Necessary and sufficient conditions for optimal flow control in multirate multicast networks

W.-H. Wang, M. Palaniswami and S.H. Low

Abstract: The authors consider the optimal flow control problem in multirate multicast networks where all receivers of the same multicast group can receive service at different rates with different QoS. The objective is to achieve the fairness transmission rates that maximise the total receiver utility under the capacity constraint of links. They first propose necessary and sufficient conditions for the optimal solution to the problem, and then derive a new optimal flow control strategy using the Lagrangian multiplier method. Like the unicast case, the basic algorithm consists of a link algorithm to update the link price, and a receiver algorithm to adapt the transmission rate according to the link prices along its path. In particular if some groups contain only one receiver and become unicast, the algorithm will degrade to their previously proposed unicast algorithm.

1 Introduction

With the rapid growth and development of computer network technologies, the conventional telephone networks and television broadcasting networks are gradually merging into computer networks. It has become desirable to transmit high quality multimedia (data, audio and video) information through one multi-service network, such as the Internet.

Many present-day real-time applications, like teleconferencing, audio and video broadcasting, require a source to send data information to the members of a multicast group. In conventional unirate multicasting, all receivers of the same multicast group receive services at the same rate. Thus a single rate transmission within a multicast group is likely to overwhelm the slow receivers and starve the fast ones. It is therefore desirable to use multirate multicasting strategy, where the receivers in the same group can receive data at different rates with different quality of service (QoS), depending on the receiver's own characteristics and requirements and different link capacities leading to the receivers. In this case, each link needs to match the fastest downstream receivers in each group and the total transmission rate equals the sum of the maximum downstream rates within the different groups.

One way of achieving multirate multicast transmission is through hierarchical encoding of real time signals. In this approach, a signal is encoded into a number of layers that can be combined incrementally to provide progressive refinement. This layered transmission scheme can be used for both audio and video transmissions over the Internet [1, 2], and also has potential use in ATM networks [3]. In the case of the Internet, each layer can be transmitted as a separate multicast group and receivers can adapt to congestion by joining and leaving these groups (see [4, 5] for Internet protocols for adding and dropping layers).

To ensure that the traffic offered in a network by different sources remains within the limits that the network can carry, an effective flow control strategy is required, and this motivates a recent extensive study on the topic of network flow control based on the optimisation method, e.g. [6–12]. In this formulation, each source is characterised by a utility function of its transmission rate and the goal is to maximise aggregate utility. Indeed, one can interpret major TCP congestion control protocols, such as Reno [13], Vegas [14] and RED [15], within this framework where different protocols are merely different algorithms to solve the same prototype problem with different utility functions [16].

Even though there are tremendous advances in solving the optimal flow control problem in data networks, most of these works focus at the single-path or unicast transmissions case, with some extensions to the multiple-path problem [17–19]. In general, the solutions for the unicast problem can be directly extended to the conventional unirate multicast problem, but designing the optimal flow control strategy in multirate multicast networks remains an open and interesting problem.

The problem is first studied by Kar *et al.* [20, 21]. In [20], the authors first propose an optimisation model for the optimal multirate multicast problem. It is difficult to solve the problem by traditional optimisation methods since the constraints contain maximum functions which are not differentiable. To overcome this difficulty, they introduce the additional pseudo-rates and replace each constraint by a set of linear inequalities to simplify the original optimisation problem. They further use the dual method to derive the optimal flow control algorithms to solve the simplified problem. Their algorithm seems to be complicated, and the psuedo-rates not only introduce a communication overhead in the network, but also cause difficulties in practical implementations. They give a simple algorithm in [21], which is based on the subgradient method and behaves like a discrete version of the sliding mode control technique, but the step size must decrease to zero to ensure the convergence of the algorithm. If a constant step size is used, their algorithm only converges to a neighbourhood of the

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optimum and has a significant chattering. Moreover, unlike the unicast networks, the optimality condition of flow control in multirate multicast networks is still unclear.

In this paper, we will revisit the optimal flow control problem in multirate multicast networks and solve the problem in a different way. We first use a continuously differentiable function to approximate the maximum function in the constraint of the original optimisation problem. The approximation can be made arbitrarily accurate. Then we give a necessary and sufficient condition for the optimal solution of the multirate multicast problem. Using the Lagrangian multiplier method, we further derive a distributed optimal flow control algorithm. In the special case where all the groups contain only one receiver, our algorithm reduces to the previously proposed unicast algorithm in [8].

2 Optimisation problem in multirate multicast networks

Consider a network consisting of a set $\mathcal{L} = \{1, 2, \ldots, L\}$ of links, and each link $l \in \mathcal{L}$ has a transmission capacity c_l. The network is shared by a set $\mathcal{S} = \{1, 2, \ldots, S\}$ of multicast groups. For each multicast group $s \in S$, there is a unique source s, a set of receivers R_s which consists of n_s receivers $\{R_{s,1}, R_{s,2}, \ldots, R_{s,n_s}\}$, and a set of links $\mathcal{L}_s \subset \mathcal{L}$ which forms a multicasting tree in group s, where source s locates at the root, and receivers R_s locate at n_s different leaves. For each receiver $R_{s,i} \in R_s$ in multicast group s, there is a specified path $\mathcal{L}_{s,i} \subset \mathcal{L}_s$ leading to the multicast source s.

Each receiver $R_{s,i} \in R_s$ in group s is characterised by a strictly concave increasing and continuously differentiable utility function $U_{s,i}$ $(x_{s,i})$ as a function of its receive rate $x_{s,i} \geq 0$. Let

$$x = [x_{1,1}, \ldots, x_{1,n_1}, x_{2,1}, \ldots, x_{2,n_2}, \ldots, x_{S,1}, \ldots, x_{S,n_S}]^{\mathrm{T}}$$

Our objective is to find an optimal solution of x, so as to solve the following optimisation problem:

P1 :

$$\max_{x \geq 0} \quad U(x) = \sum_{s \in S} \sum_{i=1}^{n_s} U_{s,i}(x_{s,i}) \tag{1}$$

subject to

$$\sum_{s \in S} x_s^l \leq c_l, \quad \forall\, l \in \mathcal{L} \tag{2}$$

$$x_s^l = \max_{\{i|\ l \in \mathcal{L}_{s,i}\}} x_{s,i} \tag{3}$$

Note that $\{R_{s,i}|\ l \in \mathcal{L}_{s,i}\}$ is the set of receivers in group s that use the link l. Thus the term $x_s^l = \max_{\{i|\ l \in \mathcal{L}_{s,i}\}} x_{s,i}$ denotes the traffic rate of multicast group s at link l, which equals the rate of the fastest downstream receiver at link l in group s. Furthermore, the constraint (2) says that the aggregate traffic rates of different groups at link l does not exceed the link capacity c_l. Clearly, the optimisation problem P1 is feasible and there exists a unique maximisation solution for the source rates x_s since the objective function (1) is strictly concave and the constraint set is convex.

Since the constraints in the optimisation P1 contain the maximum function which is not differentiable, it is difficult to solve the problem by traditional optimisation methods. Here we present a simple approximate solution to the problem P1.

It is well known that

$$x_s^l = \max_{\{i|\ l \in \mathcal{L}_{s,i}\}} x_{s,i} = \lim_{N \to \infty} \left(\sum_{\{i|\ l \in \mathcal{L}_{s,i}\}} x_{s,i}^N \right)^{\frac{1}{N}} \tag{4}$$

We can approximate each maximum function in (3) by

$$x_s^l = \left(\sum_{\{i|\ l \in \mathcal{L}_{s,i}\}} x_{s,i}^N \right)^{\frac{1}{N}} \tag{5}$$

where N is a sufficiently large integer. Then the problem P1 can be approximated by the following optimisation problem:

P2 :

$$\max_{x \geq 0} U(x) = \sum_{s \in S} \sum_{i=1}^{n_s} U_{s,i}(x_{s,i}) \tag{6}$$

subject to

$$\sum_{s \in S} \left(\sum_{\{i|\ l \in \mathcal{L}_{s,i}\}} x_{s,i}^N \right)^{\frac{1}{N}} \leq c_l, \quad \forall\, l \in \mathcal{L} \tag{7}$$

When N goes to ∞, the approximating problem P2 coincides with the original problem P1.

3 Necessary and sufficient conditions for optimal solution

Consider the following Lagrangian multiplier form for problem P2 [Note 1]

$$L(x, p) = \sum_{s \in S} \sum_{i=1}^{n_s} U_{s,i}(x_{s,i}) - \sum_{l \in \mathcal{L}} p_l \left[\sum_{s \in S} \left(\sum_{\{i|\ l \in \mathcal{L}_{s,i}\}} x_{s,i}^N \right)^{\frac{1}{N}} - c_l \right] \tag{8}$$

where $p = [p_1, p_2, \ldots, p_L]^{\mathrm{T}} \geq 0$ is the Lagrangian multiplier, which has the interpretation of link prices as in the unicast case [8].

Since the objective function in P2 is strictly concave, the optimal solution of x is unique. There is an associated $p \geq 0$ such that (x, p) is a saddle-point of (8). Moreover, when N goes to ∞, the optimal solution of P2 will also be the optimal solution for the original problem P1. Thus we have the following theorem on the optimal conditions.

Theorem 1: A solution x is optimal for problem P1 if and only if there exists a Lagrangian multiplier $p = [p_1, p_2, \ldots, p_L]^{\mathrm{T}} \geq 0$ and each receiver $R_{s,i}$ has a price weighting coefficient $w_{s,i}^l \geq 0$ associated with link l, such that:

$$x_{s,i} = [U'^{-1}_{s,i}(p_{s,i})]^+ \tag{9}$$

$$p_{s,i} = \sum_{l \in \mathcal{L}_{s,i}} w_{s,i}^l p_l \tag{10}$$

$$x_s^l = \max_{\{i|\ l \in \mathcal{L}_{s,i}\}} x_{s,i} \tag{11}$$

Note 1: For simplicity, we assume here that the optimal solution x of P1 and P2 is non-negative even without the original constraint condition $x \geq 0$. Otherwise, we should include the additional Lagrangian multiplier term $\sum_{s \in S} \sum_{i=1}^{n_s} \mu_{s,i} x_{s,i}$, where $\mu_{s,i} \geq 0$ in (8) to obtain a similar result.

$$p_l = 0 \quad \text{if} \sum_{s\in\mathcal{S}} x_s^l < c_l \tag{12}$$

$$p_l \geq 0 \quad \text{if} \sum_{s\in\mathcal{S}} x_s^l = c_l \tag{13}$$

where $[z]^+ = \max\{0, z\}$, and $w_{s,i}^l$ satisfies:

$$w_{s,i}^l > 0 \quad \text{if} \quad x_{s,i} = x_s^l \tag{14}$$

$$w_{s,i}^l = 0 \quad \text{if} \quad x_{s,i} < x_s^l \tag{15}$$

$$\sum_{\{i|l\in\mathcal{L}_{s,i}\}} w_{s,i}^l = 1, \quad \forall\, s \in S, \quad l \in \mathcal{L}_s \tag{16}$$

Proof for necessary conditions: Applying the Kuhn–Tucker theorem to the Lagrangian multiplier (8):

$$\frac{\partial L(x, p)}{\partial x} = 0 \tag{17}$$

$$p_l \frac{\partial L(x, p)}{\partial p_l} = 0, \quad \forall\, l \in \mathcal{L} \tag{18}$$

we have the following necessary and sufficient condition for the optimal solution (x, p) of P2:

$$\begin{aligned} &U'_{s,i}(x_{s,i}) \\ &= \frac{\partial(\sum_{l\in\mathcal{L}} p_l(\sum_{s\in S}(\sum_{\{j|\ l\in\mathcal{L}_{s,j}\}} x_{s,j}^N)^{\frac{1}{N}} - c_l))}{\partial x_{s,i}} \\ &= \sum_{l\in\mathcal{L}_{s,i}} p_l \frac{\partial((\sum_{\{j|\ l\in\mathcal{L}_{s,j}\}} x_{s,j}^N)^{\frac{1}{N}})}{\partial x_{s,i}} \\ &= \sum_{l\in\mathcal{L}_{s,i}} p_l \left(\sum_{\{j|\ l\in\mathcal{L}_{s,j}\}} x_{s,j}^N\right)^{\frac{1-N}{N}} x_{s,i}^{N-1} \\ &= \sum_{l\in\mathcal{L}_{s,i}} \left(\frac{x_{s,i}^N}{\sum_{\{j|\ l\in\mathcal{L}_{s,j}\}} x_{s,j}^N}\right)^{\frac{N-1}{N}} p_l \end{aligned} \tag{19}$$

$$p_l \left[\sum_{s\in S}\left(\sum_{\{j|\ l\in\mathcal{L}_{s,j}\}} x_{s,j}^N\right)^{\frac{1}{N}} - c_l\right] = 0, \quad \forall\, l \in \mathcal{L} \tag{20}$$

Equation (20) says, at optimum, that a link price $p_l \geq 0$ is associated with each link l if it is a bottleneck link, otherwise $p_l = 0$, and (19) indicates that the optimal rate $x_{s,i}$ is given by

$$x_{s,i} = [U'^{-1}_{s,i}(p_{s,i})]^+ \tag{21}$$

where $[z]^+ = \max\{0, z\}$ since x must be non-negative and

$$p_{s,i} = \sum_{l\in\mathcal{L}_{s,i}} \left(\frac{x_{s,i}^N}{\sum_{\{j|\ l\in\mathcal{L}_{s,j}\}} x_{s,j}^N}\right)^{\frac{N-1}{N}} p_l \tag{22}$$

is the path price along with the links from receiver $R_{s,i}$ to the source s.

When N goes to ∞, the problem P2 converts to the original problem P1. In this case, we define the price weighting coefficient $w_{s,i}^l$ of the downstream receiver $R_{s,i}$ at link l as follows

$$\begin{aligned} w_{s,i}^l &= \lim_{N\to\infty}\left(\frac{x_{s,i}^N}{\sum_{\{j|\ l\in\mathcal{L}_{s,j}\}} x_{s,j}^N}\right)^{\frac{N-1}{N}} \\ &= \lim_{N\to\infty} \frac{x_{s,i}^N}{\sum_{\{j|\ l\in\mathcal{L}_{s,j}\}} x_{s,j}^N} \end{aligned} \tag{23}$$

Then the path price

$$p_{s,i} = \sum_{l\in\mathcal{L}_{s,i}} w_{s,i}^l p_l \tag{24}$$

and we have the necessary conditions (9)–(13) for P1. Equations (14)–(16) result directly from the definition of $w_{s,i}^l$ in (23).

Proof for sufficient conditions: Since the objective function (1) in P1 is strictly concave and the constraint set is convex, the optimal solution is unique. The necessary optimal conditions for P1 are also the sufficient conditions.

From theorem 1, we can see that the only difference between multirate multicasting and unicasting communications is that the path price for each receiver $R_{s,i}$ is calculated in a different way. Unlike the unicast system, in which the path price is just the sum of link prices along the path, in the multicast system, the path price $p_{s,i}$ is also the sum of link prices along its path $\mathcal{L}_{s,i}$, but each link price is weighted by a non-negative coefficient $w_{s,i}^l \in [0,1]$. If $R_{s,i}$ is not the fastest downstream receiver in group s at link l, then $w_{s,i}^l$ is zero since receiver $R_{s,i}$ receives a service rate $x_{s,i} < x_s^l$ which does not congest the link l. Otherwise, the link price p_l is shared among all the fastest downstream receivers in group s whose rates equal x_s^l since only the fastest downstream receivers have the responsibility for the congestion at link l. However, the responsibilities of the fastest downstream receivers may not be the same and their link price weighting coefficients $w_{s,i}^l$ may not be equal to each other since $w_{s,i}^l$ is calculated from a limit result in (23). This will be seen clearly in the simulations.

4 Optimal flow control algorithm and implementations

In this Section, we will first give an optimisation algorithm to solve the problem P2, from which we will present a practical optimal flow control algorithm to solve the problem P1.

4.1 Approximative algorithm

From the analysis and result of the preceding Section, we have the following optimisation algorithm based on the Lagrangian dual method, which solves the maximisation problem P1 approximately:

$$x_{s,i}(t+1) = [U'^{-1}_{s,i}(p_{s,i}(t))]^+ \tag{25}$$

$$p_l(t+1) = \left[p_l(t) + \gamma\left(\sum_{s\in S} x_s^l(t) - c_l\right)\right]^+ \tag{26}$$

$$p_{s,i}(t+1) = \sum_{l\in\mathcal{L}_{s,i}} w_{s,i}^l(t+1)\, p_l(t+1) \tag{27}$$

$$w_{s,i}^l(t+1) = \frac{x_{s,i}(t)^N}{\sum_{\{j|\ l\in\mathcal{L}_{s,j}\}} x_{s,j}(t)^N} \tag{28}$$

where γ is a sufficiently small step size and N is a sufficiently large integer.

There always needs to be a very large integer N to make P2 close enough to P1. From (28), we can see that when $x_{s,i}$ becomes the fastest group downstream, it will result in a sudden change of its related coefficient $w^l_{s,i}$, its path price $p_{s,i}$, and a significant change in its next step receiving rate. Therefore, the algorithm eventually becomes unstable, no matter how small the selected step size γ.

4.2 Stable accurate algorithm

From (14) to (16) we see that, at optimum level, for each link l, only the fastest downstream $x_{s,i}$ in each group s has a related positive $w^l_{s,i}$, and is responsible for the congestion at link l. The other slow downstream receivers within the same group have their weighting coefficients equal to 0.

Since the main problem in the approximate algorithm (25)–(28) comes from the calculation of $w^l_{s,i}$ in (28), we give the following modification to (28) in order to improve the robustness and accuracy of the algorithm:

(i) Initiate $w^l_{s,i}$ (0) to equal values that sum to 1 for each group s at link l:

$$w^l_{s,i}(0) = \frac{1}{|\{j|l \in \mathcal{L}_{s,j}\}|} \tag{29}$$

where $|\{j|l \in \mathcal{L}_{s,j}\}|$ is the number of downstream receivers in group s at link l.

(ii) Update $w^l_{s,i}$ by

$$w^l_{s,i}(t+1) = [w^l_{s,i}(t) + \gamma(x_{s,i}(t) - x^l_s(t))]^+ \tag{30}$$

(iii) Select any one of the fastest downstream receivers in group s and update its $w^l_{s,i}$ $(t+1)$:

$$w^l_{s,i}(t+1) = 1 - \sum_{\{j|j \neq i, l \in \mathcal{L}_{s,j}\}} w^l_{s,j}(t+1) \tag{31}$$

Equations (30) and (31) indicate that in each group s, $w^l_{s,i}$ is decreased among slow receivers, and the discrepancy is shifted to one of the fastest receivers. Finally, all the price weighting coefficients are only supported by the fastest receivers.

Since (30) and (31) update $w^l_{s,i}$ step by step with a small step size γ, this results in a smooth change in path price $p_{s,i}$ and the traffic rates x converge to the optimal point eventually.

4.3 Implementation of algorithm

We give below the following synchronous distributed implementation of the optimal flow control algorithm in multirate multicast networks.

OFC algorithm in multirate multicast network

Link l's algorithm: Initiate $w^l_{s,i}(0)$ equally for each group s at link l and the total sum is 1:

$$w^l_{s,i}(0) = \frac{1}{|\{j|l \in \mathcal{L}_{s,j}\}|} \tag{32}$$

At times $t = 1,2,\ldots$, link l:

Step 1. Download the information data in each group s at the service rate

$$x^l_s(t) = \max_{\{i|\ l \in \mathcal{L}_{s,i}\}} x_{s,i}(t) \tag{33}$$

Step 2. Calculate the aggregated traffic rate

$$x^l(t) = \sum_{s \in S} x^l_s(t) \tag{34}$$

Step 3. Update the new link price

$$p_l(t+1) = [p_l(t) + \gamma(x^l(t) - c_l)]^+ \tag{35}$$

Step 4. Update price weighting coefficients

$$w^l_{s,i}(t+1) = [w^l_{s,i}(t) + \gamma(x_{s,i}(t) - x^l_s(t))]^+ \tag{36}$$

Step 5. Select any one of the fastest downstream receivers and set its $w^l_{s,i}$ as

$$w^l_{s,i}(t+1) = 1 - \sum_{\{j|j \neq i, l \in \mathcal{L}_{s,j}\}} w^l_{s,j}(t+1) \tag{37}$$

Step 6. Communicate the afforded link price $p^l_{s,i}$ to each downstream receiver $R_{s,i}$:

$$p^l_{s,i}(t+1) = w^l_{s,i}(t+1)\,p_l(t+1) \tag{38}$$

Receiver $R_{s,i}$'s algorithm: At times $t = 1,2,\ldots$, receiver $R_{s,i}$:

Step 1. Receive from the network the path price

$$p_{s,i}(t) = \sum_{l \in \mathcal{L}_{s,i}} p^l_{s,i}(t) \tag{39}$$

Step 2. Update the new receiving rate and send the request of new rate to the up links and the source s along the path $\mathcal{L}_{s,i}$

$$x_{s,i}(t+1) = [U'^{-1}_{s,i}(p_{s,i}(t))]^+ \tag{40}$$

Source s's algorithm: At times $t = 1,2,\ldots$, source s downloads the information to its adjacent link at a rate x_s to match the fastest downstream receiver

$$x_s(t) = \max\{x_{s,1}(t), \ldots, x_{s,n_s}(t)\} \tag{41}$$

Remark 1: In the available bit rate (ABR) service, each receiver $R_{s,i}$ may be further required for a minimum receiving rate $m_{s,i}$, a maximum rate $M_{s,i}$ and $0 \leq m_{s,i} \leq x_{s,i} \leq M_{s,i}$. It is assumed here that the minimum transmission rates are achievable in the network. In this case, we can project the receiving rate $x_{s,i}$ of (40) into the interval [$m_{s,i}$, $M_{s,i}$] directly:

$$x_{s,i}(t+1) = [U'^{-1}_{s,i}(p_{s,i}(t))]^{M_{s,i}}_{m_{s,i}} \tag{42}$$

where $[z]^b_a = \max\{a, \min\{b, z\}\}$.

Remark 2: Suppose there is only one receiver $R_{s,1}$ in a unicast group s; then its price weighting coefficient $w^l_{s,1} \equiv 1$ in (38) and the algorithm of group s is simplified to our previously proposed unicast optimal flow control algorithm in [8].

5 Numerical example and simulation results

Consider the following multicast network in Fig. 1, which is used by Kar *et al.* [20]. The network consists of 10 links labelled L_1, $L_2, \ldots, L_{10}$ with capacities $c = (5,3,5,5,4,3,5,5,4,5)$ (in Mbit/s) shown in the Figure, and shared by two multicast groups. The utility functions of all receivers in group 1 and $R_{2,1}$ in group 2 are $\log(1+x)$, while those of the

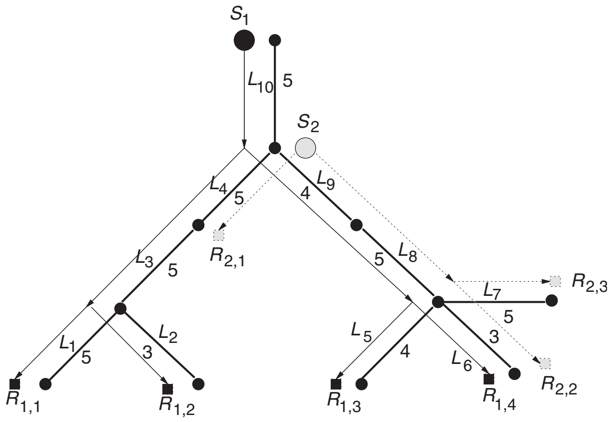

Fig. 1 *Network topology*
Numbers associated with links are link labels and link capacities in Mbit/s
— multicast group 1 (source S_1, receivers: $R_{1,1}$, $R_{1,2}$, $R_{1,3}$, $R_{1,4}$)
… multicast group 2 (source S_2, receivers: $R_{2,1}$, $R_{2,2}$, $R_{2,3}$)

remaining receivers $R_{2,2}$ and $R_{2,3}$ are $2\log(1+x)$. The minimum and maximum receiver rates are 0 and 5 Mbit/s, respectively. Assume that receivers $R_{1,1}$, $R_{1,2}$, $R_{1,3}$, $R_{1,4}$, $R_{2,2}$ and $R_{2,3}$ arrive at time $t=0$. Receiver $R_{2,1}$ joins later at $t=60$ s, while receiver $R_{1,2}$ leaves at $t=120$ s. The multicasting terminates at $t=180$ s.

The simulation is based on Matlab, in which the algorithm is updated at a interval of 0.1 s, with a constant step size $\gamma=0.01$ in (35) and (36).

Figures 2*a* and 2*b* show the requested receiver rates of groups 1 and 2, respectively; all the rates converge to the same optimal values as in Kar *et al.* [20], but here the convergence speed is more than 10 times faster. Compared with the results given in [21], our algorithm also has a smoother convergence.

Figure 2*c* shows the convergence of link prices of bottleneck links L_2, L_4, L_6 and L_9. The other link prices remain at 0.

Figure 2*d* gives the price weighting coefficients of receivers $R_{1,1}$ and $R_{1,2}$ at link L_4, and we discuss their property next.

With a close study of the example at $t=60$–120 s, we can see that the receive rates x converge to the optimum (3, 3, 1, $\frac{2}{3}$, 2, $2\frac{1}{3}$, 3). In particular, the receivers $R_{1,1}$ and $R_{1,2}$ in group 1 have the same rate at 3. According to link L_4, both are the fastest downstream receivers in group 1. From Fig. 2*d* we see clearly that their price weighting coefficients are not identical, but with a different $w^4_{1,1}=0.75$ and $w^4_{1,2}=0.25$. Thus their path prices are:

$$
\begin{aligned}
p_{1,1} &= w^4_{1,1} p_4 = 0.75 \times \frac{1}{3} = \frac{1}{4} = U'_{1,1}(3) \\
p_{1,2} &= w^2_{1,2} p_2 + w^4_{1,2} p_4 \\
&= 1 \times \frac{1}{6} + 0.25 \times \frac{1}{3} = \frac{1}{4} = U'_{1,2}(3)
\end{aligned}
$$

and the optimal condition is satisfied.

The other link capacity of 2 at link L_4 is used by $R_{2,1}$ in group 2, whose path price is

$$
p_{2,1} = p_4 = \frac{1}{3} = U'_{2,1}(2)
$$

For the other receivers, the optimal condition based on receiver rates and their path prices can be verified in the same way.

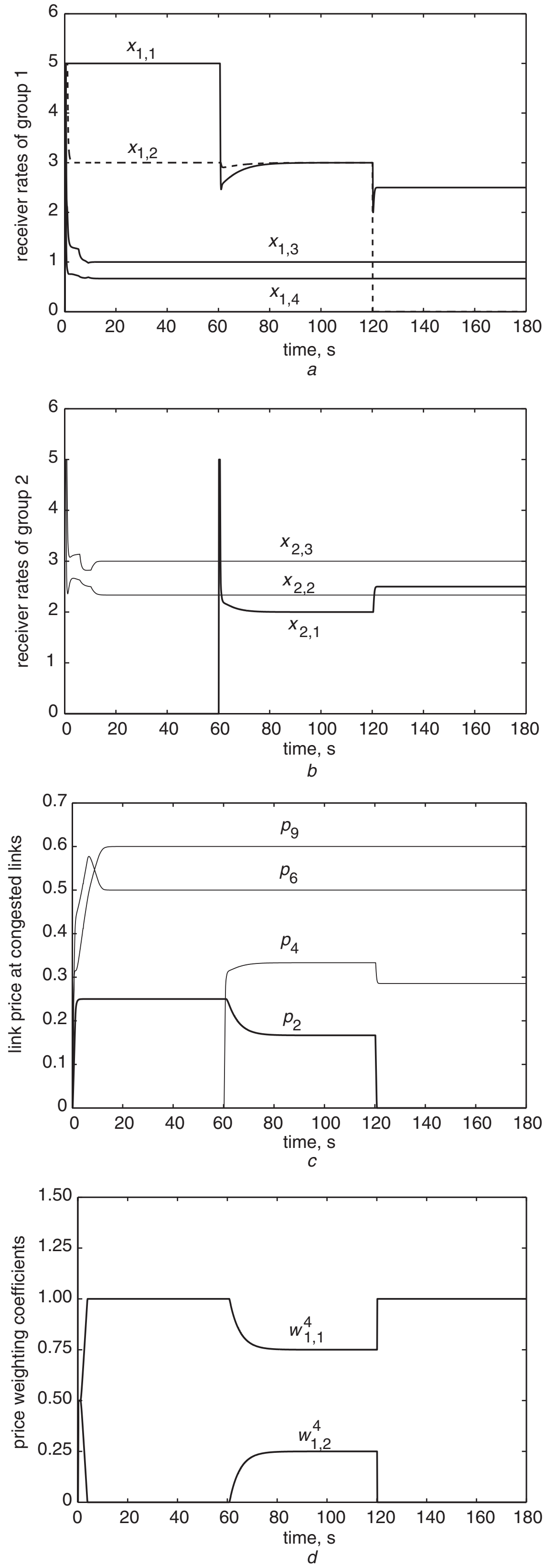

Fig. 2 *Simulation results of OFC algorithm, $\gamma=0.01$*

6 Conclusion

In this paper, we have investigated the optimal flow control problem in multirate multicast networks. We give the necessary and sufficient conditions for the optimal solution to the flow control problem. Now we know that the link price is only shared by its fastest downstream receivers in each group and the slow downstream receivers are not responsible for the link congestion. A better understanding between the multirate multicast transmission and the extensively studied unicast transmission is achieved. Therefore, some recently proposed techniques for unicast transmission, such as random exponential marking (REM) [22] (in which the sum of link prices is fed back to the source/receiver by marking the ECN (explicit congestion notification [23]) bit in the arrival packet with a probability that is exponentially increasing in current link price, and the source/receiver estimates the path price from the end-to-end ECN marking probability), can be used here directly to eliminate the communication problem of link prices from each link to the receivers. We further derive an optimal flow control algorithm for the multirate multicast networks. The algorithm is distributive and can easily be implemented in practical networks. For some special groups which are unicasting, the receiver algorithm reduces to the unicast algorithm we have studied before.

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