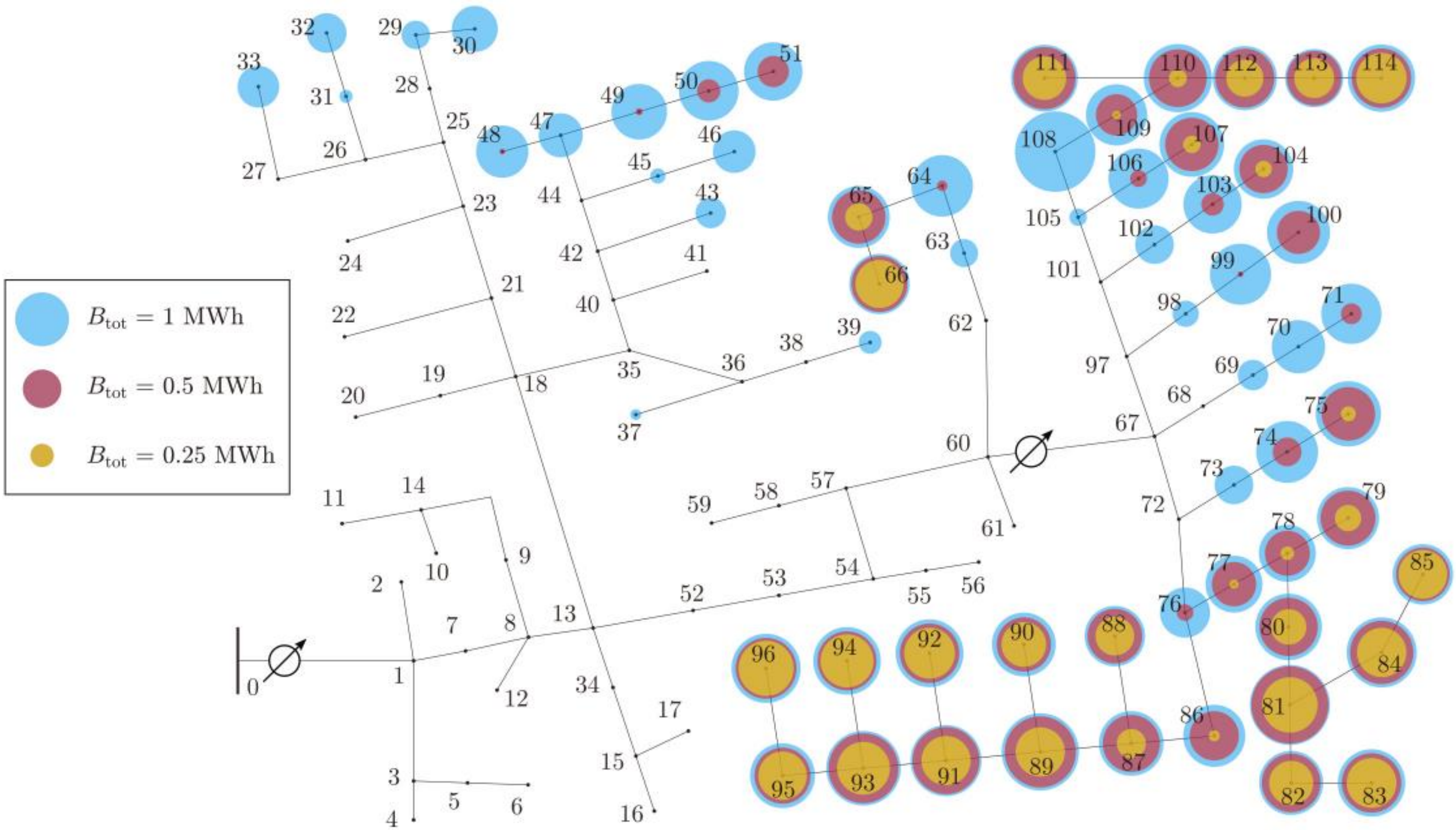
Compare with optimal placement

- Monotone property
- Total loss

Optimal placement solved using SOCP relaxation of OPF

Simulations

Optimal placement & sizing from SOCP relaxation of OPF:



Scaled monotone property roughly holds

Simulations

$B_{tot} = 1$ MWh		
	Loss reduction with $\hat{B}_i^{\leftarrow}$	Optimal loss reduction
Instance 1	45.457 kWh	45.484 kWh
Instance 2	45.037 kWh	45.054 kWh
Instance 3	46.303 kWh	46.323 kWh
$B_{tot} = 0.5$ MWh		
	Loss reduction with $\hat{B}_i^{\leftarrow}$	Optimal loss reduction
Instance 1	32.123 kWh	32.148 kWh
Instance 2	31.828 kWh	31.846 kWh
Instance 3	32.546 kWh	32.556 kWh
$B_{tot} = 0.25$ MWh		
	Loss reduction with $\hat{B}_i^{\leftarrow}$	Optimal loss reduction
Instance 1	19.639 kWh	19.640 kWh
Instance 2	19.485 kWh	19.489 kWh
Instance 3	19.880 kWh	19.882 kWh

Loss with assumptions (same load shape, linear PF) ~ optimal

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- L_2 contraction
- $L_{¥}$ contraction

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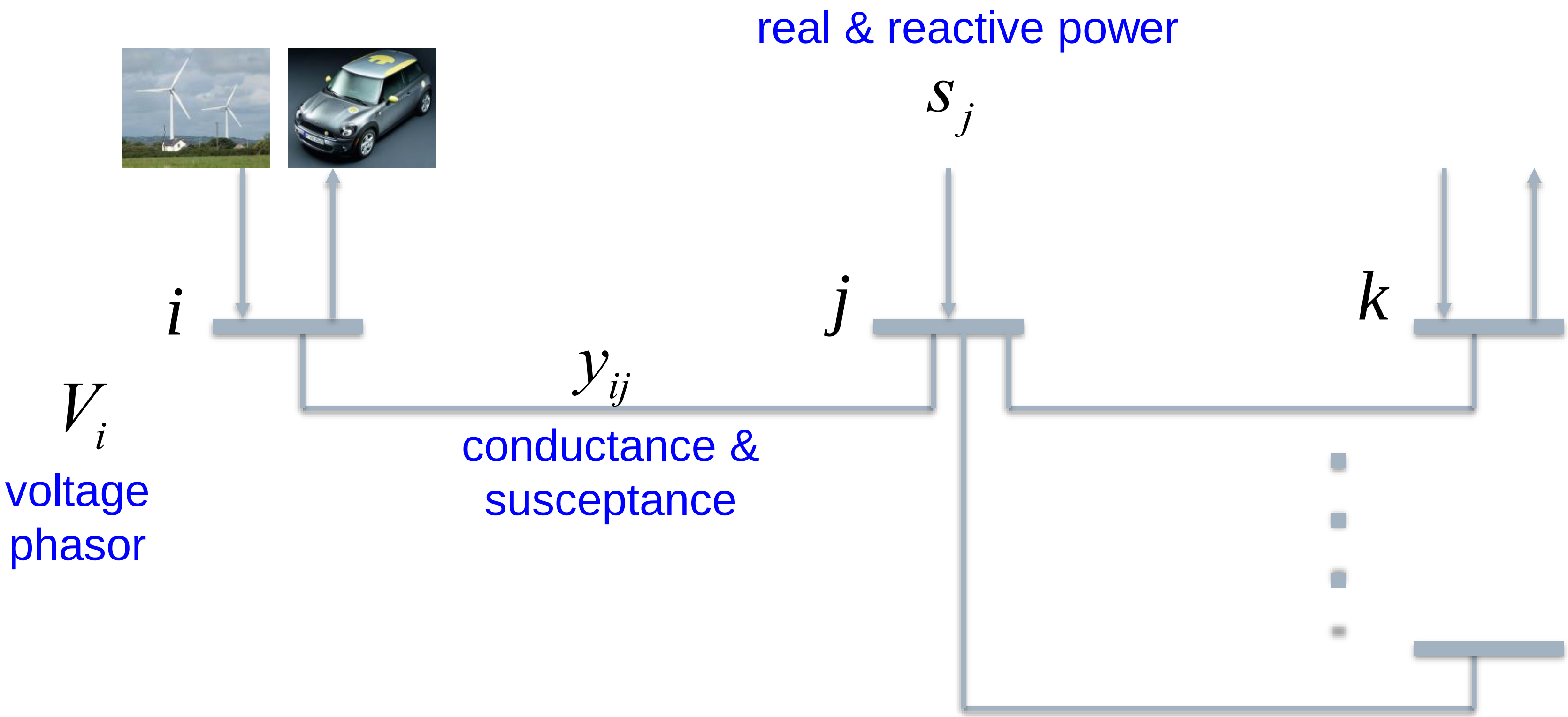
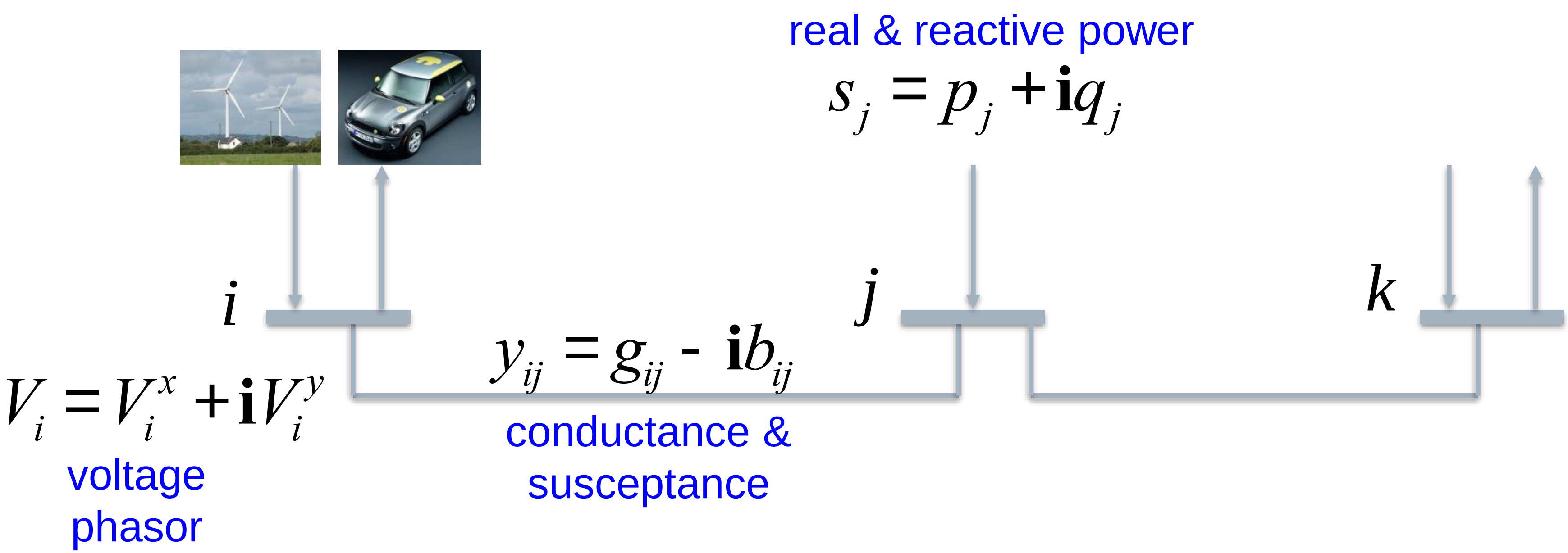
Power flow models



Given s, find V that satisfies

$$s_j = \sum_{k:k\sim j} y_{jk}^H \left(|V_j|^2 - V_j V_k^H \right) \qquad s_j, V_j \in \mathbb{C}$$

Power flow models

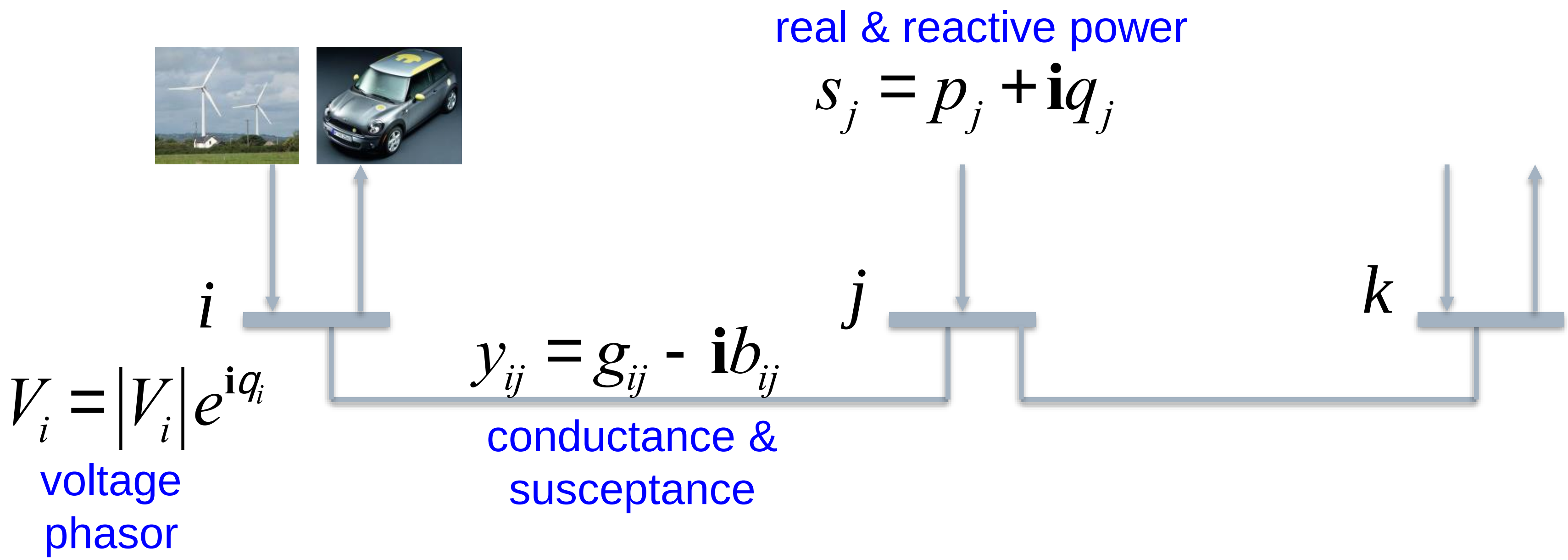


Given (p,q), find (V^x, V^y) that satisfies

$$p_i = \sum_{k:(i,k)\in E} \left(g_{ik} \left((V_i^x)^2 + (V_i^y)^2 \right) - g_{ik}(V_i^x V_k^x + V_i^y V_k^y) + b_{ik}(V_i^y V_k^x - V_i^x V_k^y) \right)$$

$$q_i = \sum_{k:(i,k)\in E} \left(b_{ik} \left((V_i^x)^2 + (V_i^y)^2 \right) - b_{ik}(V_i^x V_k^x + V_i^y V_k^y) - g_{ik}(V_i^y V_k^x - V_i^x V_k^y) \right)$$

Power flow models



In polar form (can be reparametrized into quadratic equations):

$$p_i = \left(\sum_{k=0}^{n} g_{ik}\right) |V_i|^2 - \sum_{k:(i,k)\in E} |V_i||V_k| \left(g_{ik}\cos\theta_{ik} - b_{ik}\sin\theta_{ik}\right)$$

$$q_i = \left(\sum_{k=0}^{n} b_{ik}\right) |V_i|^2 - \sum_{k:(i,k)\in E} |V_i||V_k| \left(b_{ik}\cos\theta_{ik} + g_{ik}\sin\theta_{ik}\right)$$

Power flow solutions

Given s, find x that satisfies

$$F(x) = s$$

where $s, x \in \mathbf{R}^n$ and $F : \mathbf{R}^n \rightarrow \mathbf{R}^n$ is quadratic

Different power flow equations lead to different F

- Different solution properties
- Different computational efficiencies

Power flow solutions

Given s, find x that satisfies

$$F(x) = s$$

where $s, x \in \mathbf{R}^n$ and $F : \mathbf{R}^n \rightarrow \mathbf{R}^n$ is quadratic

Let $J_F(x)$ denote its Jacobian

$$[J_F(x)]_{ij} := \frac{\partial F_i}{\partial x_j}(x)$$

Power flow solutions

Classical algorithms

- Newton-Raphson, Gauss-Seidel, …
- Advantage: simple
- No guarantee: convergence, solution with desired properties, or all solutions

Homotopy-based algorithms

- Can find all PF solutions
- Computationally very expensive

Ours: compromise of 2 methods

- Characterize PF solution region with desirable properties (operation regime)
- Prove there is at most one solution in the region
- Fast computation to find the unique solution if exists or certify none exists in the region

L_2 contraction

Let $D \subseteq \mathbf{R}^n$ be nonempty, closed, convex, over which

1. $J_F(x)$ is *d* -strongly positive definite, i.e.

$$\mathrm{sym}\left(J_F(x)\right) \text{ ³ } dI, \quad x \text{ Î } D$$

implying F is over D

- *d*-strongly monotone

L_2 contraction

Let $D \subseteq \mathbf{R}^n$ be nonempty, closed, convex, over which

1. $J_F(x)$ is *d* -strongly positive definite, i.e.

$$\mathrm{sym}\left(J_F(x)\right)^{3}\ dI, \quad x\ Î\ D$$

1. $J_F(x)$ is L_2 -bounded, i.e.,

$$\left\| J_F(x) \right\|_2 \leq \mathrm{D}, \qquad x \in D$$

implying F is over D

- *d*-strongly monotone
- D-Lipschitz (because F is quadratic)

L_2 contraction

Let $D \subseteq \mathbf{R}^n$ be nonempty, closed, convex, over which

1. $J_F(x)$ is δ-strongly positive definite, i.e.

$$\text{sym}\left(J_F(x)\right) \geq \delta I, \quad x \in D$$

1. $J_F(x)$ is L_2-bounded, i.e.,

$$\left\|J_F(x)\right\|_2 \leq \Delta, \qquad x \in D$$

Theorem

- There is at most one PF solution in D

how to **efficiently** find it if exists, or certify otherwise ?

L_2 contraction

Consider $T(x) := \mathrm{P}_D\left(x - \dfrac{d}{\mathrm{D}^2}(F(x) - s) \right)$

Theorem

- T is a contraction over D with rate $\sqrt{1 - d^2/\mathrm{D}^2}$ and a unique fixed point x^*

L_2 contraction

Consider $T(x) := \Pi_D\left(x - \frac{\delta}{\Delta^2}(F(x) - s) \right)$

Theorem

- T is a contraction over D with rate $\sqrt{1 - \delta^2/\Delta^2}$ and a unique fixed point x^*
- If $F\left(x^*\right) = s$ then x^* is the unique PF solution in D
- ε-approx solution can be computed in $2\log\left(\frac{\text{diam}(D)}{\varepsilon}\right) \Big/ -\log\left(1 - \frac{\delta^2}{\Delta^2}\right)$ fixed point iterations

L_2 contraction

Consider $$T(x) := \Pi_D\left(x - \frac{\delta}{\Delta^2}(F(x) - s)\right)$$

Theorem

- T is a contraction over D with rate $\sqrt{1 - \delta^2/\Delta^2}$ and a unique fixed point x^*
- If $F\left(x^*\right) = s$ then x^* is the unique PF solution in D
- ϵ-approx solution can be computed in $2\log\left(\frac{\text{diam}(D)}{\epsilon}\right) \Big/ -\log\left(1 - \frac{\delta^2}{\Delta^2}\right)$ fixed point iterations
- Otherwise, there is no PF solution in D

Solution strategy

Consider $T(x) := \mathrm{P}_D\left(x - \frac{d}{\mathrm{D}^2}\left(F(x) - s\right)\right)$

Application

- Let $\tilde{D}$ be desirable operation regime

e.g.

$$\tilde{D} := \left\{ V \in \mathbf{C}^n \,\middle|\, \underline{v}_i \le |V_i| \le \overline{v}_i, \;\; |y_{ij}|\,|V_i - V_j| \le \ell_{ij} \right\}$$

voltage magnitudes close to nominal values

line currents lower than capacities

Solution strategy

Consider $$T(x) := \mathrm{P}_D\left(x - \frac{d}{\mathrm{D}^2}(F(x) - s)\right)$$

Application

- Let $\tilde{D}$ be desirable operation regime

Solution strategy

Consider $T(x) := \Pi_D\left(x - \frac{\delta}{\Delta^2}(F(x) - s) \right)$

Application

- Let $\tilde{D}$ be desirable operation regime
- Compute D s.t. $\tilde{D} \subseteq D$ and $J_F(x)$ satisfies conditions 1 and 2 over D

Solution strategy

Consider $T(x) := \Pi_D\left(x - \frac{\delta}{\Delta^2}(F(x) - s)\right)$

Application

- Let $\tilde{D}$ be desirable operation regime
- Compute D s.t. $\tilde{D} \subseteq D$ and $J_F(x)$ satisfies conditions 1 and 2 over D
- Compute $x(t+1) := T(x(t)), \; x(0) \in D$

s

Solution strategy

Consider $T(x) := \Pi_D\left(x - \frac{\delta}{\Delta^2}(F(x) - s) \right)$

Application

- Let $\tilde{D}$ be desirable operation regime
- Compute D s.t. $\tilde{D} \subseteq D$ and $J_F(x)$ satisfies conditions 1 and 2 over D
- Compute $x(t+1) := T(x(t)), \quad x(0) \in D$
- If fixed point $x^* \in \tilde{D}$ and $F\left(x^*\right) = s$ then it is the unique desirable PF solution

Solution strategy

Consider $T(x) := \Pi_D\left(x - \frac{d}{\Delta^2}(F(x) - s)\right)$

Application

- Let $\tilde{D}$ be desirable operation regime
- Compute D s.t. $\tilde{D} \subseteq D$ and $J_F(x)$ satisfies conditions 1 and 2 over D
- Compute $x(t+1) := T(x(t)),\ \ x(0) \in D$
- If fixed point $x^* \in \tilde{D}$ and $F\left(x^*\right) = s$ then it is the unique desirable PF solution
- Otherwise, there is no PF solution in D

Enhancement through scaling

- Instead of solving $F(x) = s$

  solve, for invertible W, $WF(x) = Ws$

- Maximize monotonicity region D by choice of W

Example

Instead of checking if $d := \min_{z \in \mathbf{R}^n,\, x \in D} \; z^T \mathrm{sym}\left(J_F(x)\right) z \;>\; 0$

Optimally choose W by solving:

$$\max_{W} \; \min_{z \in \mathbf{R}^n,\, x \in D} \; z^T \mathrm{sym}\left(W J_F(x)\right) z$$

Optimal W^* yields largest monotonicity region D

Outline

Introduction

Contractive operator method

- L_2 contraction
- L_∞ contraction

L_∞ contraction

Let $D := \{ x \in \mathbf{R}^n \mid l_i \le x_i \le u_i \}$. Suppose, over D,

1. $J_F(x)$ is row δ-diagonally dominant, i.e.

$$[J_F(x)]_{ii} \geq \sum_{k:k\neq i} |[J_F(x)]_{ik}| + \delta, \qquad x \in D$$

L_∞ contraction

Let $D := \{x \in \mathbf{R}^n \mid l_i \le x_i \le u_i\}$. Suppose, over D,

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L_∞ contraction

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1. $J_F(x)$ is L_∞-bounded, i.e.,

$$\|J_F(x)\|_\infty \le \Delta, \qquad x \in D$$

Consider $T(x) := \Pi_D \left(x - \frac{1}{\Delta}(F(x) - s) \right)$

$L_{¥}$ contraction

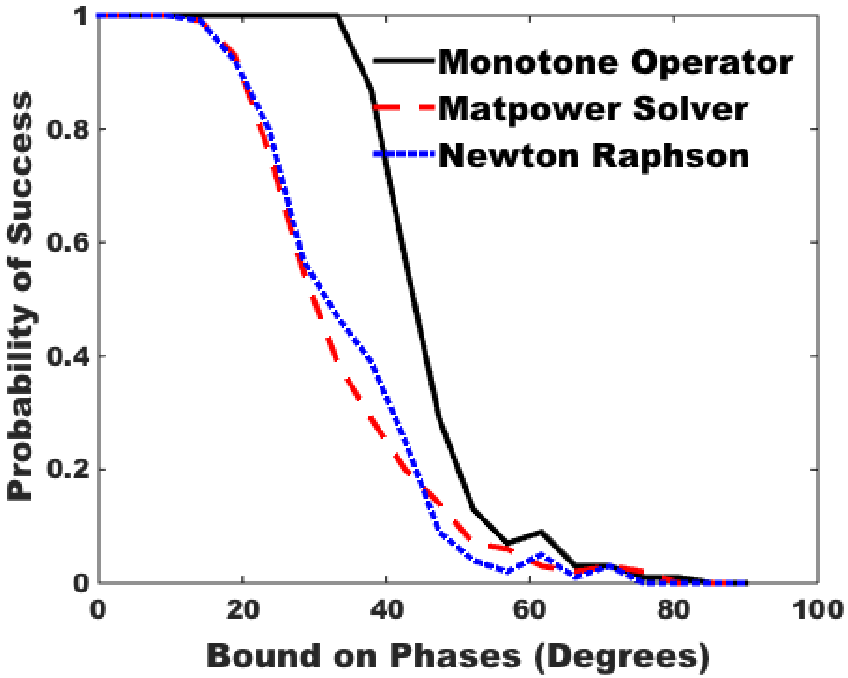
Consider $T(x) := \mathrm{P}_D\left(x - \frac{1}{\mathrm{D}}\left(F(x) - s\right)\right)$

Theorem

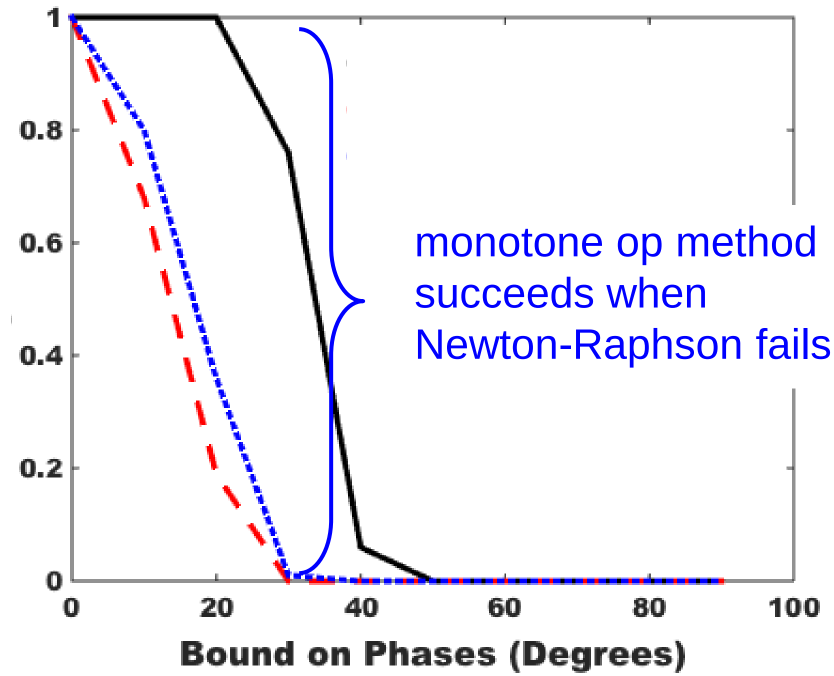
- T is a contraction over D with rate $1 - d/\mathrm{D}$ and a unique fixed point x^*
- If $F\left(x^*\right) = s$ then x^* is the unique PF solution in D
- Otherwise, there is no PF solution in D

Theorem suggests a similar solution strategy

Performance



IEEE 14-bus network



IEEE 39-bus network

Conclusion

PF solution through contraction

- Characterize PF solution region with desirable properties
- Prove there is at most one solution in the region
- Fixed-point iteration computes efficiently…
- … either finds the unique solution or certifies none exists in the region