Autonomous Energy Grid
optimization

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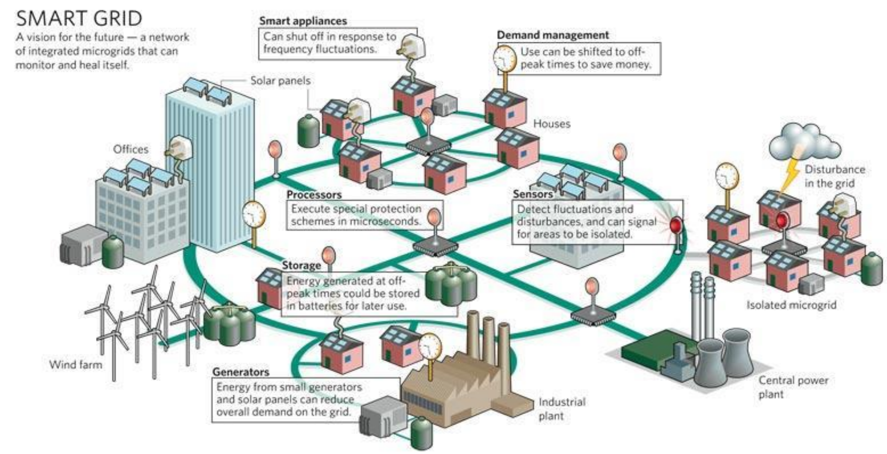
Caltech

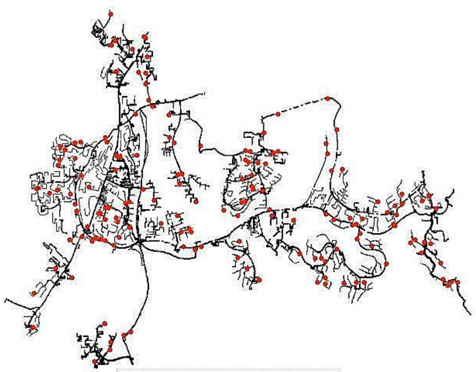
NREL, September 2017

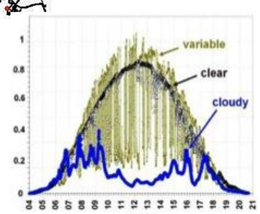
Risk: active DERs introduce rapid random fluctuations in supply, demand, power quality increasing risk of blackouts

Opportunity: active DERs enables realtime dynamic network-wide feedback control, improving robustness, security, efficiency



Caltech research: distributed control of networked DERs





- Foundational theory, practical algorithms, concrete applications
- Integrate engineering and economics
- Active collaboration with industry

Autonomous energy grid

Computational challenge

- nonlinear models, nonconvex optimization

Scalability challenge

- billions of intelligent DERs

Increased volatility

- in supply, demand, voltage, frequency

Limited sensing and control

- design of/constraint from cyber topology

Incomplete or unreliable data

- local state estimation & system identification

Data-driven modeling and control

- real-time at scale

many other important problems, inc. economic, regulatory, social, ...

Autonomous energy grid

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a sample of our work for illustration

Outline

Relaxations of AC OPF

- Dealing with nonconvexity

Distributed AC OPF

- Dealing with scalability

Realtime AC OPF

- Dealing with volatility

Optimal placement

- Dealing with limited sensing/control

SOUTHERN CALIFORNIA EDISON
RESNICK INSTITUTE science + energy + sustainability
Skoltech Skolkovo Institute of Science and Technology

Relaxations of AC OPF

dealing with nonconvexity


Bose (UIUC)


Chandy


Farivar (Google)


Gan (FB)


Lavaei (UCB)


Li (Harvard)

many others at & outside Caltech …

Low, Convex relaxation of OPF, 2014
http://netlab.caltech.edu

Optimal power flow (OPF)

Computational challenge

- OPF underlies numerous power system applications but is nonconvex (and NP-hard)

Scalability challenge

- Future smart grid will have billions of intelligent distributed energy resources (DERs)

Our approach

- Computation: developed relaxation theory that exploits hidden convexity structure
- Scalability: developed distributed algorithms implementable by DERs based on relaxation

Optimal power flow (OPF)

$$\min_{V \in \mathbf{C}^n} \quad \mathrm{tr}\left(CVV^H\right)$$

$$\text{s. t.} \quad \underline{s}_j \le \mathrm{tr}\left(Y_j^H V V^H\right) \le \overline{s}_j$$

$$\underline{v}_j \le \left|V_j\right|^2 \le \overline{v}_j$$

min generation cost, network loss

generation limits

voltage constraints

$$C,\ Y_j \in \mathbf{C}^{n \times n},\quad \underline{s}_j, \overline{s}_j \in \mathbf{C},\ \underline{v}_j, \overline{v}_j \in \mathbf{R}$$

power flow equations: $s_j = \mathrm{tr}\left(Y_j^H V V^H\right)$ for node j

- Y_j^H describes network topology and impedances
- s_j is net power injection (generation) at node j
- "power balance at each node j" (Kirchhoff's law)

Optimal power flow (OPF)

$$\min_{V \in \mathbf{C}^n} \quad \mathrm{tr}\left(CVV^H\right)$$

$$\text{s. t.} \quad \underline{s}_j \le \mathrm{tr}\left(Y_j^H VV^H\right) \le \overline{s}_j$$

$$\underline{v}_j \le \left|V_j\right|^2 \le \overline{v}_j$$

min generation cost, network loss

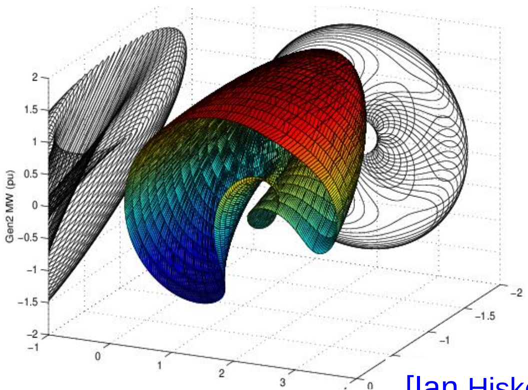
generation limits

voltage constraints

nonconvex feasible set

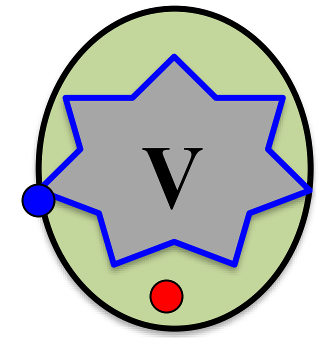
- Y_j^H not Hermitian (nor positive semidefinite)
- C is positive semidefinite (and Hermitian)

nonconvex QCQP



[Ian Hiskens]

Equivalent feasible sets



$$\min \quad \mathrm{tr}\, C V V^H$$

$$\text{subject to} \quad \underline{s}_j \le \mathrm{tr}\left(Y_j^H V V^H\right) \le \bar{s}_j \qquad \underline{v}_j \le |V_j|^2 \le \bar{v}_j$$

quadratic in V
linear in W

Equivalent problem:

$$\min \quad \mathrm{tr}\, C W$$

$$\text{subject to} \quad \underline{s}_j \le \mathrm{tr}\left(Y_j^H W\right) \le \bar{s}_j \qquad \underline{v}_j \le W_{jj} \le \bar{v}_j$$

$$W \ge 0, \quad \mathrm{rank}\, W = 1$$

convex in W
except this constraint

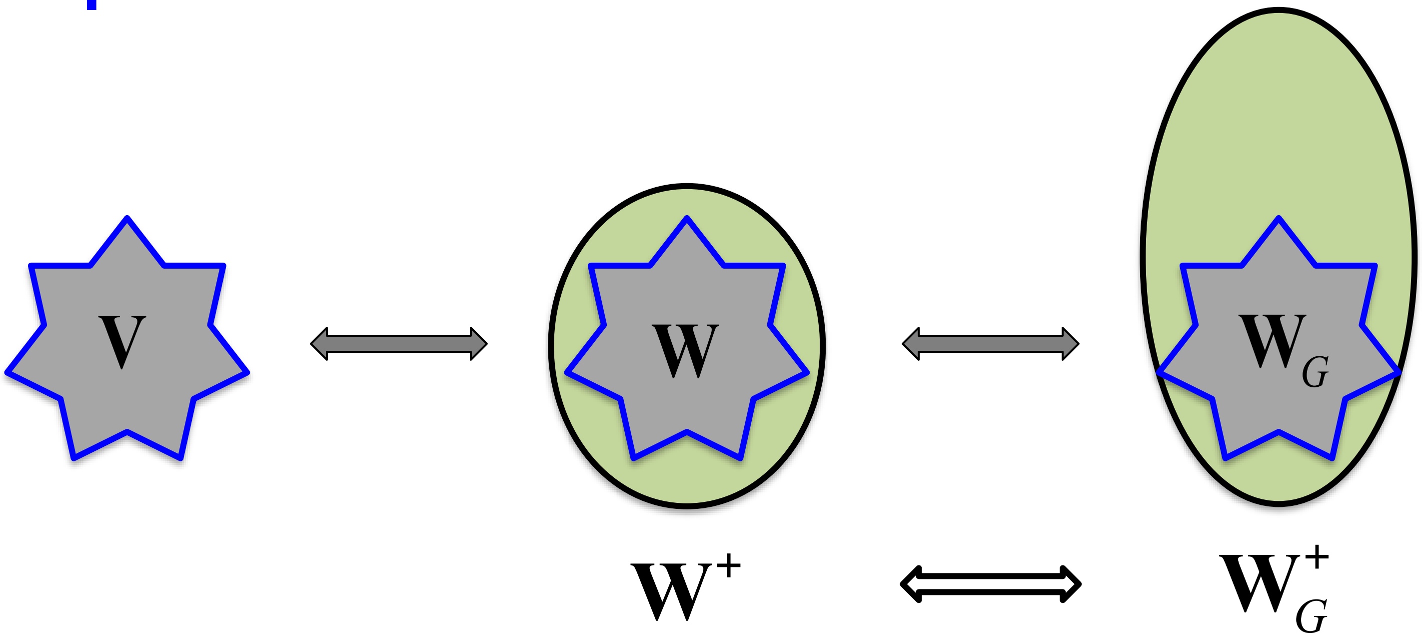
Solution strategy

OPF: $$\min_{x \in \mathbf{X}} f(x)$$

relaxation: $$\min_{\hat{x} \in \mathbf{X}^{+}} f(\hat{x})$$

If optimal solution $\hat{x}^*$ satisfies easily checkable conditions, then optimal solution x^* of OPF can be recovered

Equivalent relaxations



Theorem

- Radial G: SOCP is equivalent to SDP ($\mathbb{V} \subseteq \mathbb{W}^+ \cong \mathbb{W}_G^+$)
- Mesh G: SOCP is strictly coarser than SDP

For radial networks: always solve SOCP !

Exact relaxation

For radial networks, sufficient conditions on

- power injections bounds, or
- voltage upper bounds, or
- phase angle bounds

Exact relaxation

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- power injections bounds, or
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- phase angle bounds

Exact relaxation

QCQP $\left(C, C_k\right)$

$$\begin{aligned} &\min && \mathrm{tr}\left(C x x^H\right) \\ &\text{over} && x \in \mathbf{C}^n \\ &\text{s.t.} && \mathrm{tr}\left(C_k x x^H\right) \le b_k \qquad k \in K \end{aligned}$$

graph of QCQP

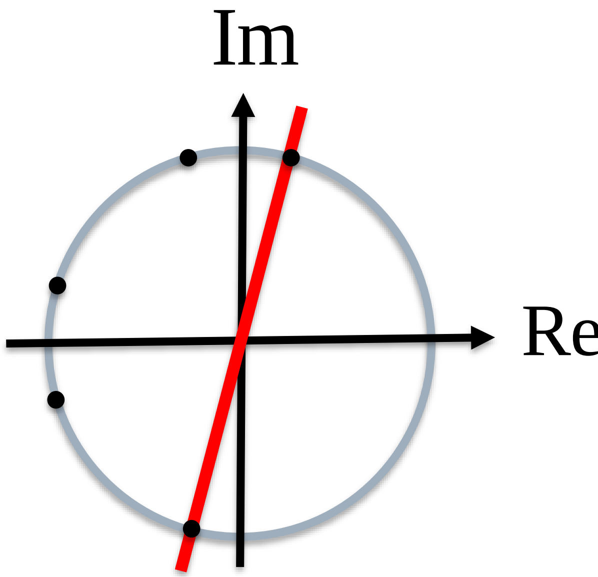
$G\left(C, C_k\right)$ has edge (i, j) $\Leftrightarrow$

$C_{ij} \neq 0$ or $\left[C_k\right]_{ij} \neq 0$ for some k

QCQP over tree

$G\left(C, C_k\right)$ is a tree

Exact relaxation



QCQP (C, C_k)

$$\begin{aligned} &\min && \mathrm{tr}\left(Cxx^H\right) \\ &\text{over} && x \in \mathbf{C}^n \\ &\text{s.t.} && \mathrm{tr}\left(C_k xx^H\right) \le b_k \qquad k \in K \end{aligned}$$

Key condition

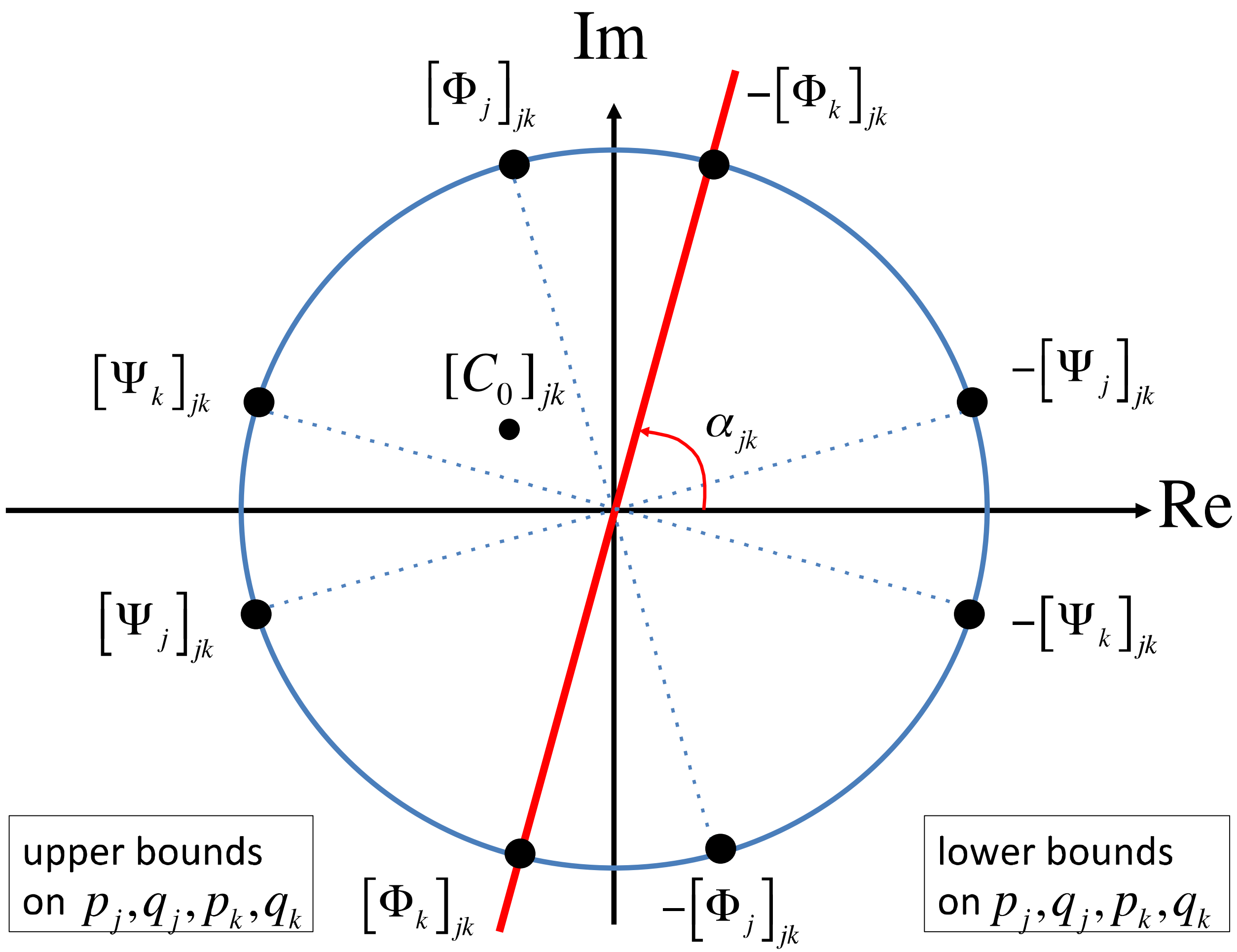
$$i \sim j: \quad \left(C_{ij}, [C_k]_{ij}, \ \forall k\right) \text{ lie on half-plane through } 0$$

Theorem

SOCP relaxation is exact for QCQP over tree

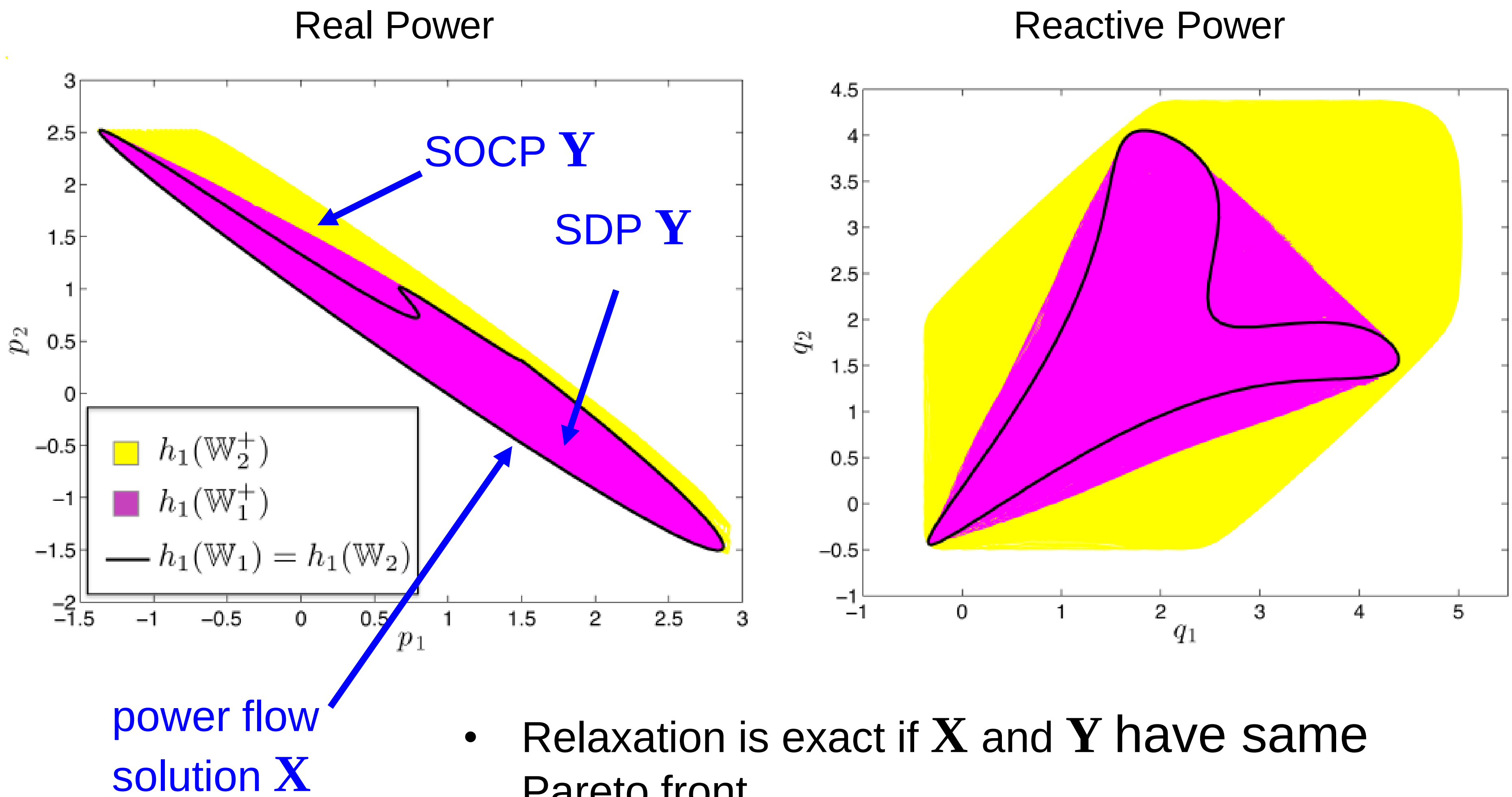
Bose et al 2012, 2014
Sojoudi, Lavaei 2013

Implication on OPF



Not both lower & upper bounds on real & reactive powers at both ends of a line can be finite

Example



- Relaxation is exact if $\mathbf{X}$ and $\mathbf{Y}$ have same Pareto front
- SOCP is faster but coarser than SDP

Bose, Low, Teeraratkul, Hassibi TAC 2015

Potential benefits

IEEE test systems		SDP cost	MATPOWER cost
Syst.	$\text{rank}(\overline{X}_0)$	J°	$\overline{J}$
9	1	5296.7	5296.7
30	1	576.9	576.9
118	1	129661	129661
14A	1	8092.8	9093.8

12.4% lower cost than solution from nonlinear solver MATPOWER

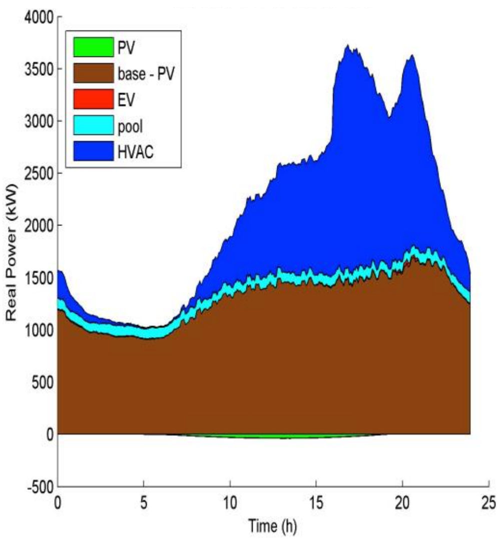
[Louca, Seiler, Bitar 2013]

Potential benefits

Our research

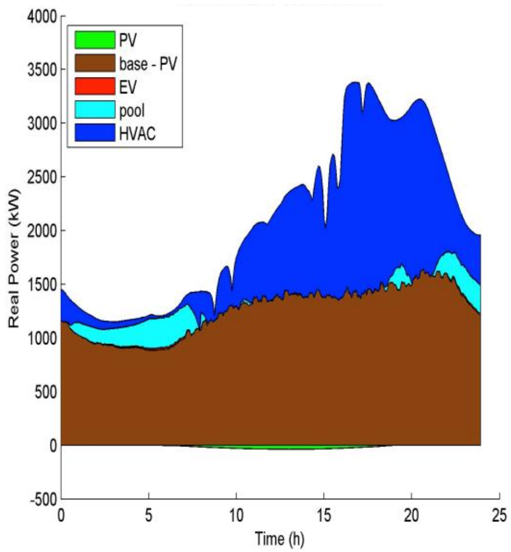
- Computation: developed relaxation theory that exploits hidden convexity structure
- Scalability: developed distributed algorithms implementable by DERs based on relaxation theory
- Benefits: captures values to both utility and users

baseline



peak load reduction: 8%
energy cost reduction: 4%

optimized



Challenges

Challenges for practical application

- Relaxation may not be exact
  - Practical application demands a feasible solution
  - No known sufficient condition for exact relaxation for general mesh (transmission) networks
- Semidefinite relaxation (as is) is not scalable

Distributed AC OPF

for scalability

Gan (FB)

Peng (Google)

Gan & L, PSCC 2014
Peng & L, TSG 2017

Summary: 3 ideas

1. Solve semidefinite relaxation using branch-flow model (BFM)
   - BFM much more numerically stable
   - assume relaxation is exact (radial nk)
2. Decouple into operations at each bus
   - introduce decoupling variables and consensus constraints
   - message passing between neighboring buses
3. Apply ADMM
   - derive closed-form solution or 6x6 eigenvalue problem for each ADMM subproblem
   - greatly speeds up each ADMM iteration

Summary: simulations

network	BIM-SDP			BFM-SDP		
	value	time	ratio	value	time	ratio
IEEE 13-bus	152.7	1.05	8.2e-9	152.7	0.74	2.8e-10
IEEE 34-bus	~~-100.0~~	~~2.22~~	~~1.0~~	279.0	1.64	3.3e-11
IEEE 37-bus	212.3	2.66	1.5e-8	212.2	1.95	1.3e-10
IEEE 123-bus	~~-8917~~	~~7.21~~	~~3.2e-2~~	229.8	8.86	0.6e-11
Rossi 2065-bus	~~-100.0~~	~~115.50~~	~~1.0~~	19.15	96.98	4.3e-8

numerically unstable

numerically stable

BFM is much more numerically stable

SDP relaxations are exact (wye loads)

[Gan & Low 2014 PSCC]

Summary: comparison (single phase)

Network size N	Total Time S	Avg time (= S/N)	Centralized (CPU)	Centralized (elapsed)
IEEE 123 buses	39.5 sec	0.32 sec	1.18 sec	11.4 sec
Rossi 2,065	1,153	0.56	14.38	157.3
1,313	471	0.36	8.88	91.2
792	226	0.29	5.13	50.3
363	66	0.18	3.08	24.5
108	16	0.14	0.78	6.5

- Parallel implementation of our distributed algorithm is much faster than solving OPF centrally

footnote: "Centralized" times reported by CVS in Matlab

- Solving SOCP using CVX (not ADMM)
- "CPU" time excludes problem set up before calling convex solver
- "elapsed" time includes setup time in CVX

Summary: simulations

Network (unbalanced)

- IEEE 13, 34, 37, 123 bus systems

Objective

- loss minimization

Convergence time (computation only)

Network	Diameter	Iterations	Total Time	Avg Time
13 Bus	6	289	17.11	1.32
34 Bus	20	547	78.34	2.30
37 Bus	16	440	75.67	2.05
123 Bus	30	608	306.3	2.49

Details: 3 ideas

1. Solve semidefinite relaxation using branch-flow model (BFM)
   - BFM much more numerically stable
   - assume relaxation is exact (radial nk)
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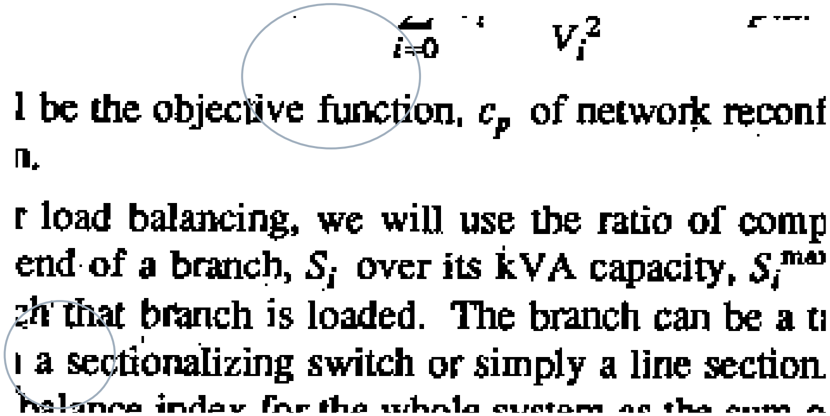
BFM and relaxations

DistFlow model (Baran & Wu 1989)



nonconvex !

OPF

$$\min_x f(x) \quad \text{subject to} \quad \text{DistFlow equations}$$

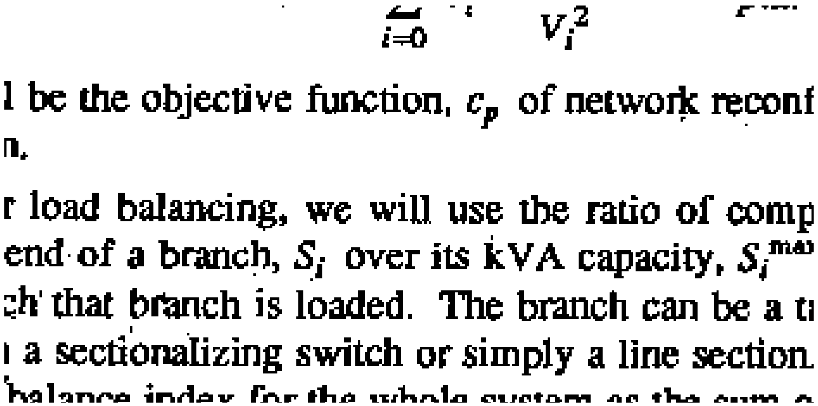
$$\text{operation constraints} \quad g(x) \le 0$$

SOCP relaxation (Farivar & Low 2013)

- Equivalent re-formulation of DistFlow equations (linear + quadratic term)
- SOCP relaxation is often exact, yielding global optimal
- Much more numerically stable than bus injection model

BFM and relaxations

DistFlow model (Baran & Wu 1989)


l be the objective function, c_p of network reconf
n.

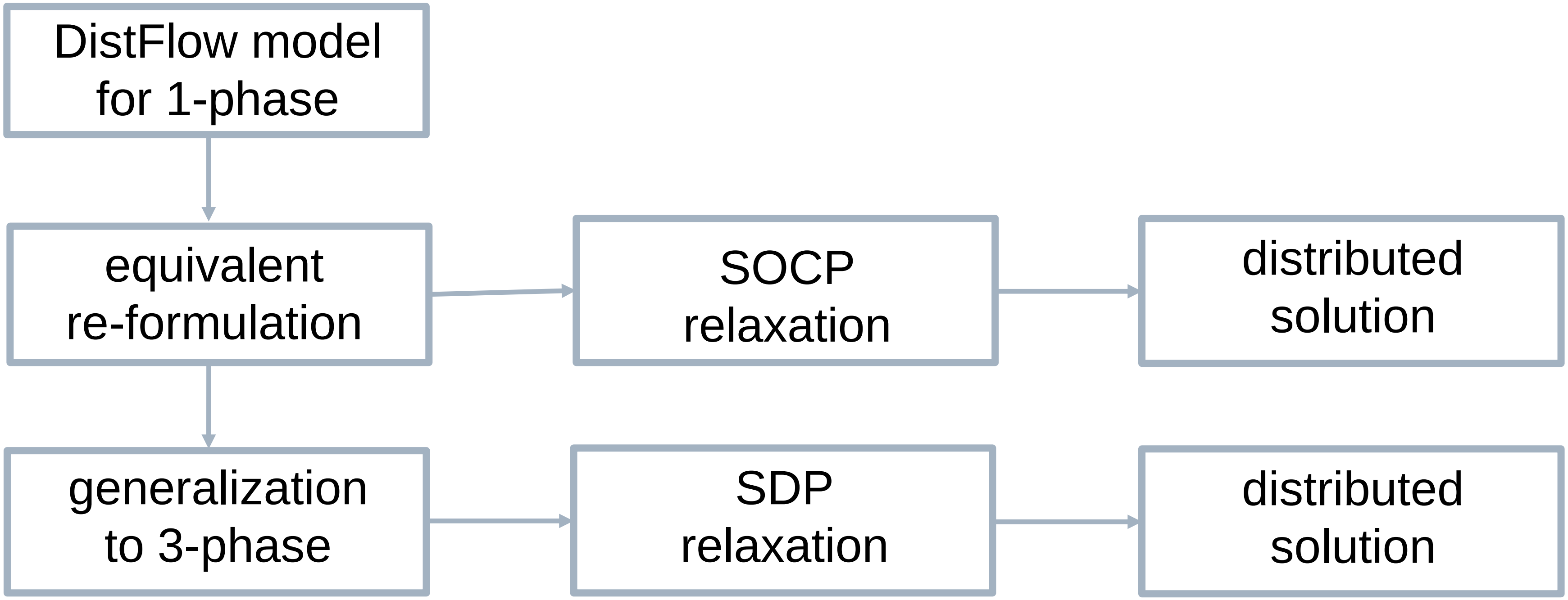
r load balancing, we will use the ratio of comp
end of a branch, S_i over its kVA capacity, S_i^{ma}
ch that branch is loaded. The branch can be a tr
a sectionalizing switch or simply a line section.

But DistFlow model is single-phase !

How to generalize to 3-phase unbalanced system?

- Preserve simple analytical structure of 1-phase model
- Preserve superior numerical stability of 1-phase model

Multiphase generalization



radial, multiphase, wye **+ delta**

Dall'Anese et al 2013 TSG
Gan & Low 2014 PSCC (above approach)

Zhao et al 2017 IREP

Peng & Low 2017 TSG
Peng & Low 2015 CDC

3phase model

3-phase balanced (positive sequence)

$$\begin{bmatrix} I_{jk}^a \\ I_{jk}^b \\ I_{jk}^c \end{bmatrix} = \begin{bmatrix} y_{jk}^{aa} & 0 & 0 \\ 0 & y_{jk}^{bb} & 0 \\ 0 & 0 & y_{jk}^{cc} \end{bmatrix} \left(\begin{bmatrix} V_j^a \\ V_j^b \\ V_j^c \end{bmatrix} - \begin{bmatrix} V_k^a \\ V_k^b \\ V_k^c \end{bmatrix} \right)$$

per-phase analysis

$$I_{jk}^a = y_{jk}^{aa} \left(V_j^a - V_k^a \right)$$

3-phase unbalanced (phase frame)

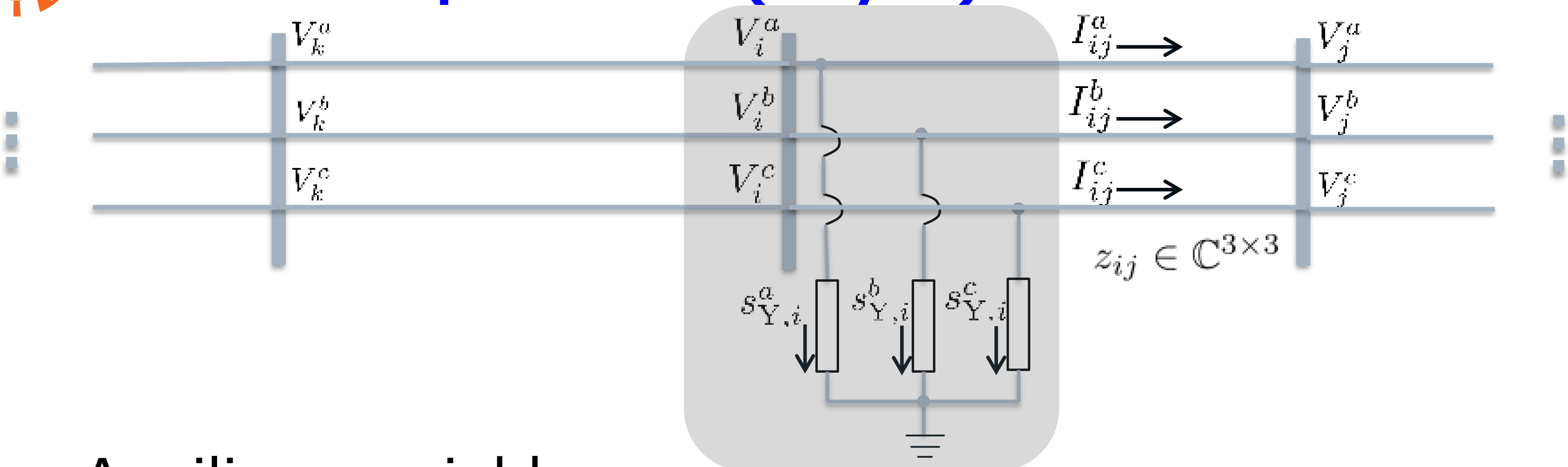
$$\begin{bmatrix} I_{jk}^a \\ I_{jk}^b \\ I_{jk}^c \end{bmatrix} = \begin{bmatrix} y_{jk}^{aa} & y_{jk}^{ab} & y_{jk}^{ac} \\ y_{jk}^{ba} & y_{jk}^{bb} & y_{jk}^{bc} \\ y_{jk}^{ca} & y_{jk}^{cb} & y_{jk}^{cc} \end{bmatrix} \left(\begin{bmatrix} V_j^a \\ V_j^b \\ V_j^c \end{bmatrix} - \begin{bmatrix} V_k^a \\ V_k^b \\ V_k^c \end{bmatrix} \right)$$

3-phase analysis

$$I_{jk} = y_{jk} \left(V_j - V_k \right)$$

3x3 matrix

BFM: 3phase (wye)



Auxiliary variables:

$$\begin{bmatrix} v_i & S_{ij} \\ S_{ij}^H & \ell_{ij} \end{bmatrix} \geq 0, \quad \text{rank} \begin{bmatrix} v_i & S_{ij} \\ S_{ij}^H & \ell_{ij} \end{bmatrix} = 1$$

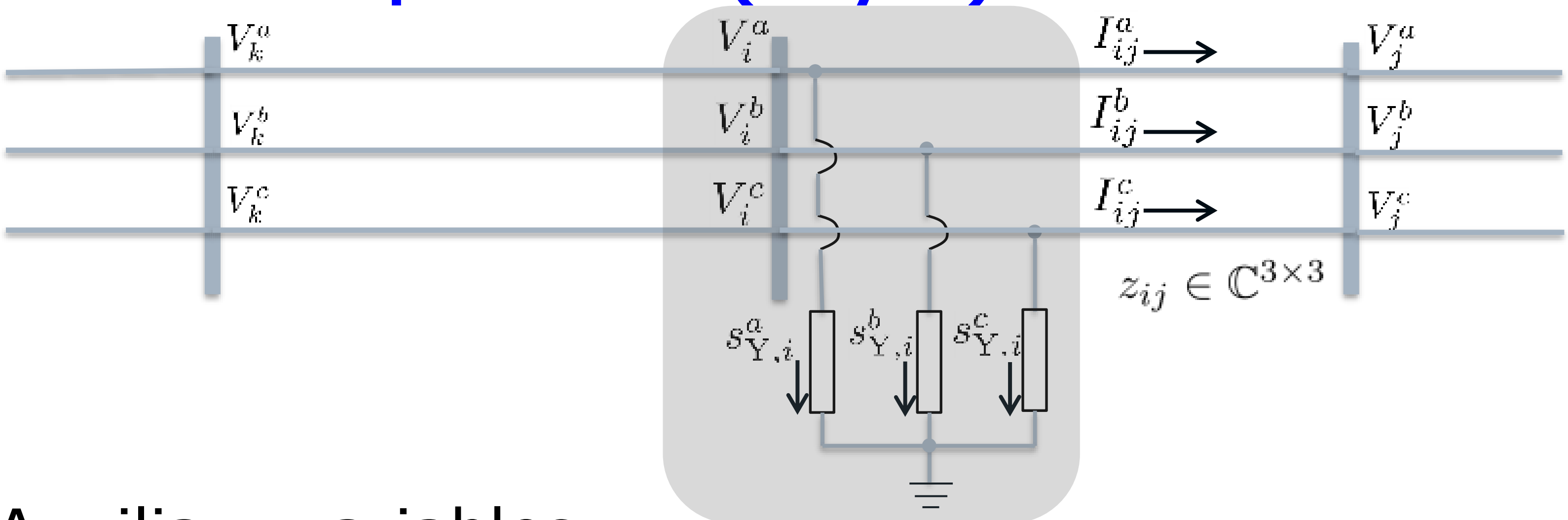
$$v_i = V_i V_i^H \qquad \ell_{ij} = I_{ij} I_{ij}^H$$
$$S_{ij} = V_i I_{ij}^H$$

3x3 rank-1 matrices

Ohm's law:

Power balance:

BFM: 3phase (wye)



Auxiliary variables:

$$\begin{bmatrix} v_i & S_{ij} \\ S_{ij}^H & \ell_{ij} \end{bmatrix} \succeq 0, \quad \text{rank} \begin{bmatrix} v_i & S_{ij} \\ S_{ij}^H & \ell_{ij} \end{bmatrix} = 1$$

6x6 matrix

Ohm's law:

$$v_j = v_i - \left(S_{ij} z_{ij}^H + z_{ij} S_{ij}^H\right) + z_{ij} \ell_{ij} z_{ij}^H$$

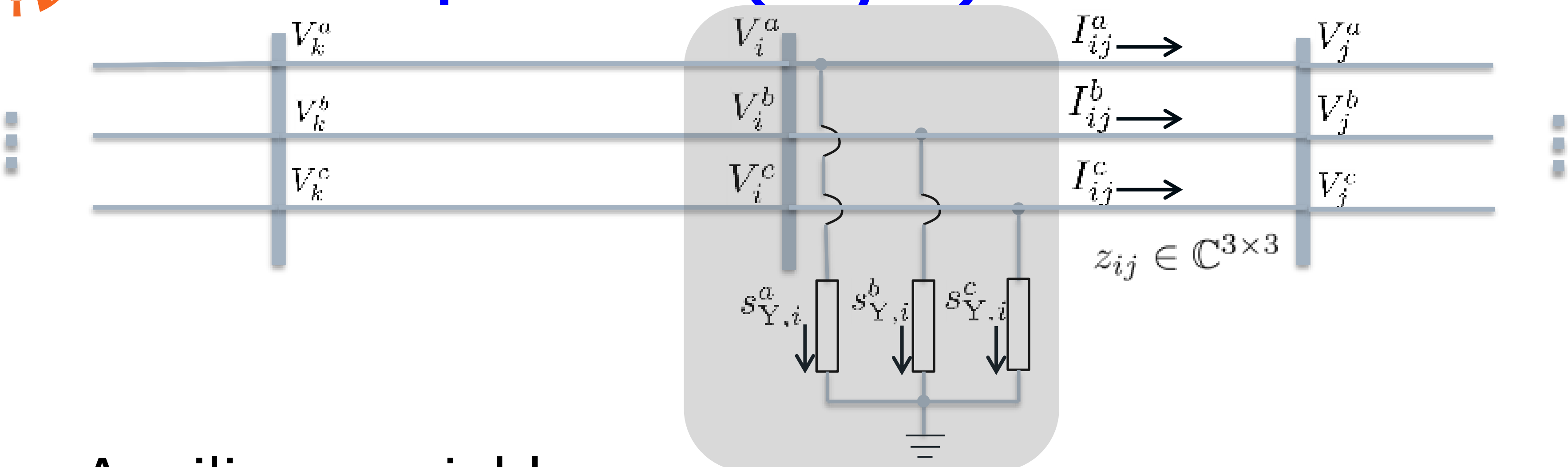
3x3 matrices

Power balance:

$$\sum_{k:k\to i} \text{diag}\left(S_{ki} - z_{ki}\ell_{ki}\right) = \sum_{j:i\to j} \text{diag}\left(S_{ij}\right) + s_{Y,i}$$

3-vectors (a,b,c)

BFM: 3phase (wye)



Auxiliary variables:

$$\begin{bmatrix} v_i & S_{ij} \\ S_{ij}^H & \ell_{ij} \end{bmatrix} \succeq 0, \quad \text{rank} \begin{bmatrix} v_i & S_{ij} \\ S_{ij}^H & \ell_{ij} \end{bmatrix} = 1$$

Ohm's law:

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Power balance:

$$\sum_{k:k\rightarrow i} \text{diag}\left(S_{ki} - z_{ki}\ell_{ki}\right) = \sum_{j:i\rightarrow j} \text{diag}\left(S_{ij}\right) + s_{Y,i}$$

equivalent to DistFlow equations if single-phase

OPF (3phase, wye)

$$\min \quad f(s_{\mathrm{Y}})$$

$$\text{over} \quad (s_{\mathrm{Y}}, v, \ell, S)$$

$$\text{s.t.} \quad \underline{v}_i \le v_i \le \overline{v}_i, \quad s_{\mathrm{Y},i} \in \mathcal{S}_{\mathrm{Y},i}, \quad \forall i$$

branch flow model

$$v_j = v_i - (S_{ij} z_{ij}^H + z_{ij} S_{ij}^H) + z_{ij} \ell_{ij} z_{ij}^H, \ \forall i \to j$$

$$\sum_{k:k\to i} \operatorname{diag}(S_{ki} - z_{ki}\ell_{ki}) = \sum_{j:i\to j} \operatorname{diag}(S_{ij}) + s_{\mathrm{Y},i}, \quad \forall i$$

$$\begin{bmatrix} v_i & S_{ij} \\ S_{ij}^H & \ell_{ij} \end{bmatrix} \succeq 0, \quad \operatorname{rank}\left(\begin{bmatrix} v_i & S_{ij} \\ S_{ij}^H & \ell_{ij} \end{bmatrix}\right) = 1, \quad \forall i \to j$$

non-convex

SDP relaxation (3phase, wye)

$$\min \quad f(s_Y)$$

$$\text{over} \quad (s_Y, v, \ell, S)$$

$$\text{s.t.} \quad \underline{v}_i \le v_i \le \overline{v}_i, \quad s_{Y,i} \in \mathcal{S}_{Y,i}, \quad \forall i$$

branch flow model

$$v_j = v_i - (S_{ij} z_{ij}^H + z_{ij} S_{ij}^H) + z_{ij} \ell_{ij} z_{ij}^H, \ \forall i \to j$$

$$\sum_{k:k\to i} \text{diag}\,(S_{ki} - z_{ki}\ell_{ki}) = \sum_{j:i\to j} \text{diag}\,(S_{ij}) + s_{Y,i}, \quad \forall i$$

$$\begin{bmatrix} v_i & S_{ij} \\ S_{ij}^H & \ell_{ij} \end{bmatrix} \succeq 0, \quad \text{rank}\left(\begin{bmatrix} v_i & S_{ij} \\ S_{ij}^H & \ell_{ij} \end{bmatrix}\right) = 1, \quad \forall i \to j$$

6x6: semidefinite constraint

Gan, Low 2014 PSCC

Partition & decouple

$$\min_{x,y} \sum_{i \in \mathcal{N}} f_i(x_{i0})$$

$x_i := (v_i, \mathbf{s_i}, \ell_{iA_i}, \mathbf{S_{iA_i}})$

y_{ji} : decoupling vars

$$\text{s.t.} \quad \sum_{j \in N_i} A_{ij} y_{ji} = 0 \quad i \in \mathcal{N}$$

power balance & voltage eqtns

$$x_{i0} \in \mathcal{K}_{i0} \quad i \in \mathcal{N}$$
$$x_{i1} \in \mathcal{K}_{i1} \quad i \in \mathcal{N}$$

PSD & injection constraints
voltage magnitude constraints

$$x_{i0} = y_{ij} \quad j \in N_i \; i \in \mathcal{N}$$
$$x_{i1} = y_{ii} \quad i \in \mathcal{N}$$

consensus constraints
(coupling across i)

ADMM

$$\min_{x,y} \; f(x) + g(y)$$
$$\text{s.t. } x \in \mathcal{K}_x, \quad y \in \mathcal{K}_y$$
$$x = y$$

λ : Lagrangian multiplier for coupling constraint

augmented Lagrangian:

$$L_\rho(x, y, \lambda) := f(x) + g(y) + \lambda^T (x - y) + \frac{\rho}{2} (x - y)^H \Lambda (x - y)$$

ADMM update at each iteration k

$$x^{k+1} = \arg \min_{x \in \mathcal{K}_x} L_\rho(x, y^k, \lambda^k)$$
$$y^{k+1} = \arg \min_{y \in \mathcal{K}_y} L_\rho(x^{k+1}, y, \lambda^k)$$

reduce min to
- QP: closed-form soln
- SDP: 6x6 eigenvalue problems

$$\lambda^{k+1} = \lambda^k + \rho(x^{k+1} - y^{k+1})$$

[Peng & L, 2016]

ADMM

Greatly speeds up each ADMM iteration

- much faster than standard iterative solution for each ADMM subproblem

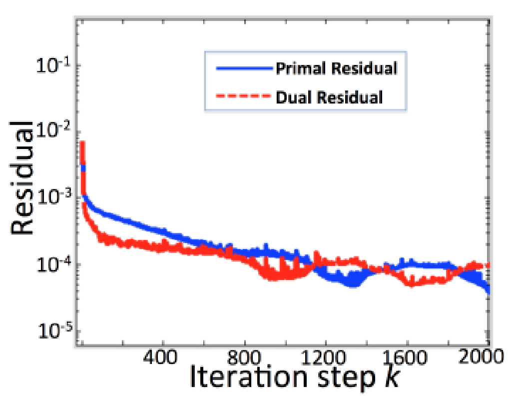
per-bus computation time	x-update	z-update
Our algorithm	1.7×10^{-4} sec	5.1×10^{-4} sec
CVX	2×10^{-1} sec	3×10^{-1} sec
speedup	1,176x	588x

per-bus computation time : time to solve 1 sample ADMM iteration for Rossi circuit with 2,065 buses, divided by 2,065, for both algorithms (single-phase)

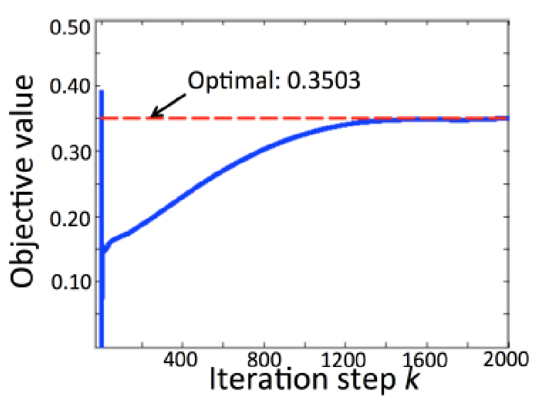
Challenges

Challenges for practical application

- ADMM too slow for high precision solution
- Relaxation (feasible power flow)
  - Wye loads: empirically exact but no proof
  - Delta loads: empirically inexact
- Offline (distributed) algorithm
  - Intermediate iterates are not feasible and cannot be applied to network



(a) Primal and dual residual



(b) Objective value

Realtime AC OPF

for tracking

Gan (FB)

Tang (Caltech)

Dvijotham (DeepMind)

See also: Dall'Anese et al, Bernstein et al, Hug & Dorfler et al, Callaway et al

Gan & L, JSAC 2016
Tang et al, TSG 2017

OPF

$$\min \quad c_0(y) + c(x)$$

$$\text{over} \quad x, \; y$$

$$\text{s. t.}$$

controllable devices

uncontrollable state

OPF

$$\min \quad c_0(y) + c(x)$$

$$\text{over} \quad x,\ y$$

$$\text{s. t.} \quad F(x, y) = 0$$

power flow equations

OPF

$$\min \quad c_0(y) + c(x)$$

$$\text{over} \quad x,\ y$$

$$\text{s. t.} \quad F(x, y) = 0 \qquad \text{power flow equations}$$

$$y \le \overline{y} \qquad \text{operational constraints}$$

$$x \in X := \{\underline{x} \le x \le \overline{x}\} \qquad \text{capacity limits}$$

$$\text{Assume: } \frac{\partial F}{\partial y} \neq 0 \quad \Rightarrow \quad y(x) \ \text{ over } X$$

Static OPF

$$\begin{array}{ll} \min & f(x, y(x);\ \mu) \\ \text{over} & x \in X \end{array}$$

gradient projection algorithm:

$$x(t+1) = \left[x(t) - \eta \frac{\partial f}{\partial x}(t) \right]_X$$ active control

$$y(t) = y(x(t))$$ law of physics

[Gan & Low, JSAC 2016]

Online (feedback) perspective

DER : gradient update

$$x(t+1) = G(x(t),\ y(t))$$

cyber network

control $x(t)$

measurement, communication $y(t)$

Network: power flow solver

$$y(t) : F(x(t),\ y(t)) = 0$$

physical network

- Explicitly exploits network as power flow solver
- Naturally tracks changing network conditions

Drifting OPF

$$\min_{x} \quad c_0(y(x)) + c(x)$$
$$\text{s. t.} \quad y(x) \le \overline{y}$$
$$x \in X$$

static OPF

$$\min_{x} \quad c_0(y(x), g_t) + c(x, g_t)$$
$$\text{s. t.} \quad y(x, g_t) \le \overline{y}$$
$$x \in X$$

drifting OPF

Drifting OPF

$$\min \quad f_t(x, y(x);\ \mu_t)$$
$$\text{over} \quad x \in X_t$$

Quasi-Newton algorithm:

$$x(t+1) = \left[x(t) - \eta \left(H(t) \right)^{-1} \frac{\partial f_t}{\partial x}(x(t)) \right]_{X_t} \qquad \text{active control}$$

$$y(t) = y(x(t)) \qquad \text{law of physics}$$

[Tang, Dj & Low, 2017]

Tracking performance

$$\text{error} := \frac{1}{T}\sum_{t=1}^{T}\left\| x^{\text{online}}(t) - x^{*}(t)\right\|$$

control error

Tracking performance

$$\text{error} \;:=\; \frac{1}{T}\sum_{t=1}^{T} \left\| x^{\text{online}}(t) - x^{*}(t) \right\|$$

Theorem

$$\text{error} \;\le\; \frac{\epsilon}{\sqrt{\lambda_M/\lambda_m} - \epsilon} \cdot \frac{1}{T}\sum_{t=1}^{T}\left(\left\| x^{*}(t) - x^{*}(t-1) \right\| + \Delta_t \right) + \alpha$$

avg rate of drifting

- of optimal solution
- of feasible injections

Tracking performance

$$\text{error} := \frac{1}{T}\sum_{t=1}^{T}\left\| x^{\text{online}}(t) - x^{*}(t) \right\|$$

Theorem

$$\text{error} \leq \frac{\epsilon}{\sqrt{\lambda_M/\lambda_m} - \epsilon} \cdot \frac{1}{T}\sum_{t=1}^{T}\left(\left\| x^{*}(t) - x^{*}(t-1) \right\| + \Delta_t \right) + \delta$$

error in Hessian approx

Tracking performance

$$\text{error} := \frac{1}{T}\sum_{t=1}^{T}\left\| x^{\text{online}}(t) - x^{*}(t) \right\|$$

Theorem

$$\text{error} \leq \frac{\epsilon}{\sqrt{\lambda_M / \lambda_m} - \epsilon} \cdot \frac{1}{T}\sum_{t=1}^{T}\left(\left\| x^{*}(t) - x^{*}(t-1) \right\| + \Delta_t \right) + \delta$$

"condition number" of Hessian

Tracking performance

$$\text{error} := \frac{1}{T}\sum_{t=1}^{T}\left\| x^{\text{online}}(t) - x^{*}(t) \right\|$$

Theorem

$$\text{error} \leq \frac{\varepsilon}{\sqrt{\lambda_M / \lambda_m} - \varepsilon} \cdot \frac{1}{T}\sum_{t=1}^{T}\left(\left\| x^{*}(t) - x^{*}(t-1) \right\| + \Delta_t \right) + \delta$$

"initial distance" from $x^{*}(t)$

Implementation

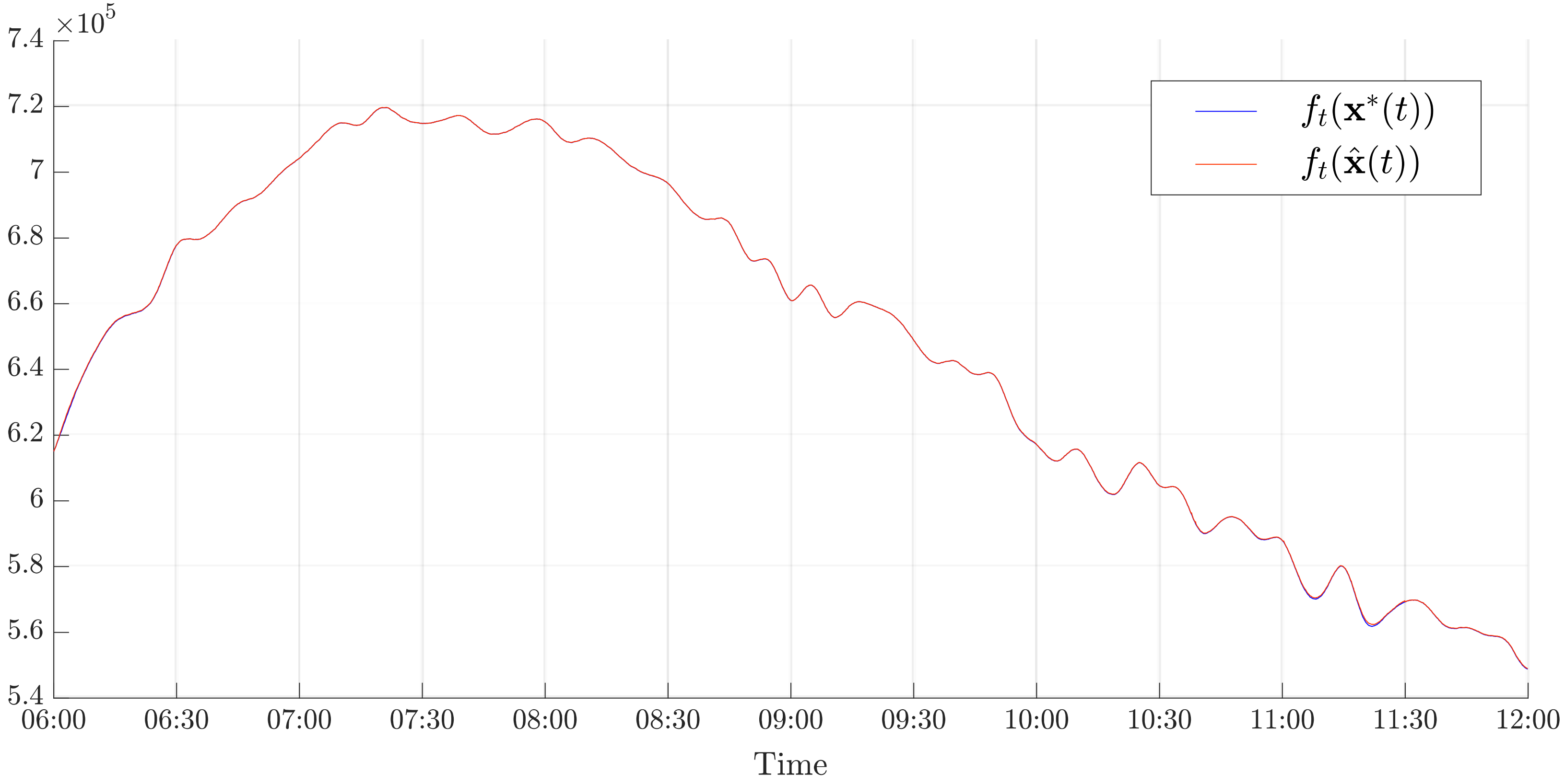
Implement L-BFGS-B

- More scalable
- Handles (box) constraints X

Simulations

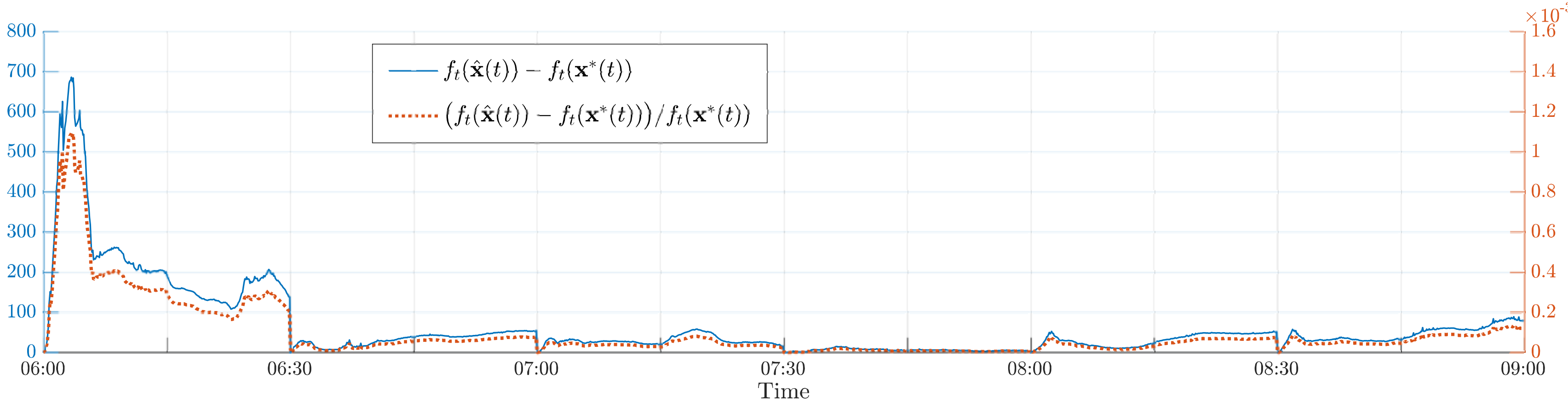
- IEEE 300 bus

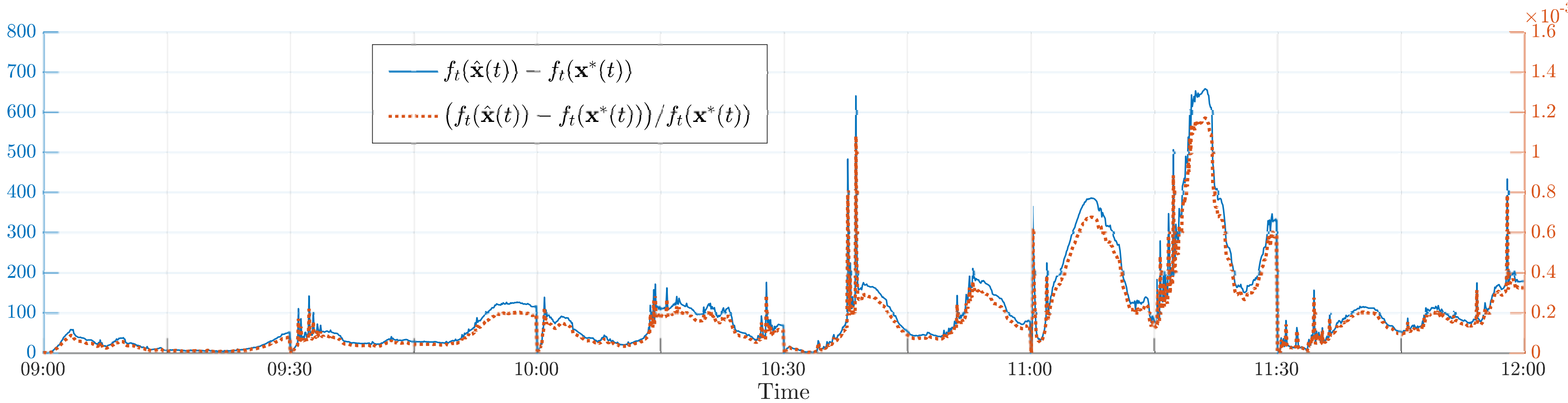
Tracking performance



IEEE 300 bus

Tracking performance





IEEE 300 bus

Challenges

Challenges for practical application

- Distributed implementation
- Tracking with lower update speed
- Not all buses have sensors/controllers

Optimal placement

dealing with limited sensing/control

Guo (Caltech)

Summary

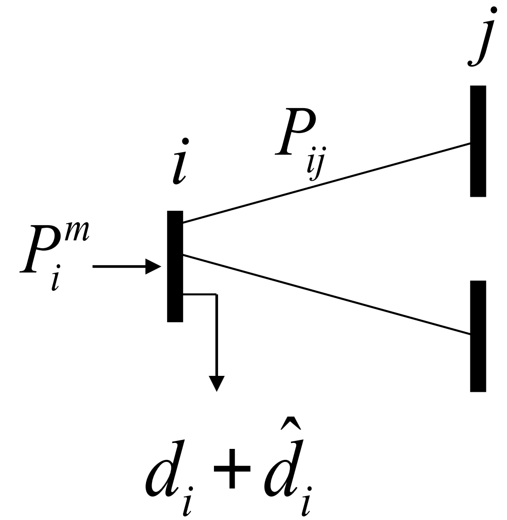
Characterization of controllability and observability

- of swing dynamics
- in terms spectrum of graph Laplacian matrix

Implications on optimal placement of controllable DERs and sensors

- set covering problem

Network model



swing dynamics:

$$-M_j \dot{\omega}_j = 1_{\mathcal{F}}(j)\hat{d}_j + 1_{\mathcal{U}}(j) d_j - P_j^m + \sum_{e \in \mathcal{E}} C_{je} P_e$$

$$\dot{P}_{ij} = B_{ij}(\omega_i - \omega_j)$$

controllable DER

$$y_j = 1_{\mathcal{S}}(j)\omega_j$$

frequency sensor

weighted Laplacian matrix

$$L = M^{-1/2} C B C^T M^{-1/2}$$

Algebraic coverage

spectral decomposition of L

$$L = Q \Lambda Q^T$$

eigenvectors of L

$$Q = \left[\beta_1 \ \cdots \ \beta_n \right]$$

algebraic coverage of bus j

$$\mathrm{cov}(j) := \left\{ s \mid b_{sj} \neq 0 \right\}$$

Controllability

<u>Theorem</u>

Swing dynamics is controllable if and only if

- L has a simple spectrum holds a.s.
- controllable DERs have full coverage

$$\underset{j \in U}{} \mathrm{cov}(j) = \{\text{all buses}\}$$

Observability

<u>Theorem</u>

Swing dynamics is observable if and only if

- L has a simple spectrum holds a.s.
- frequency sensors have full coverage

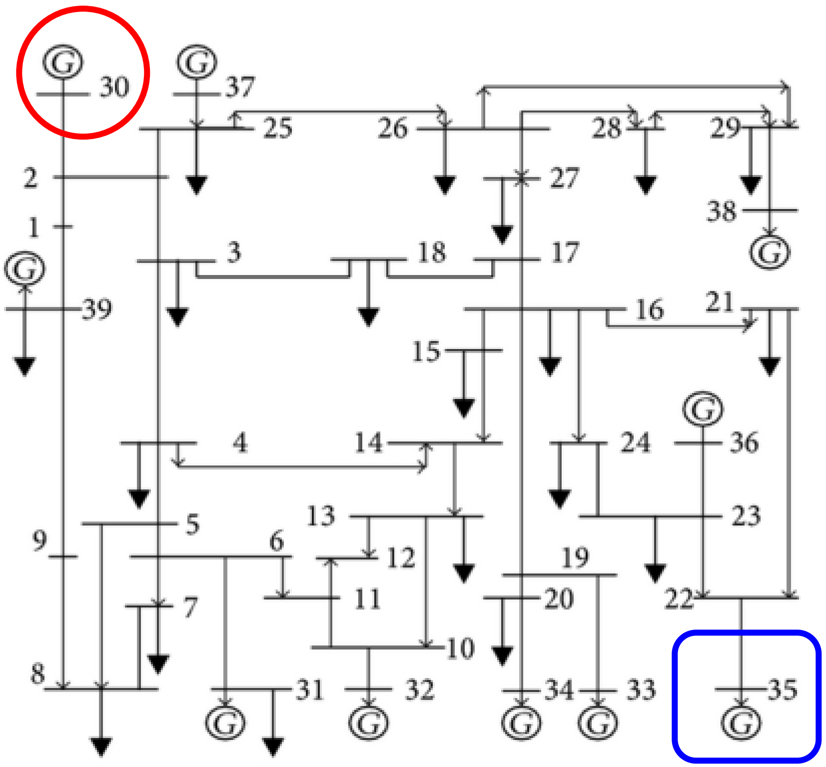
$${}_{j \in S}\,\mathrm{cov}(j) = \{\text{all buses}\}$$

Application

Optimal placement of DER & frequency sensors

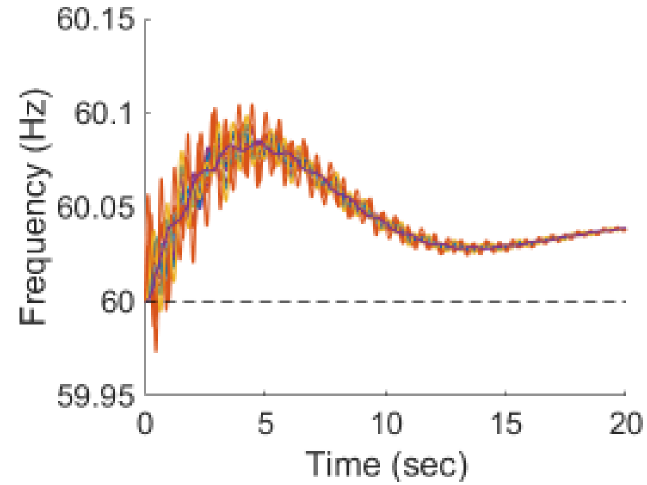
- set covering problem
- always install sensors at buses with controllable DERs, and vice versa

Application

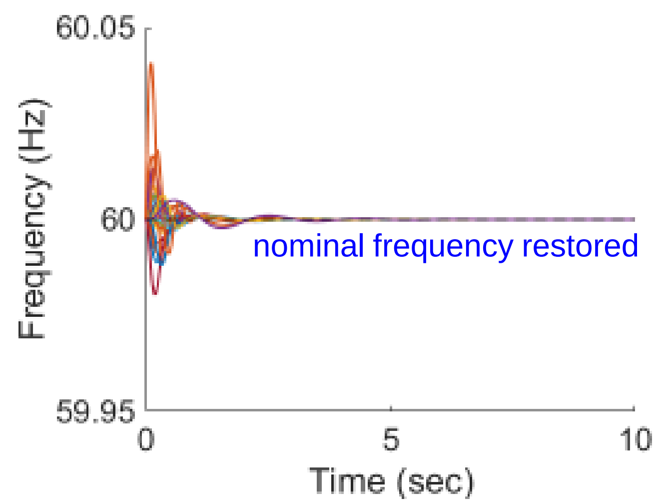


IEEE 39-bus New England system

1pu step disturbance at bus 30



without control



with local control at single bus 35

Summary

Relaxations of AC OPF

- Dealing with nonconvexity

Distributed AC OPF

- Dealing with scalability

Realtime AC OPF

- Dealing with volatility

Optimal placement

- Dealing with limited sensing/control