

Price-Based Max-Min Fair Rate Allocation in Wireless Multi-Hop Networks

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Abstract—The interaction between links in wireless multihop networks introduces extra constraints on the combinations of achievable flow rates. Algorithms have been proposed to achieve max-min fairness under these additional constraints. This letter provides a simple price-based max-min fair rate allocation scheme, building on a utility maximization scheme recently proposed for such networks.

Index Terms—Rate allocation, max-min fairness, flow control, wireless networks.

I. INTRODUCTION

IN RECENT years, allocating flow rates fairly either in wire-line networks or in mobile/wireless networks has received much attention [1]–[9]. In both wireless and wireline networks, links have a limited capacity. In wireless networks, interference from one transmitter to a nearby receiver means that some combinations of links cannot be used simultaneously. Most of these additional constraints have the form of “clique constraints” first introduced in the context of cellular telephony [10], and studied for ad-hoc networks in [5]. A simpler set of constraints arises if each link has a dedicated resource (frequency or spreading code); in that case, the additional constraints reduce to the “MAC constraint”, which requires that no node simultaneously transmit and receive packets [8].

Max-min fairness can be achieved under the MAC constraint using scheduling [8], [9], in the special case of networks with bipartite connectivity graphs [8]. We present a more general approach, which achieves max-min fairness subject to the MAC constraint in networks with arbitrary topologies, or subject to the more general clique constraints, and also allows more flexible “weighted max-min fairness”.

The approach is to use pricing as a means for allocating bandwidth as explained in Section II. This was initially proposed in [1]–[11] for wired networks, where it was shown that distributed algorithms can maximize the sum of the utilities of the users, resulting in a “proportionally fair” rate allocation. This pricing mechanism has been generalized by [5] and [7] to ad-hoc networks, as explained in Section III.

Manuscript received June 23, 2005. The associate editor coordinating the review of this letter and approving it for publication was Christos Douligeris. This research has been supported by the National Natural Science Foundation of China under Grant No. 60473085, the Australian Research Council and by a grant from City University of Hong Kong (Project No. 7001584).

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Digital Object Identifier 10.1109/LCOMM.2006.01015.

As shown by Mo and Walrand [12], rate allocations that approach max-min fairness can be achieved by defining a sequence of utility functions such that the limit of the maximum utility rate allocations is max-min fair. This approach is applied to wireless in [5]. A more direct, and realisable, approach is simply to charge users the highest price of any of the resources they use [13]–[15]. If all users have equal utility functions, this achieves max-min fairness. If they have different utility functions, the rates are the weighted max-min fair rates [16], defined as the rates such that no flow can increase its rate without reducing the rate of a flow being charged more. (For a suitable choice of demand functions, this is reduced to the simpler concept called weighted max-min fairness in [7].) Section III shows how to achieve max-min fair rate allocation in wireless networks by applying this principle to the clique and/or MAC constraints.

II. PRICING FRAMEWORK

The concept of using pricing to optimize rate allocations was originally described in a very general framework [1], [11], for the optimization of the aggregate “social welfare” in a network subject to general linear capacity constraints. It has subsequently been modified to allow max-min fairness to be achieved [13]–[15]. Both of these use the following framework.

Consider a network with a set of fixed resources R which are to be shared among a set of users S . When a user $s \in S$ transmits at a rate x_s , it places a demand $a_{rs}x_s$ on resource $r \in R$. The total demand on resource r cannot exceed a bound b_r , giving the vector constraint

$$Ax \leq b. \quad (1)$$

If each user, s , has a “utility” function, U_s , indicating how much benefit is derived from achieving a given bandwidth, then pricing can be used to optimize the sum of the users’ utilities, as follows. Each resource, r , is dynamically allocated a price, p_r , to indicate its level of congestion. User s is “charged” a price, q_s , per unit bandwidth, which aggregates the prices of the resources it uses. The greedy response from user s is to send at a rate which maximizes its net utility, defined as the utility it gets from that rate minus the cost of transmitting at that rate, $U_s(x_s) - q_s x_s$. Optimizing with respect to x_s for a given q_s causes the user to transmit at a rate $x_s = D_s(q_s)$, where $D_s = (U'_s)^{-1}$ is called the demand function, and $()'$ denotes the derivative.

The difference between the original framework and the max-min framework is in how the price q_s is calculated from the individual link prices, p_r . These will be described in turn.

A. Social Welfare Maximization

The original motivation for the pricing framework was to maximize the sum of users' utilities, subject to the capacity constraints. By applying the technique of Lagrange multipliers, it was shown [1]–[11] that the optimization is achieved if the price p_r is set to be the Lagrange multiplier arising from the constraint r , and user s is “charged” the sum of the prices of the resources it uses, $\sum_{r \in R} a_{rs} x_s p_r$. This gives a price per unit bandwidth of $q_s = \sum_{r \in R} a_{rs} p_r$.

Assuming the resource r knows both its bound, b_r , and its aggregate arrival rate, $\sum_{s \in S(r)} x_s$, a distributed way for the equilibrium prices to be evaluated is by an iterative gradient projection method,

$$p_r(t+1) = \left[p_r(t) + \gamma \left(\sum_{s \in S(r)} x_s(t) - b_r \right) \right]^+, \quad (2)$$

where γ is a step size parameter and $[z]^+ = \max(z, 0)$. This is straightforward when the resources are wired links. However, as will be discussed later, this may not be trivial for the resources used in wireless networks.

B. Max-Min Fairness

It was shown in [13]–[15] that max-min fairness can be obtained from the pricing framework simply by charging users the *maximum* price of any resource they use, $q_s = \max_{r \in R} a_{rs} p_r$, rather than the *sum* of the prices. User s will again send at rate $D_s(q_s)$, with this new form for q_s . This result holds quite generally and in particular applies to wireless networks.

In this context, the prices are no longer the Lagrange multipliers in an optimization problem. However the role of the prices is simply to enforce the capacity constraints. If the utility functions are concave and increasing, then the correct equilibrium prices will result from any price update rule which increases the price of any resource which is overloaded and decreases the price of any resource which is under-utilized (if the price is positive).

The primary contribution of this paper is to apply this price aggregation approach (called “MaxNet” in [13]–[15]) to the set of constraints imposed by wireless networks. These constraints are described in the next section.

III. WIRELESS NETWORKS: CONSTRAINTS AND PRICES

Resources R have usually been taken to be the bandwidths of links. In wireless networks, there are other constraints to include [5]–[9] which give rise to the “clique constraints” [5], [6], less restrictive “MAC constraints” [8], [9] or simply constraints on the total transmission capacity of a node [7]. These constraints will now be described, followed by a description of the price update rules each of them implies.

A. Clique and MAC Constraints

Consider a static multi-hop wireless network, with links $l \in L$ whose capacities c_l are fixed on the timescale of the congestion control scheme, and in which no node can transmit to or receive from multiple nodes simultaneously and no node can simultaneously transmit and receive. Note that wireless links are directed links, as the capacity may not be equal on

both directions, and that c_l excludes capacity wasted by any inefficiencies of the specific MAC protocol.

Define a graph, $\mathcal{G} = (V, E)$ whose vertices, V , are the links in the network. Some pairs of links will not be able to be used simultaneously, because the transmitter of one link causes excessive interference to the receiver of the other, or because the two links share a node. Define the edges of \mathcal{G} , E , to be all such pairs of links.

A clique of a graph is a fully meshed subgraph. A maximal clique is a clique which is not a subgraph of any other clique. Let \mathcal{C}_i be the set of vertices (i.e., wireless links) in the i th maximal clique of \mathcal{G} . Clearly no two links in any clique of \mathcal{G} can be active simultaneously. The fraction of time that a clique is in use becomes an additional limited resource, with bound $b_r = 1$. If user s transmits over link l in clique r , it monopolizes the clique for a proportion of time x_s/c_l . Adding up the fraction of time each clique is busy gives

$$\sum_{l \in \mathcal{C}_r} \sum_{s \in S(l)} \frac{x_s}{c_l} \leq 1, \quad (3)$$

where $S(l)$ denotes the set of sources using link l .

Note that the constraints for any maximal clique subsume the constraints from all sub-cliques, since all terms are positive. In particular, they subsume the traditional link constraints $\sum_{s \in S(l)} x_s/c_l \leq 1$.

The clique constraints are not always sufficient conditions for a set of flow rates to be feasible, a fact which is not mentioned in [5]. Indeed, they are sufficient if and only if the graph is “perfect”. If a graph is not perfect, the clique constraints can be transformed into sufficient conditions for feasibility if the capacity of each link, c_l , is replaced by $2c_l/3$ [6].

The MAC constraints arise when links cannot cause excess interference to each other, such as when all links operate on different frequencies, or use orthogonal spreading codes. In this degenerate case, cliques are simply those links which have a node in common. This constraint is imposed by most medium access control protocols, giving rise to the name “MAC constraint”. Interchanging the order of the summations in (3) then yields

$$\sum_{s \in I(r)} x_s \left(\frac{1}{c_{l_i(r,s)}} + \frac{1}{c_{l_o(r,s)}} \right) \leq 1. \quad (4)$$

Here $I(r)$ is the set of flows incident on node r , and $l_i(r, s)$ and $l_o(r, s)$ are the input and output links for flow s at node r . Flows originating or terminating at a node are deemed to enter or depart through a fictitious link of infinite capacity, causing the corresponding $1/c_l$ terms to disappear.

B. Price Update Rules

For the clique constraints, the price update rule was derived in [5] as

$$p_r(t+1) = \left[p_r(t) + \gamma \left(\sum_{l \in \mathcal{C}_r} \sum_{s \in S(l)} \frac{x_s(t)}{c_l} - 1 \right) \right]^+, \quad (5)$$

As pointed out in [5], care is required in implementing this, since a clique in general has no centralised processor to perform this update. One exception to this is when each link

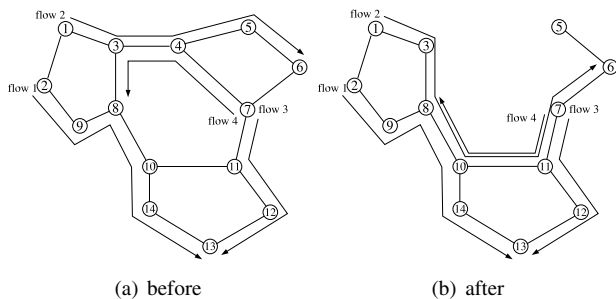


Fig. 1. Network topology, before and after reconfiguration.

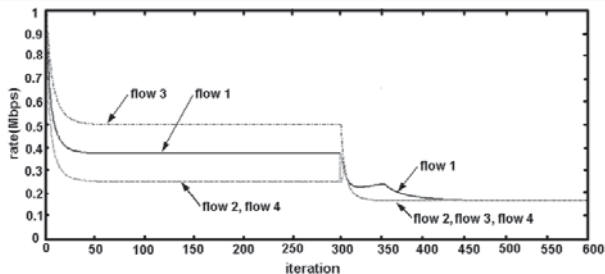


Fig. 2. Network rates.

in a clique shares a node (the MAC constraints). In that case, (5) becomes

$$p_r(t+1) = \left[p_r(t) + \gamma \left(\sum_{s \in I(r)} x_s(t) \left(\frac{1}{c_{l_i(r,s)}} + \frac{1}{c_{l_o(r,s)}} \right) - 1 \right) \right]^+ \quad (6)$$

IV. SIMULATION RESULTS

Wireless ad hoc networks change their topologies frequently, and it is important that any algorithms are able to adapt accordingly. The proposed algorithm was simulated on the network shown in Fig. 1(a) which underwent a sudden change in topology to that of Fig. 1(b), with node 4 and its corresponding connections disabled. Following [8], [9], it is assumed that links have independent transmission channels, and so only the MAC constraints apply. The capacity of each link is 1 Mb/s. The max-min rate allocation for the flows in both network topologies are listed in Table I.

The actual rates of the flows are shown in Fig. 2. Initially, all flows have price 0, and so transmit at the full link capacity. After about 50 price updates (6), the rates converged to the max-min fair values. When the topology change occurs, nodes 8, 10 and 11 are overloaded; their prices increase, and all rates decrease. The bottleneck for flows 2, 3 and 4 is always node 11; its price increases monotonically and their rates drop to 0.167 Mb/s. Until step 350, node 8's price is higher than node 10's, and it controls flow 1. As flows 2 and 4 reduce their rates, node 8 becomes underutilised near step 320, and its price drops. Flow 1 thus increases its rate until it is bottlenecked by node 10 and then it reduces its rate. Compared with the rates in Table I, Fig. 2 demonstrates the algorithm does indeed achieve max-min fairness for a range of topologies.

The actual convergence time depends on the update interval as well as the number of updates. It takes one round trip time for the incoming rate at a node to respond to the change of its price. In this simulation, it was assumed that the update

TABLE I
MAX-MIN FAIR RATES.

	flow 1	flow 2	flow 3	flow 4
before reconfiguration	0.375	0.25	0.5	0.25
after reconfiguration	0.167	0.167	0.167	0.167

interval was greater than one round trip time. If that were not the case, then it would take more iterations (but possibly less time) to reach equilibrium, and slight oscillations may occur.

V. CONCLUSIONS

In this paper, we developed a rate allocation algorithm in multi-hop wireless networks. With the time constraint of the MAC layer, max-min fairness can be achieved among multi-hop flows. We demonstrated that our algorithm realizes fair rate allocation efficiently in dynamically varying wireless networks.

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