Branch Flow Model

relaxations, convexification

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Computing + Math Sciences

Electrical Engineering

Caltech

May 2012

Acks and refs

Collaborators

- S. Bose, M. Chandy, L. Gan, D. Gayme, J. Lavaei, L. Li

BFM reference

- Branch flow model: relaxations and convexification
  M. Farivar and S. H. Low
  arXiv:1204.4865v2, April 2012

Other references

- Zero duality gap in OPF problem
  J. Lavaei and S. H. Low
  IEEE Trans Power Systems, Feb 2012
- QCQP on acyclic graphs with application to power flow
  S. Bose, D. Gayme, S. H. Low and M. Chandy
  arXiv:1203.5599v1, March 2012

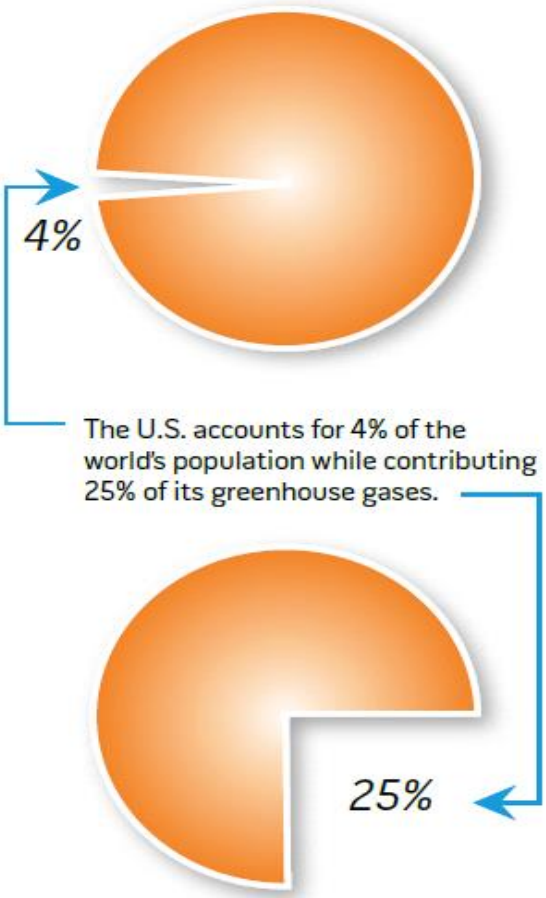
big picture

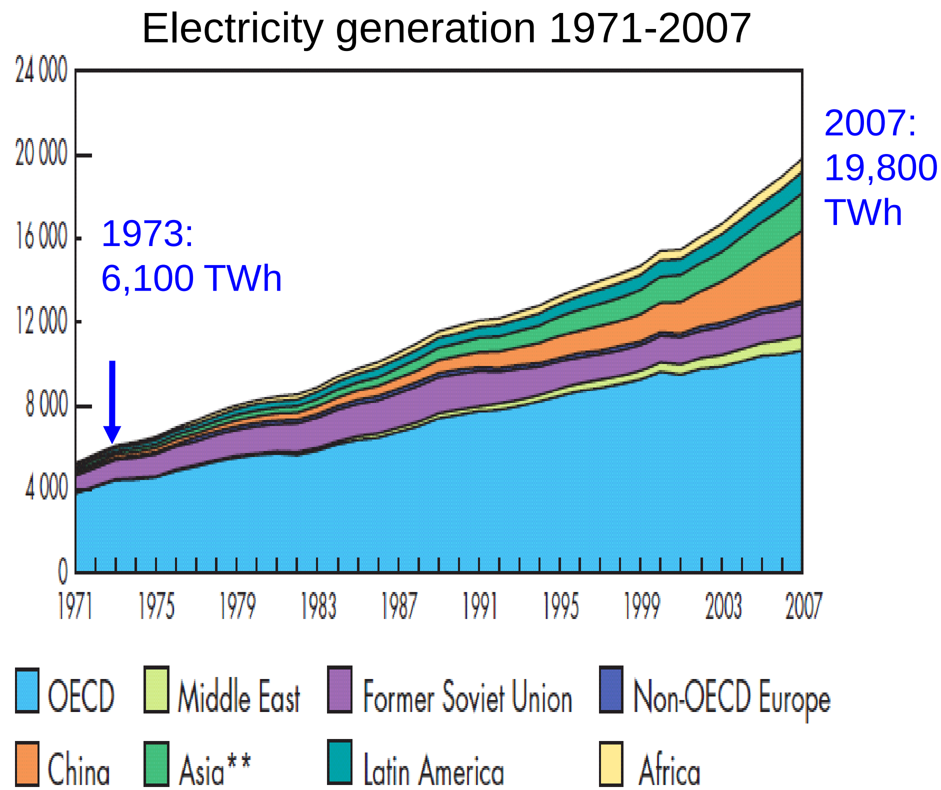
Global trends

1 Proliferation renewables

- Driven by sustainability
- Enabled by policy and investment

Sustainability challenge

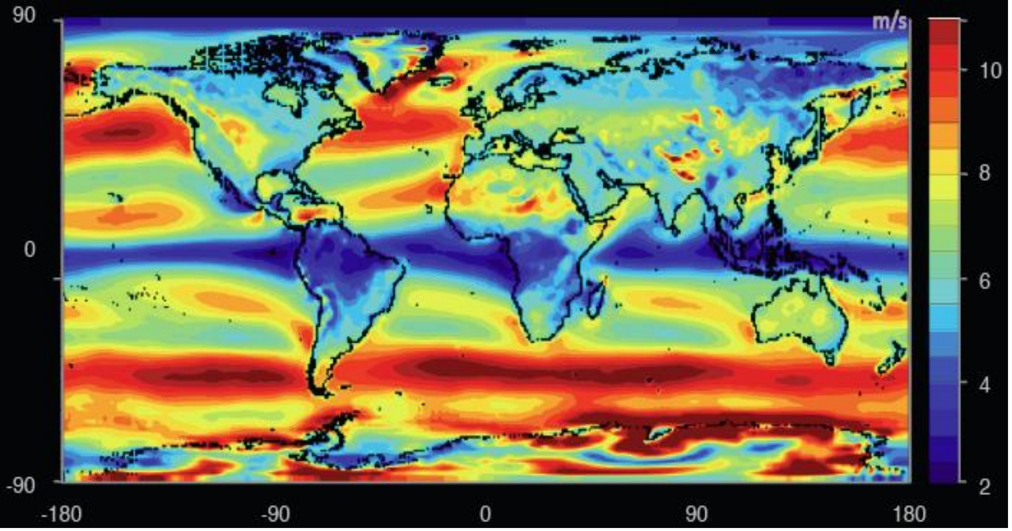




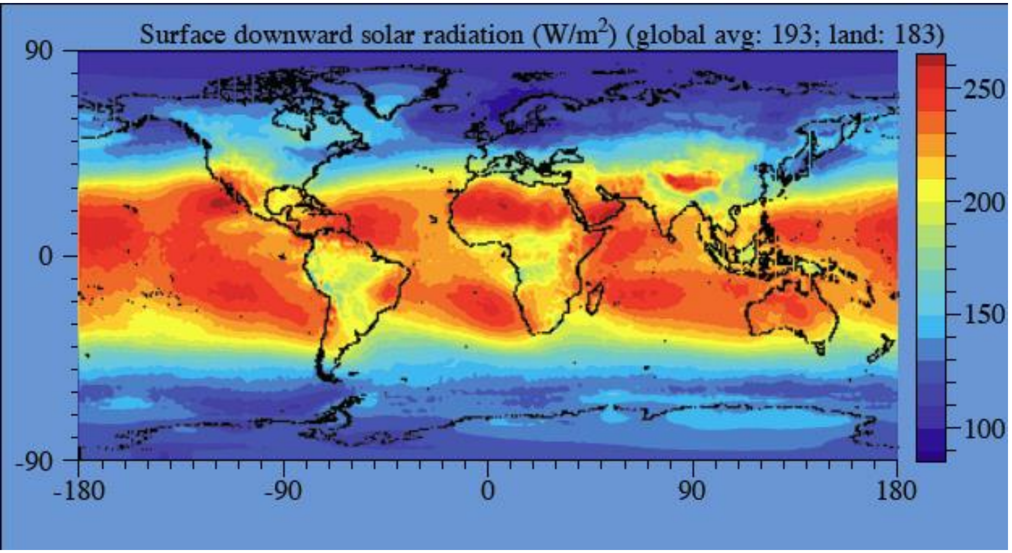
US CO_2 emission
- Elect generation: 40%
- Transportation: 20%

In 2009, 1.5B people have no electricity

Sources: International Energy Agency, 2009
DoE, Smart Grid Intro, 2008



Wind power over land (exc. Antartica)
70 – 170 TW



Solar power over land
340 TW

Worldwide

energy demand:
16 TW

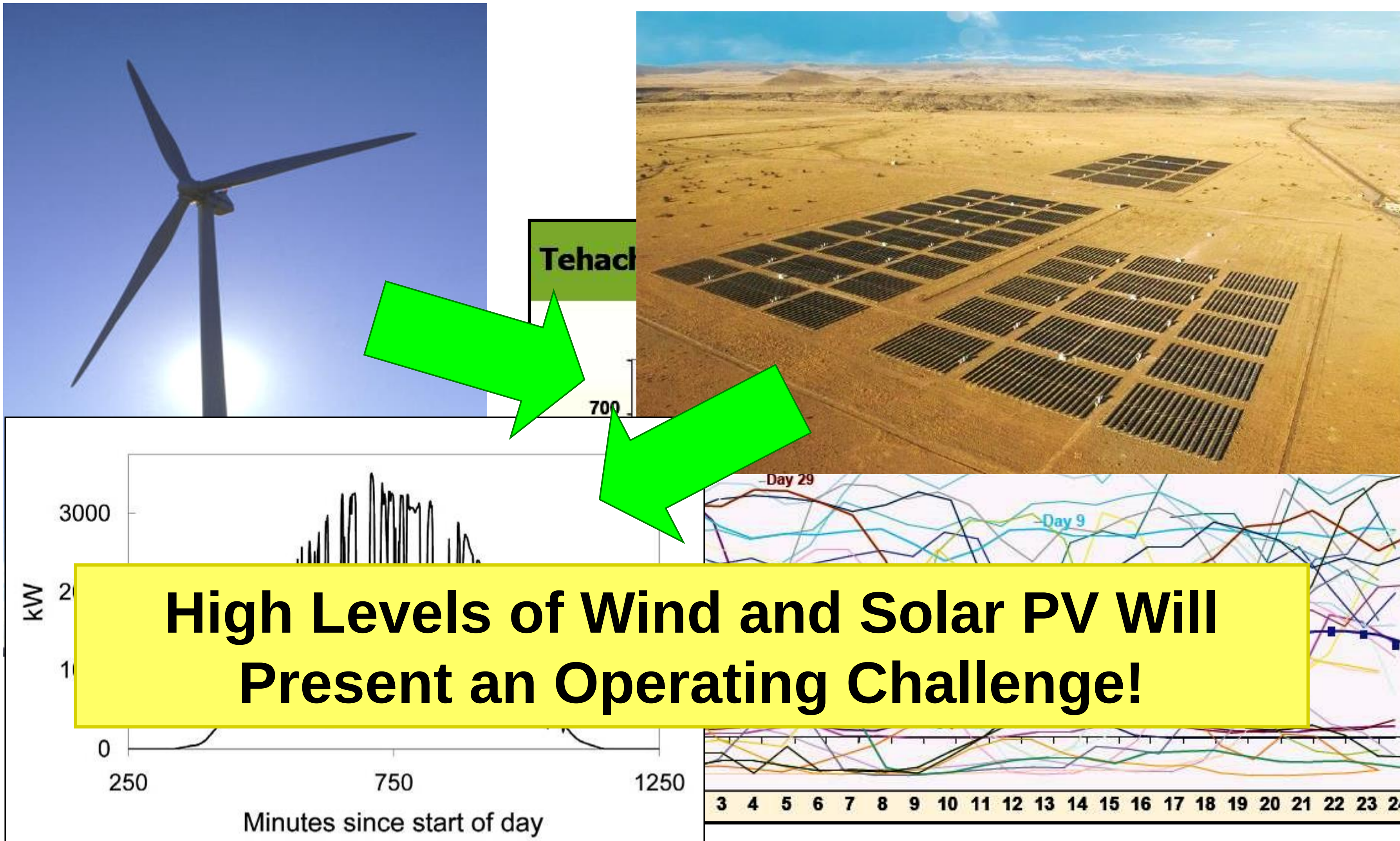
electricity demand:
2.2 TW

wind capacity (2009):
159 GW

grid-tied PV capacity (2009):
21 GW

Source: Renewable Energy
Global Status Report, 2010
Source: M. Jacobson, 2011

Uncertainty



High Levels of Wind and Solar PV Will Present an Operating Challenge!

Source: Rosa Yang, EPRI

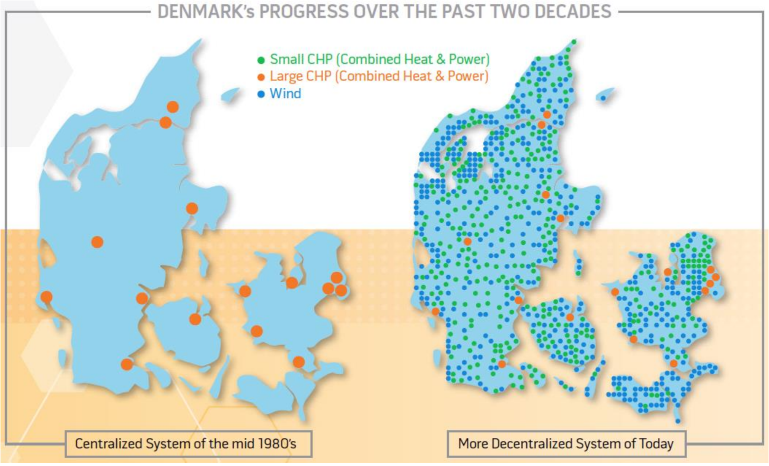
Global trends

1 Proliferation of renewables

- Driven by sustainability
- Enabled by policy and investment

2 Migration to distributed arch

- 2-3x generation efficiency
- Relief demand on grid capacity



Large active network of DER

	#nodes	capacity per node	total capacity	completion time	remarks
SCE	500	1 MW	500 MW	2015	SCE Commercial Rooftop Solar
CA	175,000	10 kW	1.75 GW	2016	CA Solar Initiative
SCE	400,000	2 kW	800 MW	--	10% penetration of SCE residential customers
CA	1,000,000	3 kW	3 GW	2017	CA Million Solar Roofs Initiative
CA	--	--	25 GW	2020	CA Renewable Portfolio Standard
US	--	--	3 TW	2035	Obama's goal for clean energy

DER: PVs, wind turbines, batteries, EVs, DR loads

Large active network of DER

	#nodes				remarks
SCE					Rooftop
CA					Portfolio
US	--		TW	2035	Obama's goal for clean energy

Millions of active endpoints introducing rapid large random fluctuations in supply and demand

DER: PVs, wind turbines, EVs, batteries, DR loads

Implications

Current control paradigm works well today

- Low uncertainty, few active assets to control
- Centralized, open-loop, human-in-loop, worst-case preventive
- Schedule supplies to match loads

Future needs

- Fast computation to cope with rapid, random, large fluctuations in supply, demand, voltage, freq
- Simple algorithms to scale to large networks of active DER
- Real-time data for adaptive control, e.g. real-time DR

Key challenges

Nonconvexity

- Convex relaxations

Large scale

- Distributed algorithms

Uncertainty

- Risk-limiting approach

Why is convexity important

Foundation of LMP

- Convexity justifies the use of Lagrange multipliers as various prices
- Critical for efficient market theory

Efficient computation

- Convexity delineates computational efficiency and intractability

A lot rides on (assumed) convexity structure

- engineering, economics, regulatory

optimal power flow

motivations

Optimal power flow (OPF)

OPF is solved routinely to determine

- How much power to generate where
- Market operation & pricing
- Parameter setting, e.g. taps, VARs

Non-convex and hard to solve

- Huge literature since 1962
- Common practice: DC power flow (LP)

Optimal power flow (OPF)

Problem formulation

- Carpentier 1962

Computational techniques

- Dommel & Tinney 1968
- Surveys: Huneault et al 1991, Momoh et al 2001, Pandya et al 2008

Bus injection model: SDP relaxation

- Bai et al 2008, 2009, Lavaei et al 2010, 2012
- Bose et al 2011, Zhang et al 2011, Sojoudi et al 2012
- Lesieutre et al 2011

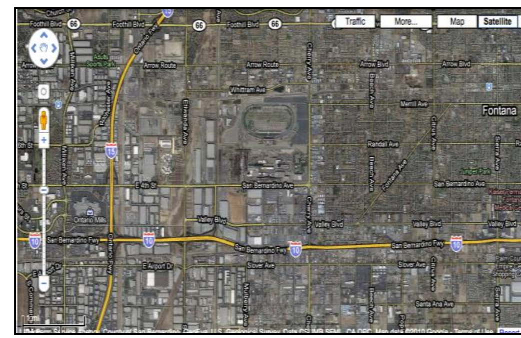
Branch flow model: SOCP relaxation

- Baran & Wu 1989, Chiang & Baran 1990, Taylor 2011, Farivar et al 2011

Application: Volt/VAR control

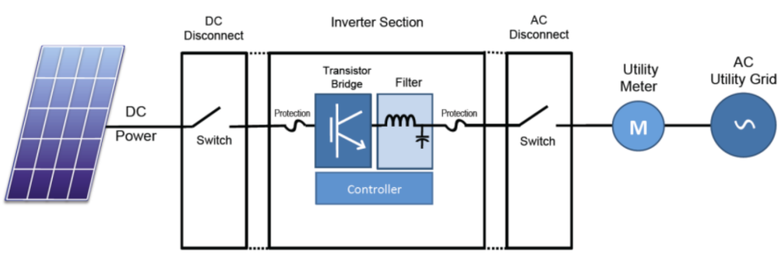
Motivation

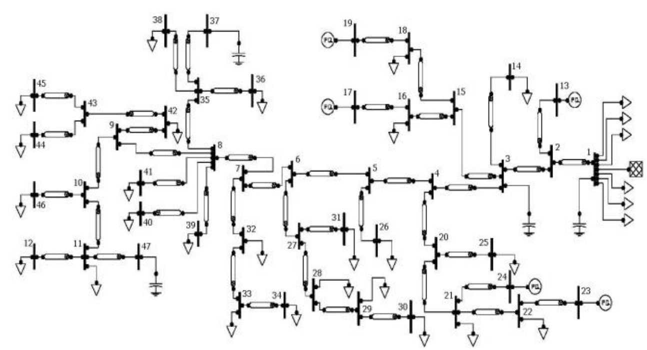
- Static capacitor control cannot cope with rapid random fluctuations of PVs on distr circuits



Inverter control

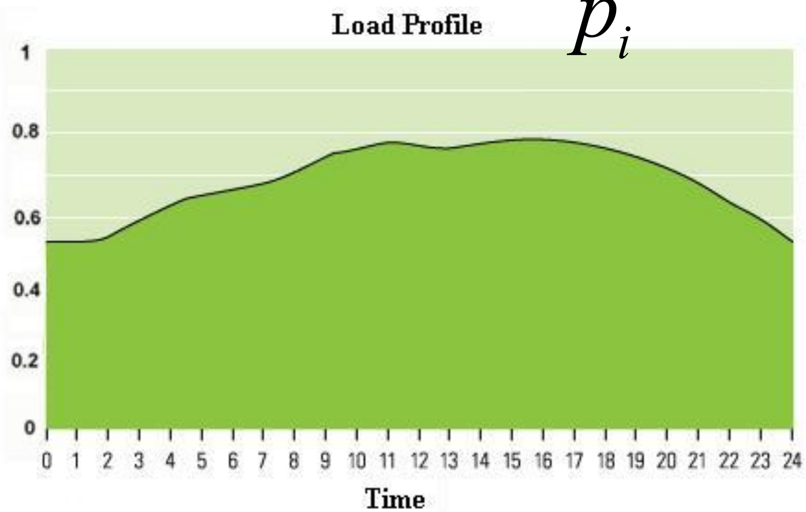
- Much faster & more frequent
- IEEE 1547 does not optimize VAR currently (unity PF)



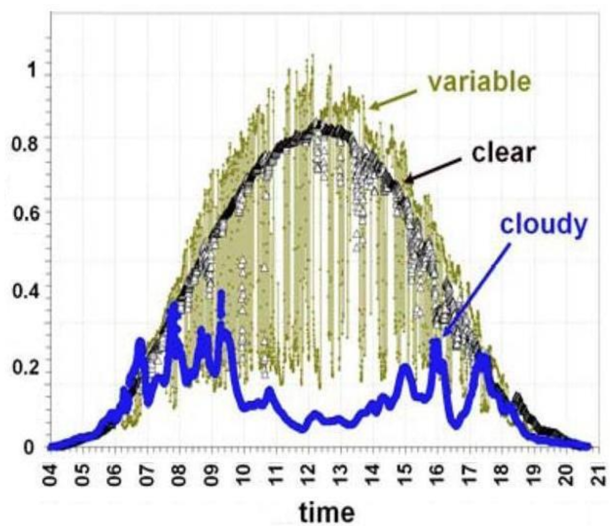


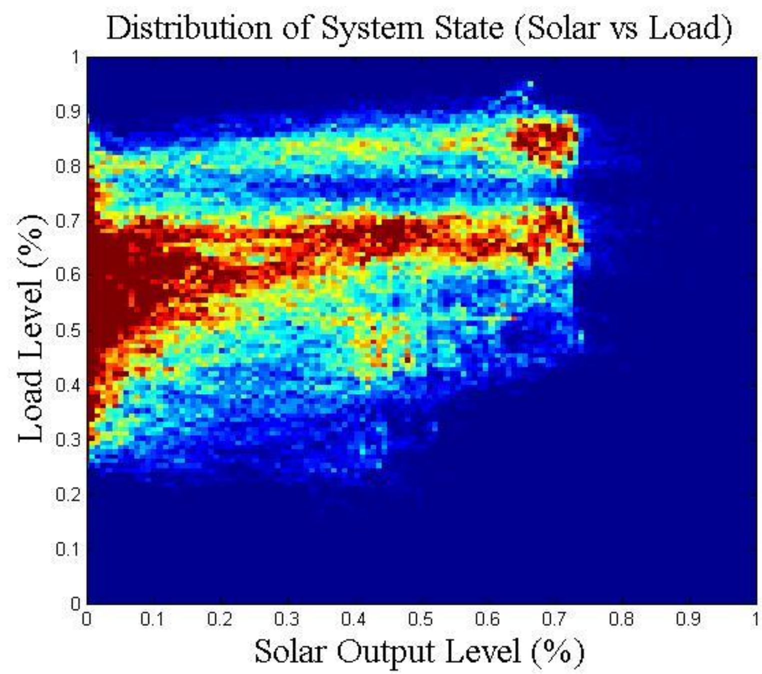
Load and Solar Variation

p_i^c



p_i^g





Empirical distribution
of (load, solar) for Calabash

Summary

RESULTS FOR SOME VOLTAGE TOLERANCE THRESHOLDS

Voltage Drop Tolerance(pu)	Annual Hours Saved Spending Outside Feasibility Region	Average Power Saving (%)
0.03	842.9	3.93%
0.04	160.7	3.67%
0.05	14.5	3.62%

- More reliable operation
- Energy savings

theory

relaxations and convexification

Outline

Branch flow model and OPF

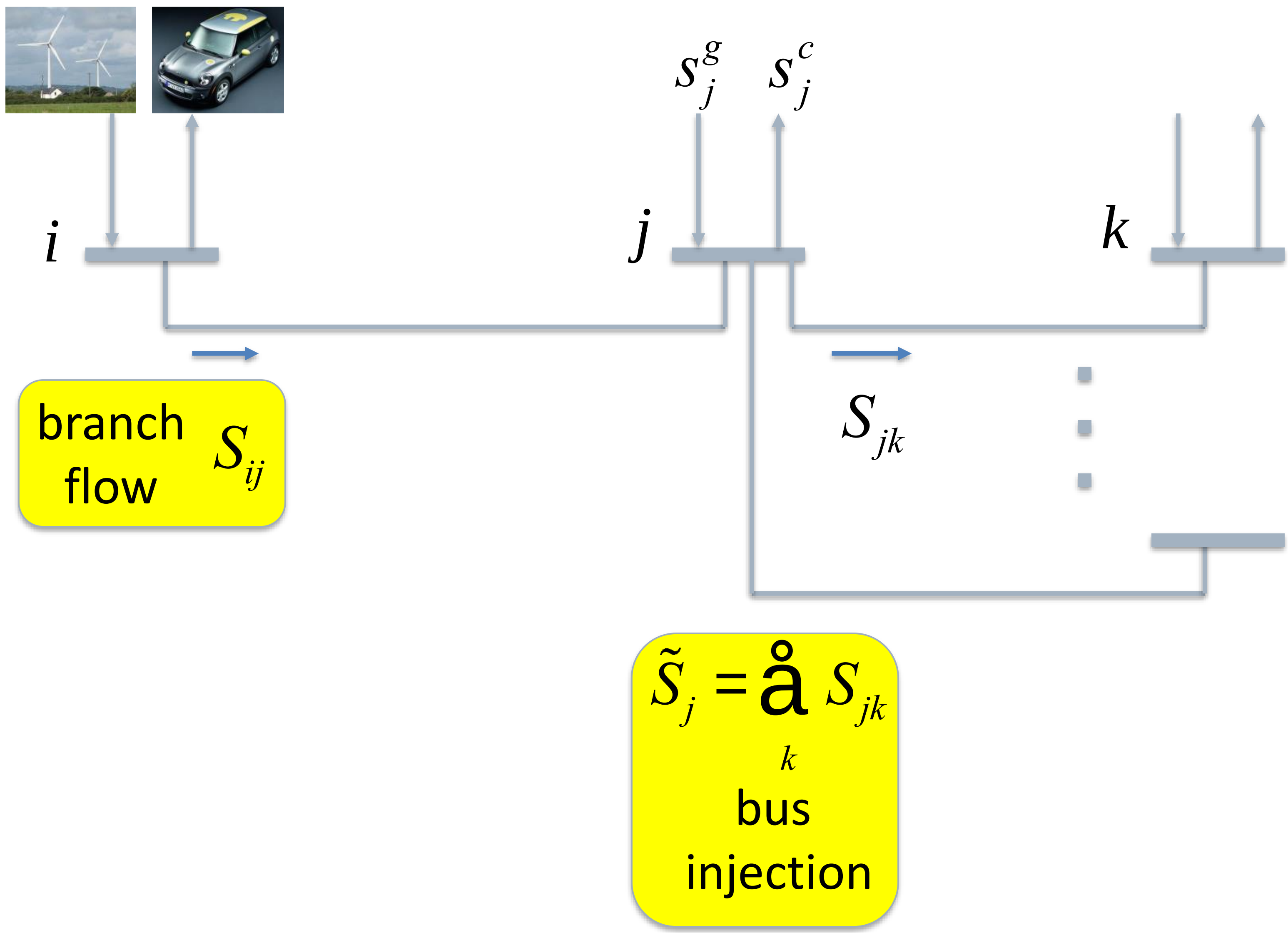
Solution strategy: two relaxations

- Angle relaxation
- SOCP relaxation

Convexification for mesh networks

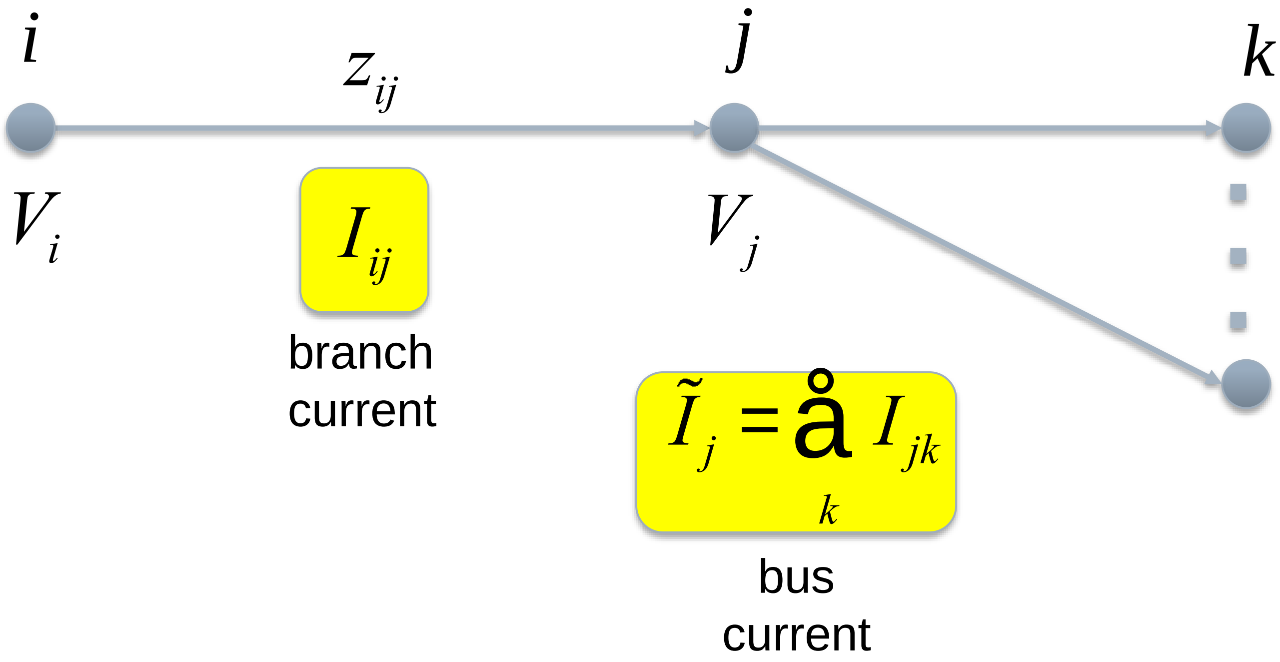
Extensions

Two models



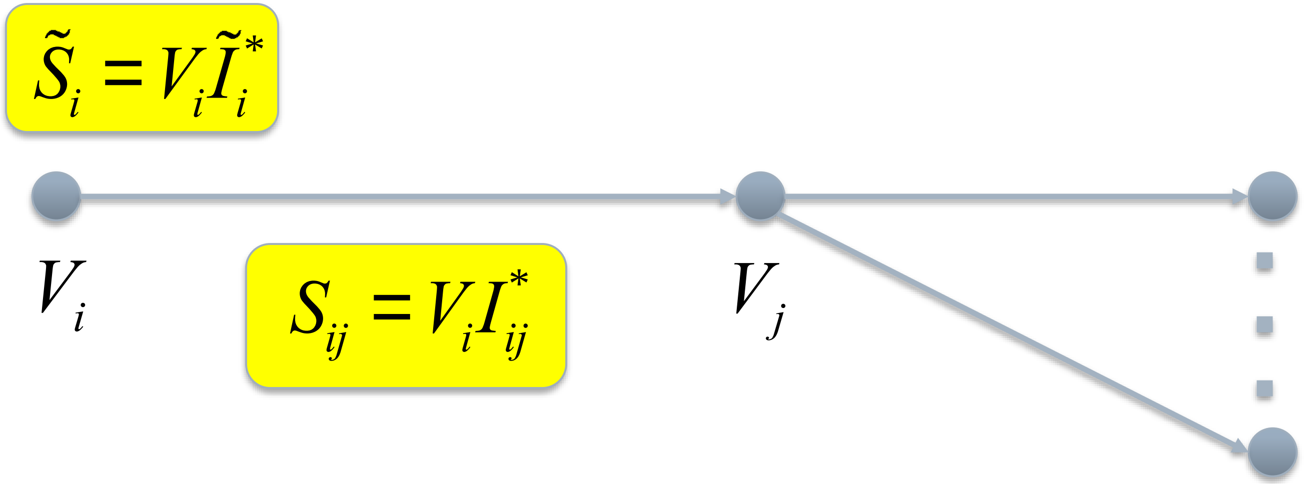
s_j^g
s_j^c
i
j
k
branch flow S_{ij}
S_{jk}
$\tilde{S}_j = \sum_k S_{jk}$
bus injection

Two models



i
z_{ij}
j
k
V_i
I_{ij}
branch current
V_j
$\tilde{I}_j = \sum_k I_{jk}$
bus current

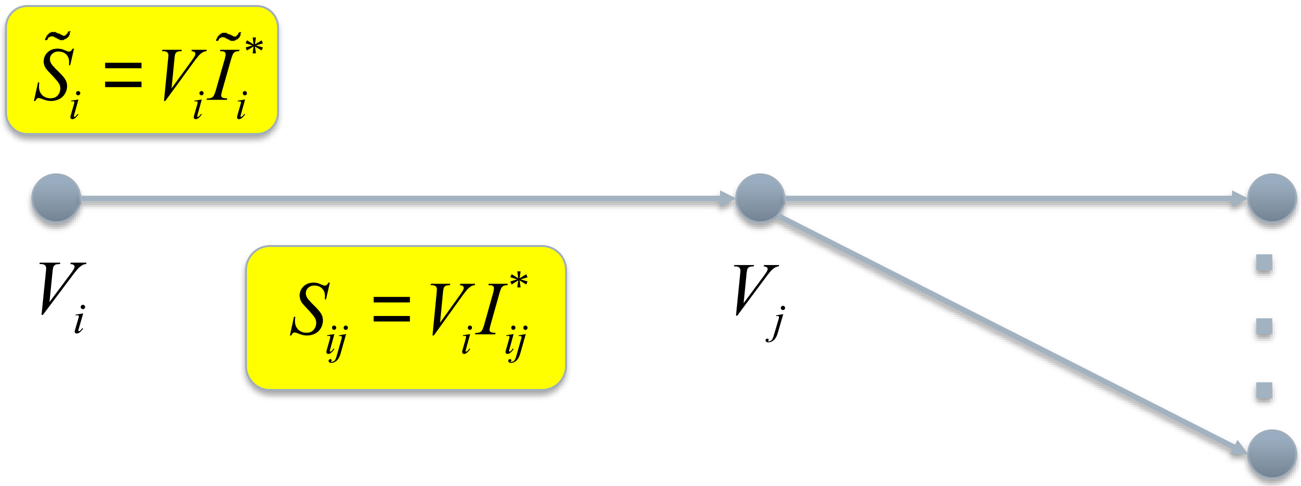
Two models



Equivalent models of Kirchhoff laws

- Bus injection model focuses on nodal vars
- Branch flow model focuses on branch vars

Two models



1. What is the model?
2. What is OPF in the model?
3. What is the solution strategy?

let’s start with something familiar

Bus injection model

$$\tilde{S}_j = V_j \tilde{I}_j^* \qquad \text{for all } j$$ power definition

$$\tilde{I} = YV$$ Kirchhoff law

$$\tilde{S}_j = -s_j \qquad \text{for all } j$$ power balance

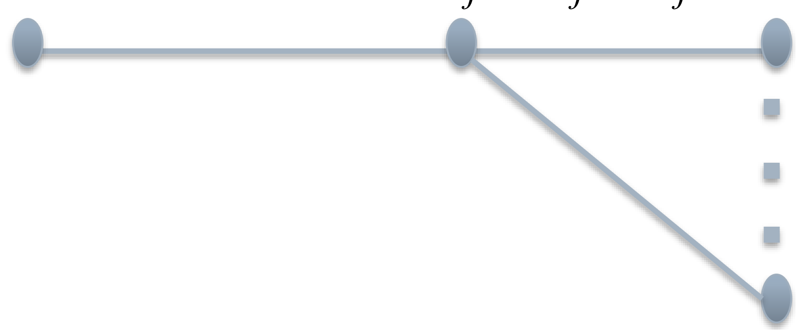
---

admittance matrix:

$$Y_{ij} := \begin{cases} \sum_{k \sim i} y_{ik} & \text{if } i = j \\ -y_{ij} & \text{if } i \sim j \\ 0 & \text{else} \end{cases}$$

$$\tilde{S}_j = V_j \tilde{I}_j^*$$

$$s_j = s_j^c - s_j^g$$



Bus injection model

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$$\tilde{I} = YV$$ Kirchhoff law

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---

$$\text{variables} \quad \tilde{x} := \left(\tilde{S}, \tilde{I}, V, s\right), \qquad s := s^c - s^g$$

Bus injection model: OPF

min $\quad \sum_j f_j\left(\mathrm{Re}\left(\tilde{S}_j\right)\right)$ — e.g. quadratic gen cost

over $\quad \tilde{x} := \left(\tilde{S}, \tilde{I}, V, s\right)$

subject to

Bus injection model: OPF

$$\min \quad \sum_j f_j\left(\mathrm{Re}\left(\tilde{S}_j\right)\right)$$

e.g. quadratic gen cost

$$\text{over} \quad \tilde{x} := \left(\tilde{S}, \tilde{I}, V, s\right)$$

$$\text{subject to} \quad \underline{s}_j \le s_j \le \overline{s}_j \qquad \underline{V}_k \le |V_k| \le \overline{V}_k$$

Bus injection model: OPF

$$\min \quad \sum_j f_j\left(\mathrm{Re}\left(\tilde{S}_j\right)\right)$$

e.g. quadratic gen cost

$$\text{over} \quad \tilde{x} := \left(\tilde{S}, \tilde{I}, V, s\right)$$

$$\text{subject to} \quad \underline{s}_j \le s_j \le \bar{s}_j \qquad \underline{V}_k \le |V_k| \le \overline{V}_k$$

$$\tilde{I} = YV$$

$$\tilde{S}_j = -s_j \qquad \tilde{S}_j = V_j \tilde{I}_j^*$$

Kirchhoff law

power balance

nonconvex, NP-hard

Bus injection model: relaxation

$$
\begin{aligned}
\min \quad & \operatorname{tr} C_0 W \\
\text{over} \quad & W \text{ matrices} \\
\text{s.t.} \quad & \operatorname{tr} C_k W \le b_k \\
& W \succeq 0 \\
& \cancel{\operatorname{rank} W = 1}
\end{aligned}
$$

convex relaxation: SDP

polynomial

Bus injection model: SDR

Non-convex QCQP

↓

Rank-constrained SDP

↓

Relax the rank constraint and solve the SDP Bai 2008

↓

Does the optimal solution satisfy the rank-constraint?

yes → We are done!

Lavaei 2010, 2012
Radial: Bose 2011, Zhang 2011
Sojoudi 2011

no → Solution may not be meaningful

Lesiertre 2011

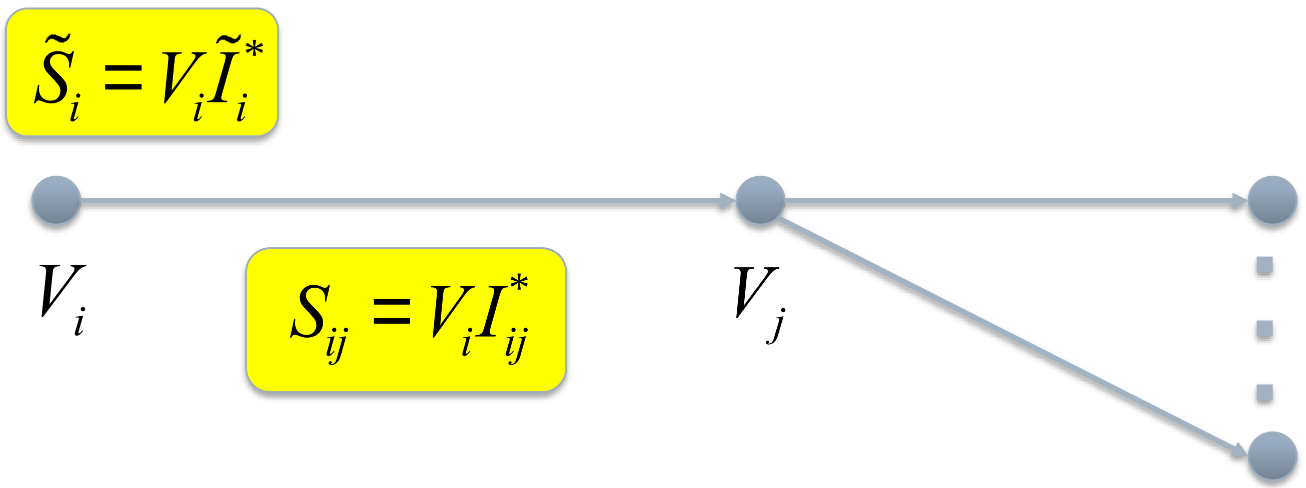
Bus injection model: summary

OPF = rank constrained SDP

Sufficient conditions for SDP to be exact

- Whether a solution is globally optimal is always easily checkable
- Mesh: must solve SDP to check
- Tree: depends only on constraint pattern or r/x ratios

Two models



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Branch flow model

$$S_{ij} = V_i I_{ij}^* \qquad \text{for all } i \to j \qquad \text{power def}$$

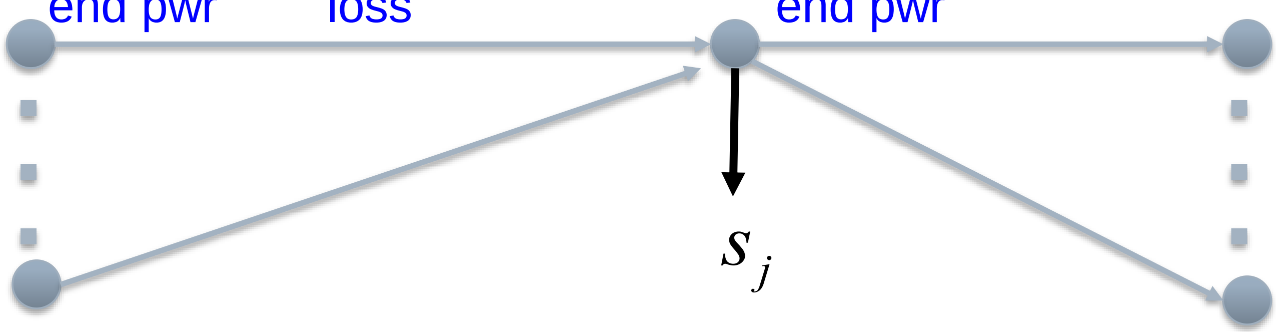
$$V_i - V_j = z_{ij} I_{ij} \qquad \text{for all } i \to j \qquad \text{Ohm's law}$$

$$\sum_{i \to j} \left(S_{ij} - z_{ij} \left| I_{ij} \right|^2 \right) - \sum_{j \to k} S_{jk} = s_j \qquad \text{for all } j \qquad \text{power balance}$$

sending end pwr

loss

sending end pwr



Branch flow model

$$S_{ij} = V_i I_{ij}^* \qquad \text{for all } i \rightarrow j$$ power def

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$$\sum_{i \rightarrow j} \left(S_{ij} - z_{ij} \left| I_{ij} \right|^2 \right) - \sum_{j \rightarrow k} S_{jk} = s_j \qquad \text{for all } j$$ power balance

---

variables $x := (S, I, V, s), \qquad s := s^c - s^g$

branch flows

Branch flow model

$$S_{ij} = V_i I_{ij}^* \qquad \text{for all } i \to j$$ power def

$$V_i - V_j = z_{ij} I_{ij} \qquad \text{for all } i \to j$$ Ohm's law

$$\sum_{i \to j} \left(S_{ij} - z_{ij} \left| I_{ij} \right|^2 \right) - \sum_{j \to k} S_{jk} = s_j \qquad \text{for all } j$$ power balance

---

variables $x := \left(S, I, V, s\right), \qquad s := s^c - s^g$

projection $\hat{y} := h(x) := \left(S, \ell, v, s\right)$

Branch flow model: OPF

$$\min \quad \sum_{i\sim j} r_{ij}\left|I_{ij}\right|^2 + \sum_{i} a_i \left|V_i\right|^2$$

real power loss

CVR (conservation voltage reduction)

Branch flow model: OPF

$$\min \quad f\big(h(x)\big)$$

$$\text{over} \quad x := (S, I, V, s^g, s^c)$$

s. t.

Branch flow model: OPF

$$\min \quad f\big(h(x)\big)$$

$$\text{over} \quad x := (S, I, V, s^g, s^c)$$

$$\text{s. t.} \quad \underline{s}_i^g \pounds s_i^g \pounds \overline{s}_i^g \qquad \underline{s}_i \pounds s_i^c \qquad \underline{v}_i \pounds v_i \pounds \overline{v}_i$$

Branch flow model: OPF

$$\min \quad f\left(h(x)\right)$$

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generation, VAR control

branch flow model

$$\sum_{i \to j} \left(S_{ij} - z_{ij} \left| I_{ij} \right|^2 \right) - \sum_{j \to k} S_{jk} = s_j^c - s_j^g$$

$$V_j = V_i - z_{ij} I_{ij} \qquad S_{ij} = V_i I_{ij}^*$$

Branch flow model: OPF

$$\min \quad f\big(h(x)\big)$$

$$\text{over} \quad x := (S, I, V, s^g, s^c)$$

$$\text{s. t.} \quad \underline{s}_i^g \le s_i^g \le \overline{s}_i^g \qquad \underline{s}_i \le s_i^c \qquad \underline{v}_i \le v_i \le \overline{v}_i$$

demand response

branch flow model

$$\sum_{i \to j} \left(S_{ij} - z_{ij} |I_{ij}|^2 \right) - \sum_{j \to k} S_{jk} = s_j^c - s_j^g$$

$$V_j = V_i - z_{ij} I_{ij} \qquad S_{ij} = V_i I_{ij}^*$$

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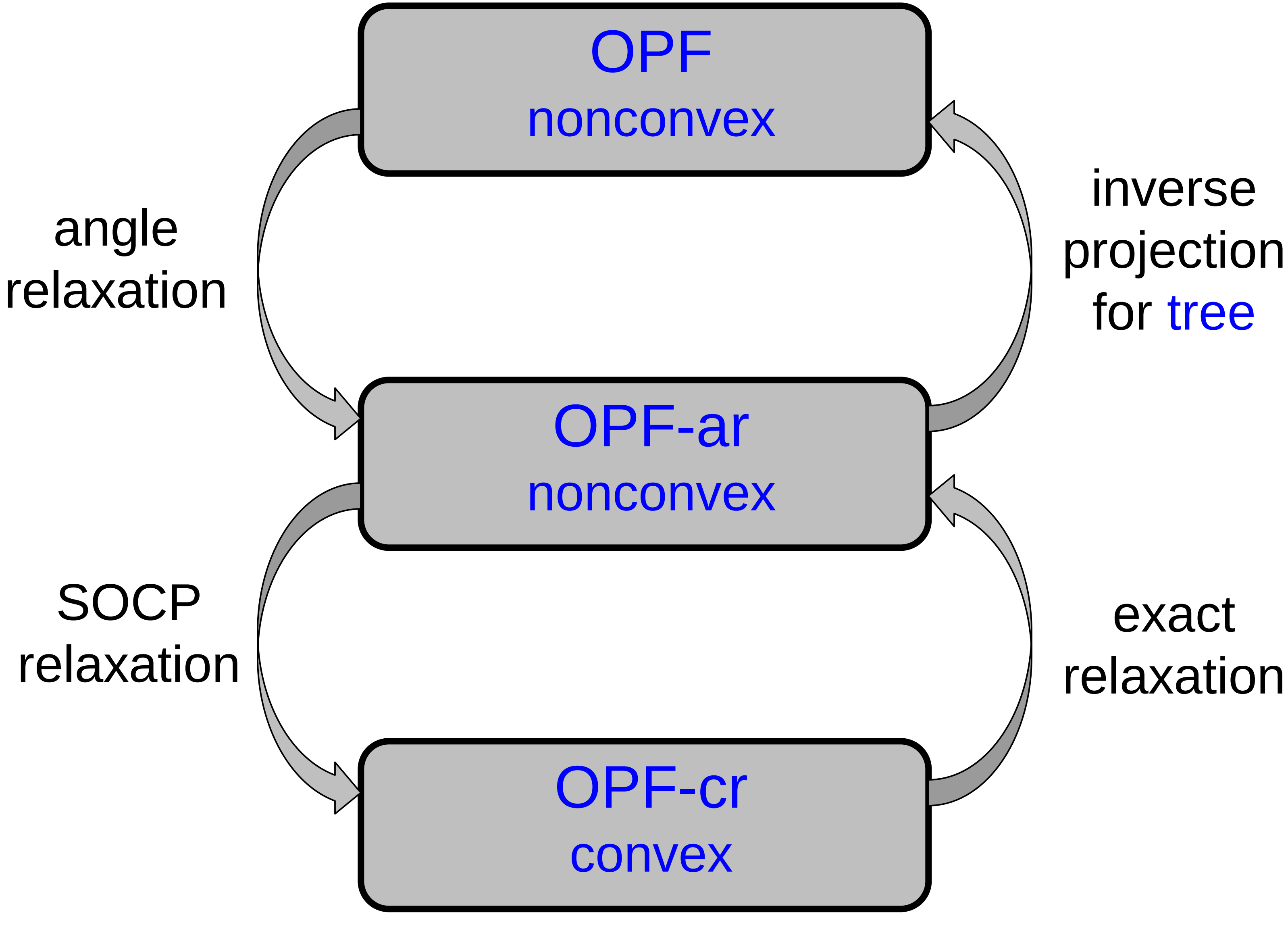
- Angle relaxation
- SOCP relaxation

Convexification for mesh networks

Extensions

Solution strategy



Angle relaxation

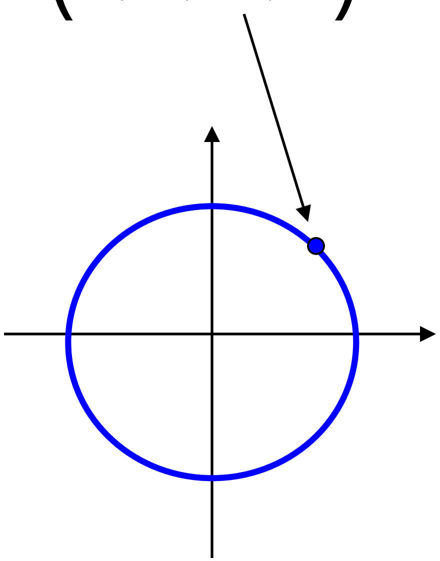
branch flow model

$$\sum_{i \to j} \left(S_{ij} - z_{ij} \left| I_{ij} \right|^2 \right) - \sum_{j \to k} S_{jk} = s_j^c - s_j^g$$

$$V_j = V_i - z_{ij} I_{ij} \qquad S_{ij} = V_i I_{ij}^*$$

Angle relaxation

$$\left(S, I, V, s\right)$$



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Angle relaxation

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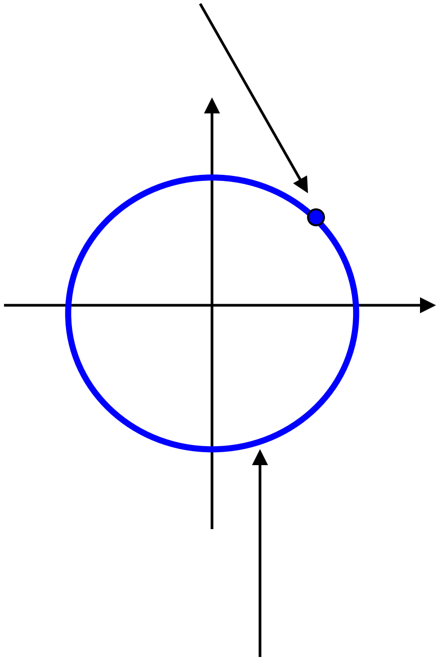
$$V_j = V_i - z_{ij} I_{ij} \qquad S_{ij} = V_i I_{ij}^*$$

(S, I, V, s)



(S, ℓ, v, s)

$$|V_i|^2 = |V_j|^2 + 2\,\mathrm{Re}\left(z_{ij}^* S_{ij}\right) - |z_{ij}|^2 |I_{ij}|^2$$

$$|I_{ij}|^2 = \frac{|S_{ij}|^2}{|V_i|^2}$$

$$\ell_{ij} := |I_{ij}|^2$$

$$v_i := |V_i|^2$$

Relaxed BF model

relaxed branch flow solutions: (S, ℓ, v, s) satisfy

$$\sum_{i \to j} \left(S_{ij} - z_{ij}\ell_{ij}\right) - \sum_{j \to k} S_{jk} = s_j^c - s_j^g$$

$$v_i = v_j + 2\,\mathrm{Re}\left(z_{ij}^* S_{ij}\right) - \left|z_{ij}\right|^2 \ell_{ij}$$

$$\ell_{ij} = \frac{\left|S_{ij}\right|^2}{v_i}$$

Baran and Wu 1989
for radial networks

OPF

min $\quad f\big(h(x)\big)$

over $\quad x := (S, I, V, s^g, s^c)$

s. t. $\quad \underline{s}_i^g \text{ £ } s_i^g \text{ £ } \overline{s}_i^g \qquad \underline{s}_i \text{ £ } s_i^c \qquad \underline{v}_i \text{ £ } v_i \text{ £ } \overline{v}_i$

branch flow model

$$\sum_{i \to j} \left(S_{ij} - z_{ij} |I_{ij}|^2 \right) - \sum_{j \to k} S_{jk} = s_j^c - s_j^g$$

$$V_j = V_i - z_{ij} I_{ij} \qquad S_{ij} = V_i I_{ij}^*$$

OPF

$$\min \quad f\big(h(x)\big)$$

$$\text{over} \quad x := (S, I, V, s^g, s^c)$$

$$\text{s. t.} \quad \underline{s}_i^g \le s_i^g \le \overline{s}_i^g \qquad \underline{s}_i \le s_i^c \qquad \underline{v}_i \le v_i \le \overline{v}_i$$

$$x \in \mathbf{X}$$

$$\mathbf{X}$$

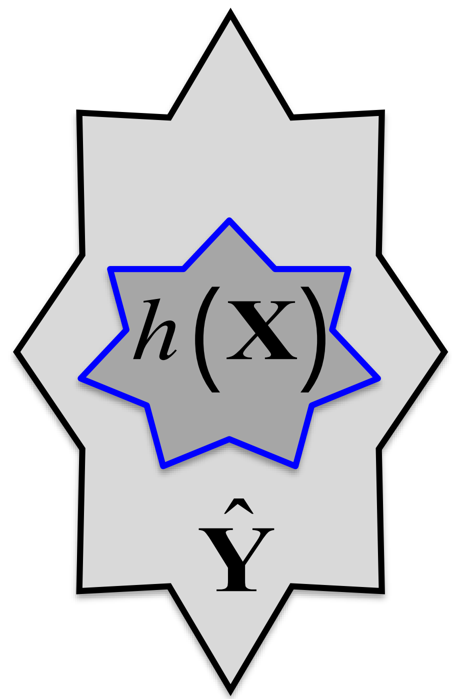
OPF-ar

min $\quad f(\hat{y})$

over $\quad \hat{y} := (S, \ell, v, s^g, s^c)$

s. t. $\quad \underline{s}_i^g \le s_i^g \le \overline{s}_i^g \qquad \underline{s}_i \le s_i^c \qquad \underline{v}_i \le v_i \le \overline{v}_i$

$$\hat{y} := h(x) \in \hat{\mathbf{Y}}$$



relax each voltage/current from a point in complex plane into a circle

OPF-ar

$$\min \quad f(\hat{y})$$

$$\text{over} \quad \hat{y} := (S, \ell, v, s^g, s^c)$$

$$\text{s. t.} \quad \underline{s}_i^g \le s_i^g \le \overline{s}_i^g \qquad \underline{s}_i \le s_i^c \qquad \underline{v}_i \le v_i \le \overline{v}_i$$

$$\sum_{i \to j} \left(S_{ij} - z_{ij} \ell_{ij} \right) - \sum_{j \to k} S_{jk} = s_j^c - s_j^g$$

$$v_i = v_j + 2 \operatorname{Re}\left(z_{ij}^* S_{ij} \right) - \left| z_{ij} \right|^2 \ell_{ij}$$

source of nonconvexity

$$\ell_{ij} = \frac{\left| S_{ij} \right|^2}{v_i}$$

- convex objective
- linear constraints
- quadratic equality

OPF-cr

$$\min \quad f(\hat{y})$$

$$\text{over} \quad \hat{y} := (S, \ell, v, s^g, s^c)$$

$$\text{s. t.} \quad \underline{s}_i^g \pounds s_i^g \pounds \overline{s}_i^g \qquad \underline{s}_i \pounds s_i^c \qquad \underline{v}_i \pounds v_i \pounds \overline{v}_i$$

$$\sum_{i \to j} \left(S_{ij} - z_{ij} \ell_{ij} \right) - \sum_{j \to k} S_{jk} = s_j^c - s_j^g$$

$$v_i = v_j + 2\ \mathrm{Re}\left(z_{ij}^* S_{ij} \right) - \left| z_{ij} \right|^2 \ell_{ij}$$

$$\ell_{ij}\ 3\ \frac{\left| S_{ij} \right|^2}{v_i}$$

→ <u>in</u>equality

OPF-cr

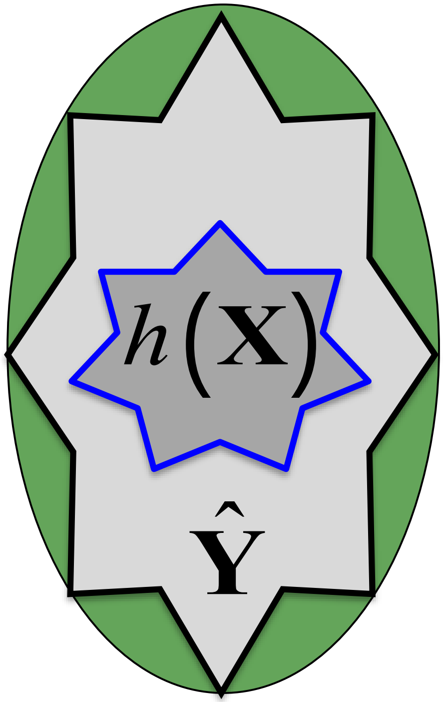
$$\min \quad f(\hat{y})$$

$$\text{over} \quad \hat{y} := (S, \ell, v, s^g, s^c)$$

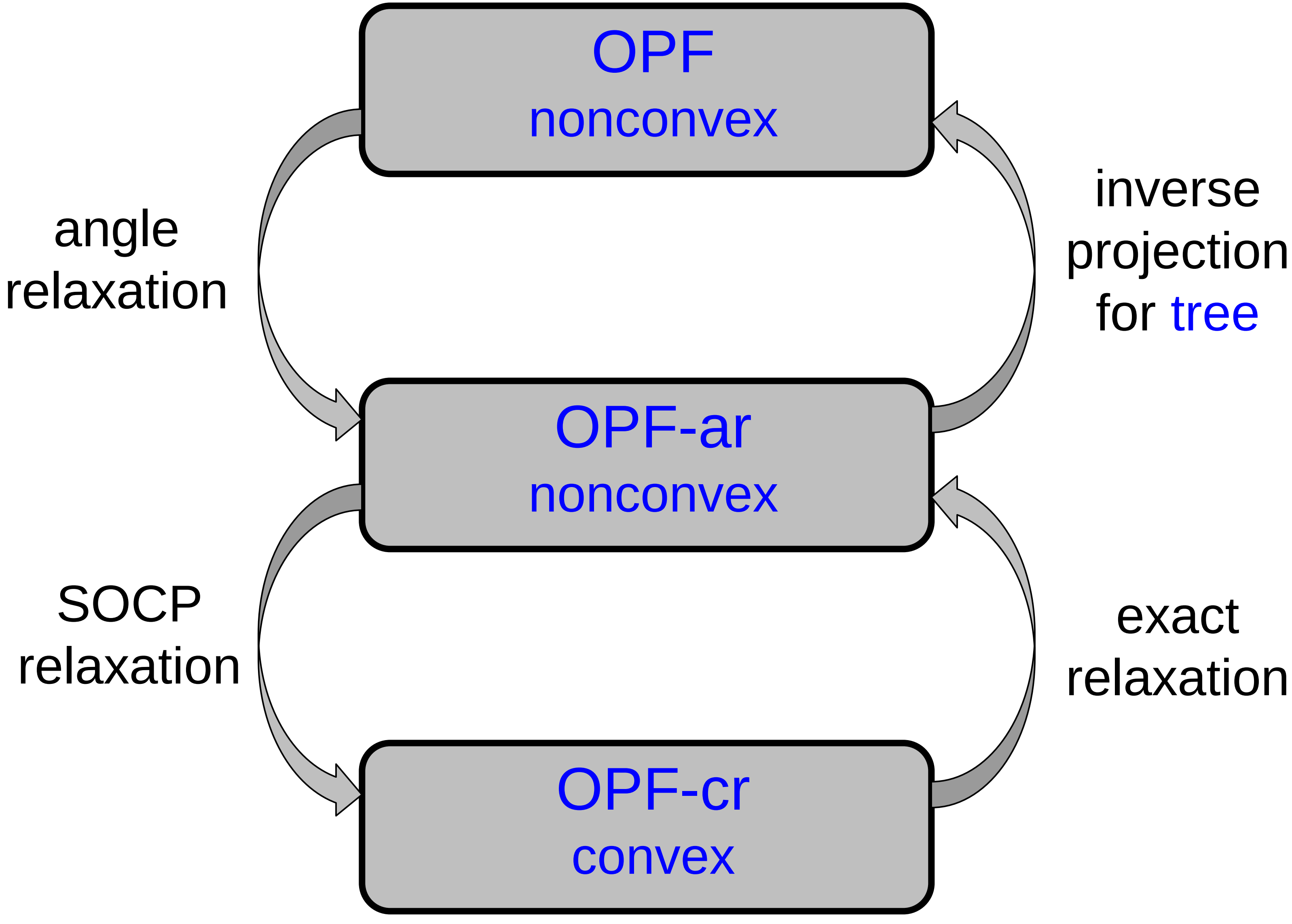
$$\text{s. t.} \quad \underline{s}_i^g \le s_i^g \le \overline{s}_i^g \qquad \underline{s}_i \le s_i^c \qquad \underline{v}_i \le v_i \le \overline{v}_i$$

$$\hat{y} \in \text{conv}\ \hat{\mathbf{Y}}$$

relax to convex hull
(SOCP)



Recap so far …


OPF
nonconvex
angle relaxation
inverse projection for tree
OPF-ar
nonconvex
SOCP relaxation
exact relaxation
OPF-cr
convex

OPF-cr is exact relaxation

<u>Theorem</u>

OPF-cr is convex

- SOCP when objective is linear

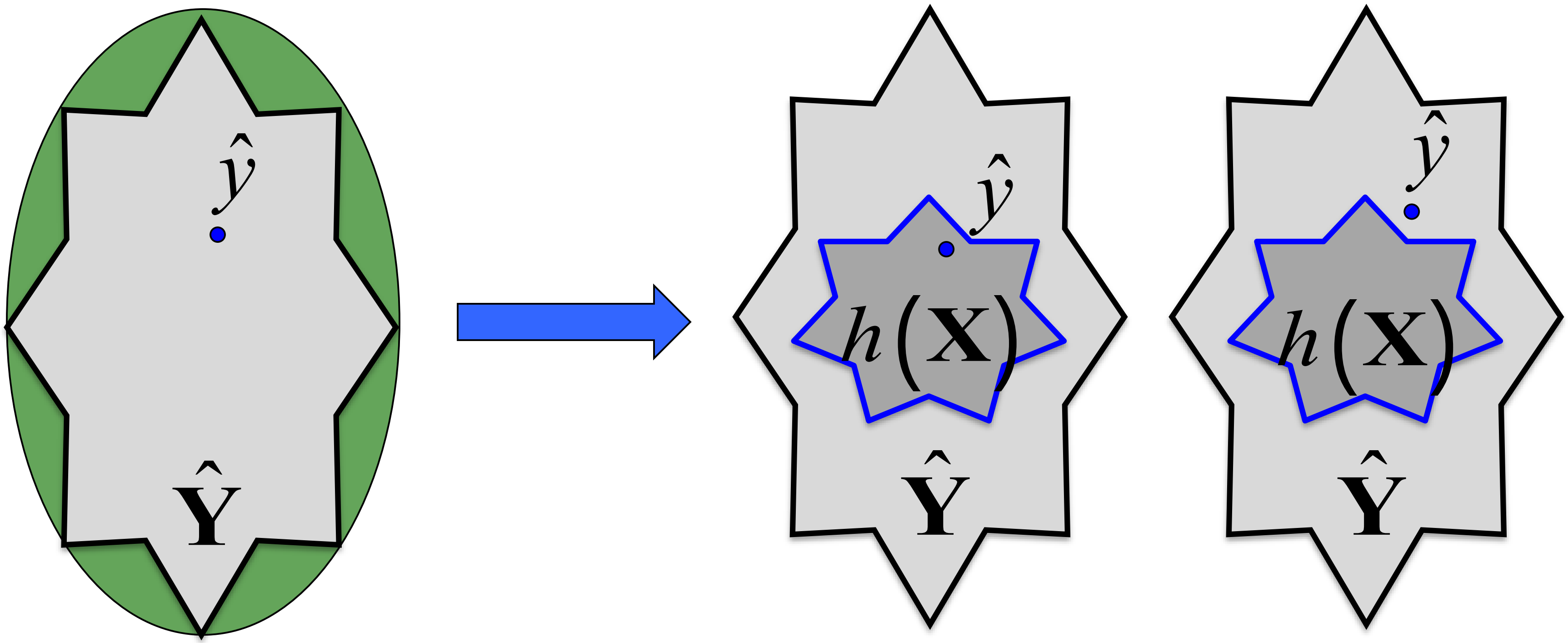
$$f\left(h(x)\right) := \sum_{i\sim j} r_{ij} l_{ij} + \sum_{i} \alpha_i v_i$$

- SOCP much simpler than SDP

OPF-cr is exact

- optimal of OPF-cr is also optimal for OPF-ar
- for mesh as well as radial networks
- real & reactive powers, but volt/current mags

Angle recovery



OPF-ar

does there exist q s.t.

$h_q^{-1}(\hat{y}) \in \mathbf{X}$?

Angle recovery

Theorem

solution x to OPF recoverable from $\hat{y}$ iff inverse projection exist iff $\$!q$ s.t.

$$Bq = b(\hat{y})$$

incidence matrix; depends on topology

depends on OPF-ar solution

Two simple angle recovery algorithms

- centralized: explicit formula
- decentralized: recursive alg

Angle recovery

Theorem

For radial network: $\$! q$

$$Bq = b(\hat{y})$$

Angle recovery

#buses - 1

#lines in T

#lines outside T

$$\begin{bmatrix} B_T \\ B_\perp \end{bmatrix} \theta = \begin{bmatrix} \beta_T \\ \beta_\perp \end{bmatrix}$$

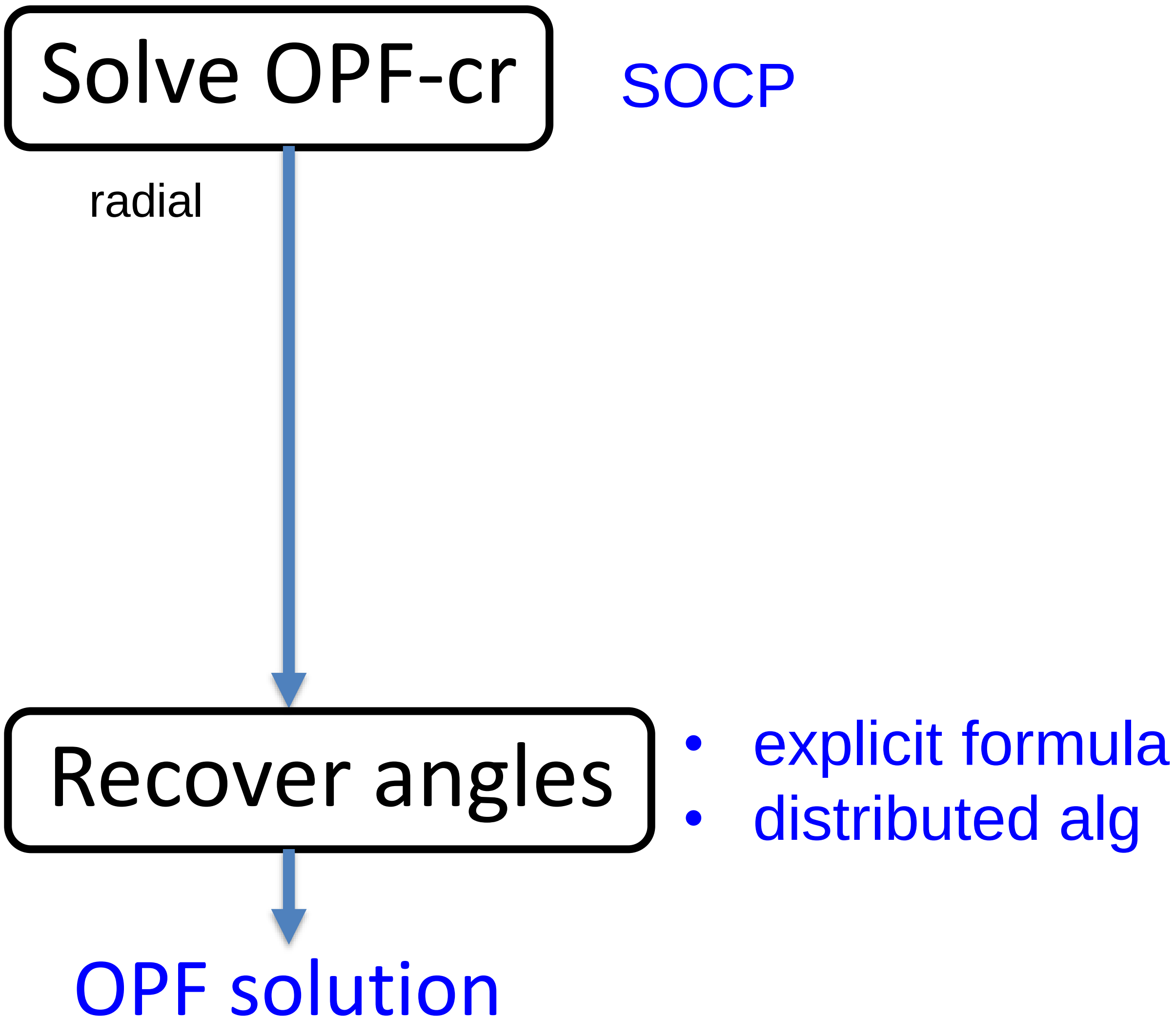
Theorem

Inverse projection exist iff $B_\perp \left(B_T^{-1} \beta_T \right) = \beta_\perp$

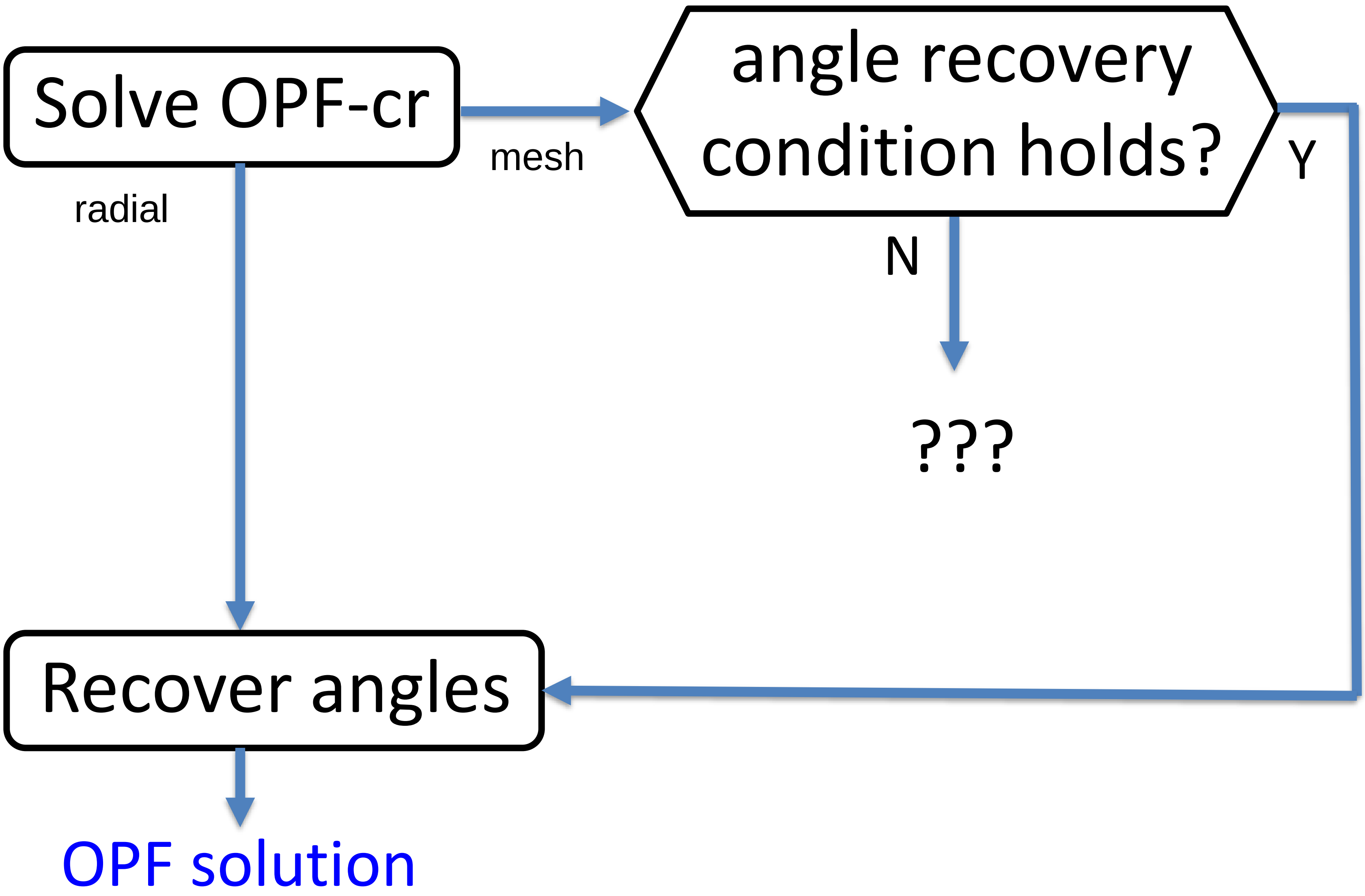
Unique inverse given by $\theta^* = B_T^{-1} \beta_T$

For radial network: $B_\perp = \beta_\perp = 0$

OPF solution



OPF solution



Outline

Branch flow model and OPF

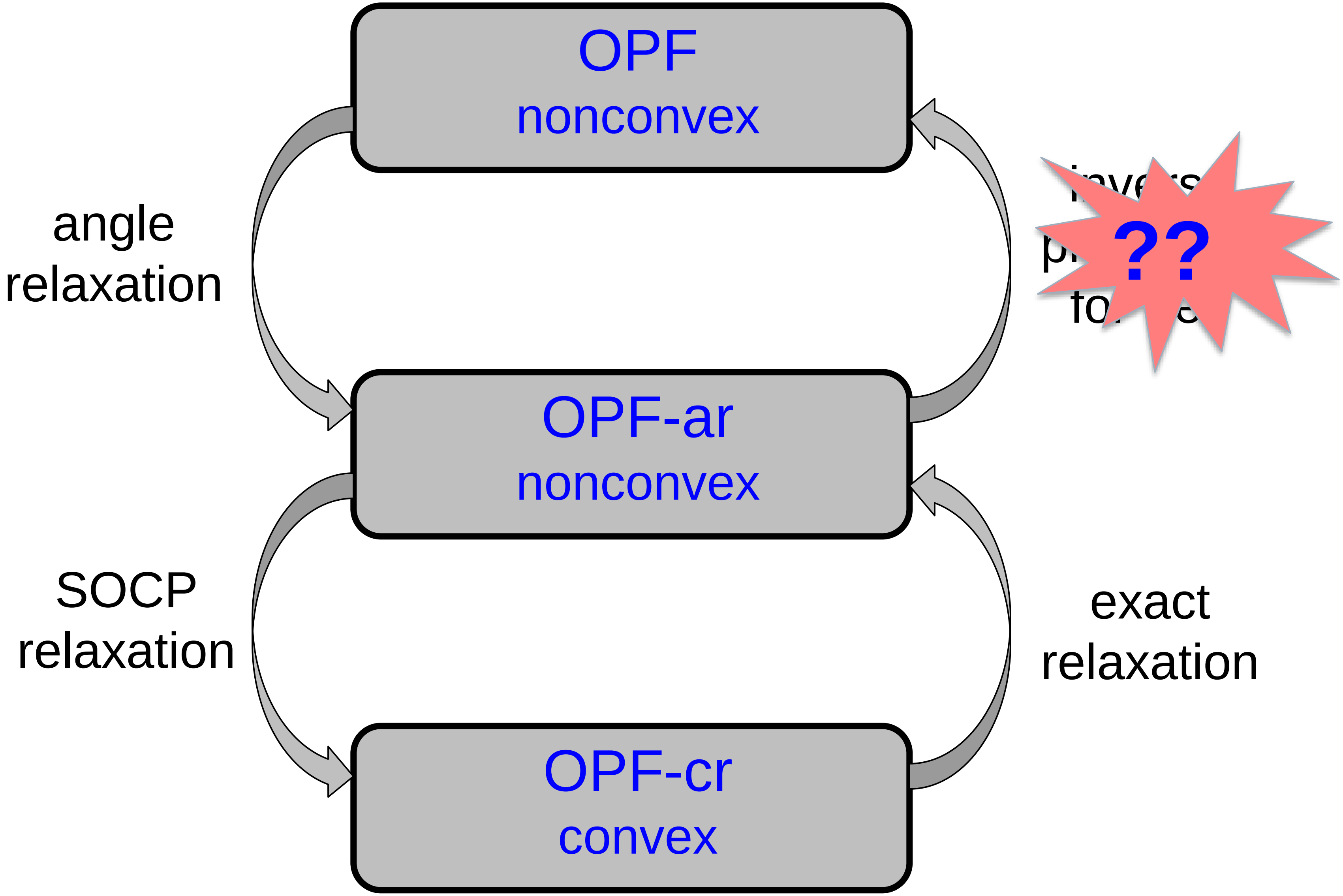
Solution strategy: two relaxations

- Angle relaxation
- SOCP relaxation

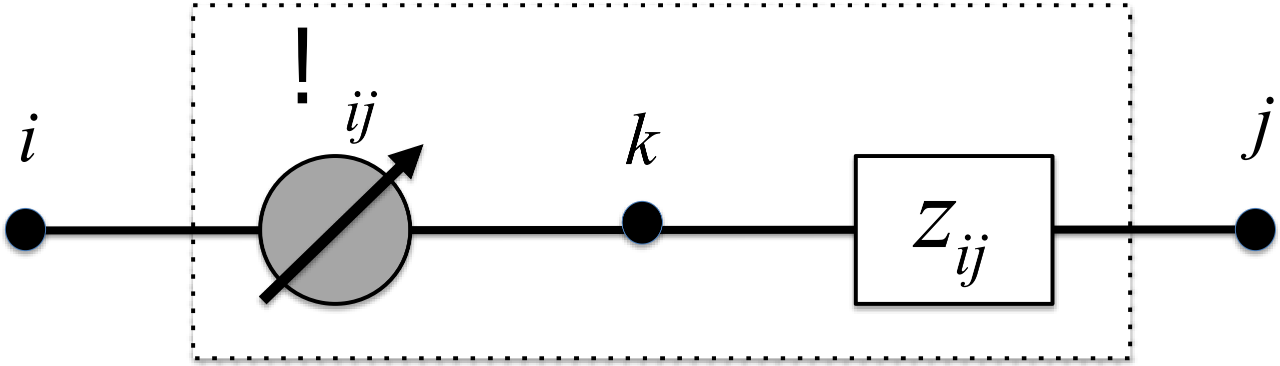
Convexification for mesh networks

Extensions

Recap: solution strategy



Phase shifter



ideal phase shifter

Convexification of mesh networks

OPF $\min_{x} \; f\left(h(x)\right) \quad \text{s.t.} \quad x \in \mathbf{X}$

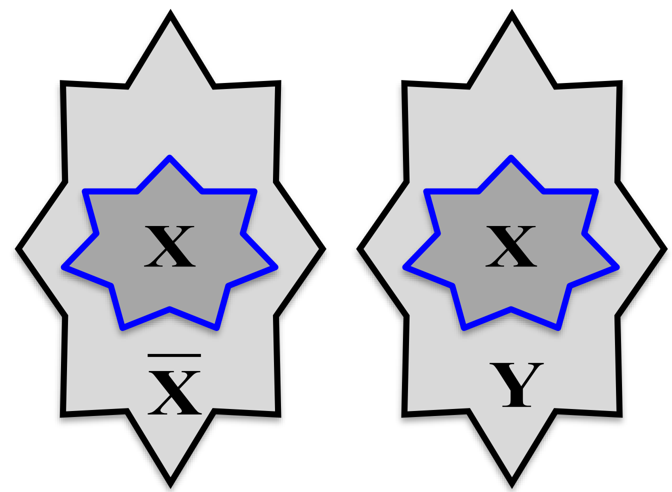
OPF-ar $\min_{x} \; f\left(h(x)\right) \quad \text{s.t.} \quad x \in \mathbf{Y}$

OPF-ps $\min_{x,\phi} \; f\left(h(x)\right) \quad \text{s.t.} \quad x \in \overline{\mathbf{X}}$

optimize over phase shifters as well

Theorem

- $\overline{\mathbf{X}} = \mathbf{Y}$
- Need phase shifters only outside spanning tree



Angle recovery with PS

$$\begin{bmatrix} B_T \\ B_\perp \end{bmatrix} \theta = \begin{bmatrix} b_T \\ b_\perp \end{bmatrix} - \begin{bmatrix} 0 \\ f_\perp \end{bmatrix}$$

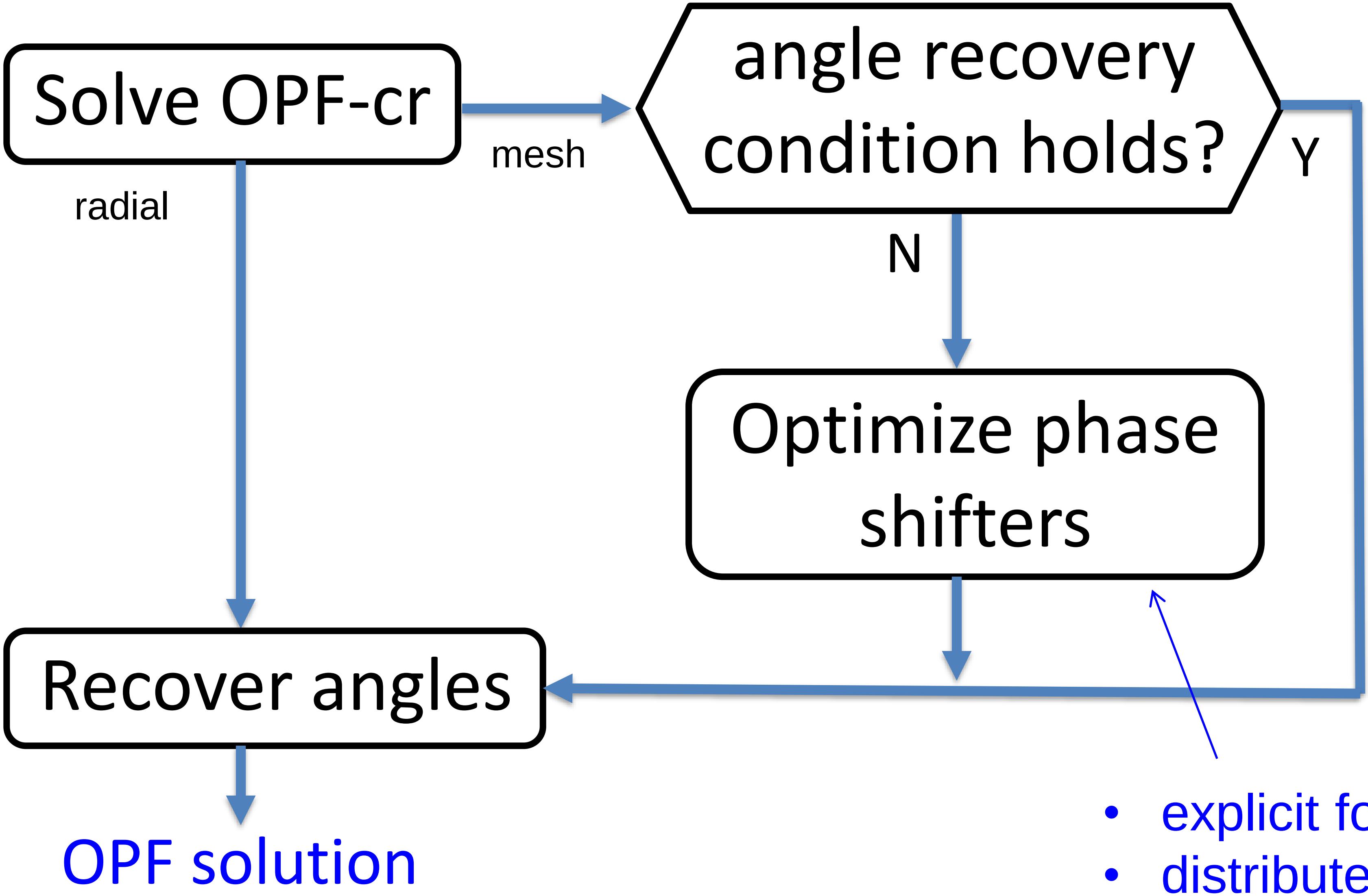
Theorem

Inverse projection always exists

Unique inverse given by $\theta^* = B_T^{-1} b_T$

Don’t need PS in spanning tree $f_\perp^* = 0$

OPF solution



- explicit formula
- distributed alg

Examples

Test cases	# links (m)	No PS Min loss (OPF, MW)	With PS Min loss (OPF-cr, MW)
IEEE 14-Bus	20	0.546	0.545
IEEE 30-Bus	41	1.372	1.239
IEEE 57-Bus	80	11.302	10.910
IEEE 118-Bus	186	9.232	8.728
IEEE 300-Bus	411	211.871	197.387
New England 39-Bus	46	29.915	28.901
Polish (case2383wp)	2,896	433.019	385.894
Polish (case2737sop)	3,506	130.145	109.905

Examples

Test cases	# links (m)	# active PS $\lvert\phi_i\rvert > 0.1^\circ$		Angle range ($^\circ$) $[\phi_{\min}, \phi_{\max}]$
IEEE 14-Bus	20	2	(10%)	$[-2.1,\ 0.1]$
IEEE 30-Bus	41	3	(7%)	$[-0.2,\ 4.5]$
IEEE 57-Bus	80	19	(24%)	$[-3.5,\ 3.2]$
IEEE 118-Bus	186	36	(19%)	$[-1.9,\ 2.0]$
IEEE 300-Bus	411	101	(25%)	$[-11.9,\ 9.4]$
New England 39-Bus	46	7	(15%)	$[-0.2,\ 2.2]$
Polish (case2383wp)	2,896	376	(13%)	$[-20.1,\ 16.8]$
Polish (case2737sop)	3,506	433	(12%)	$[-21.9,\ 21.7]$

Key message

Radial networks computationally simple

- Exploit tree graph & convex relaxation
- Real-time scalable control promising

Mesh networks can be convexified

- Design for simplicity
- Need few (?) phase shifters (sparse topology)

Outline

Branch flow model and OPF

Solution strategy: two relaxations

- Angle relaxation
- SOCP relaxation

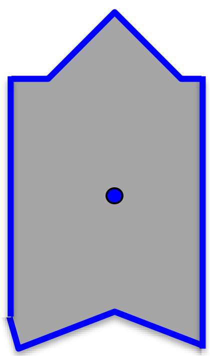
Convexification for mesh networks

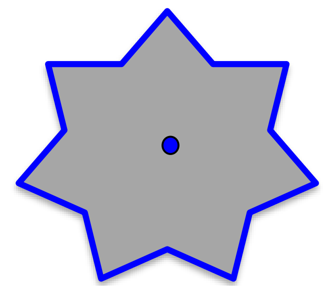
Extensions

Extension: equivalence

$$\tilde{\mathbf{X}} := \left\{ \tilde{x} = \left(\tilde{S}, \tilde{I}, V\right) \middle| \text{BI model} \right\}$$

$$\mathbf{X} := \left\{ x = \left(S, I, V\right) \middle| \text{BF model} \right\}$$





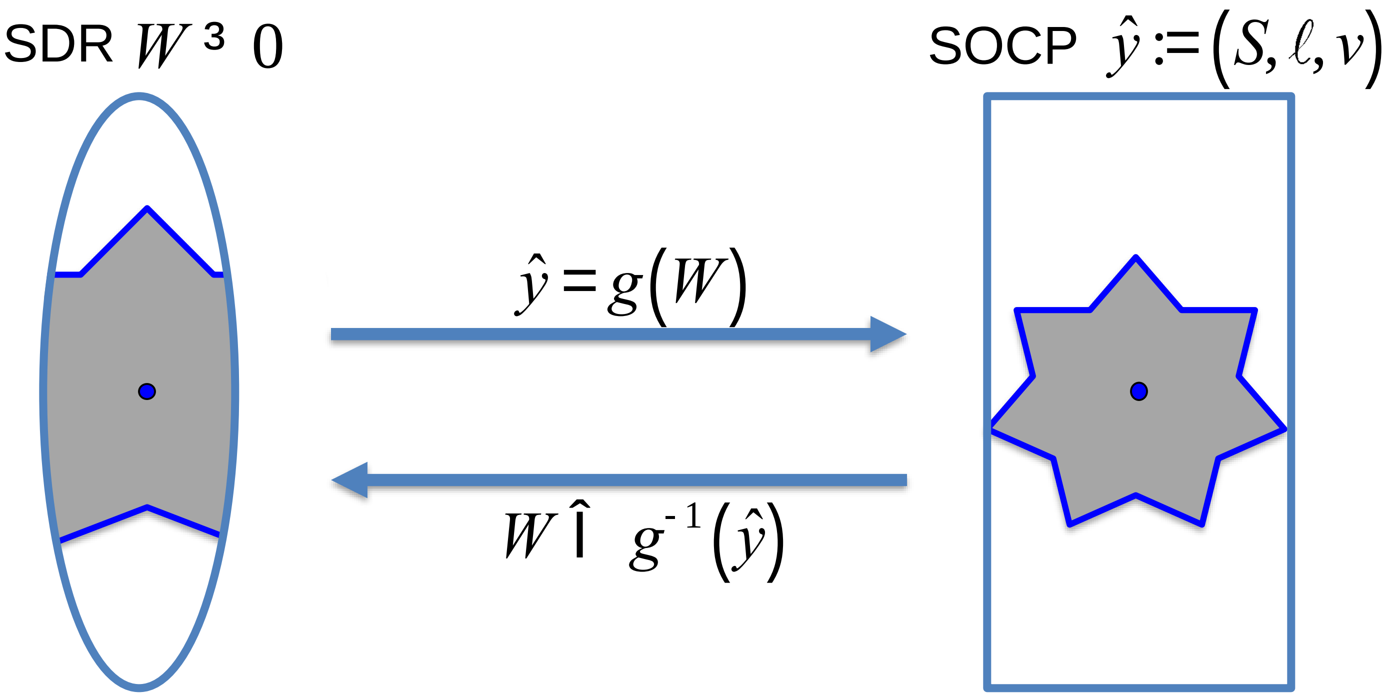
<u>Theorem</u>

BI and BF model are equivalent

(there is a bijection between $\tilde{\mathbf{X}}$ and $\mathbf{X}$)

Work in progress with Subhonmesh Bose, Mani Chandy

Extension: equivalence

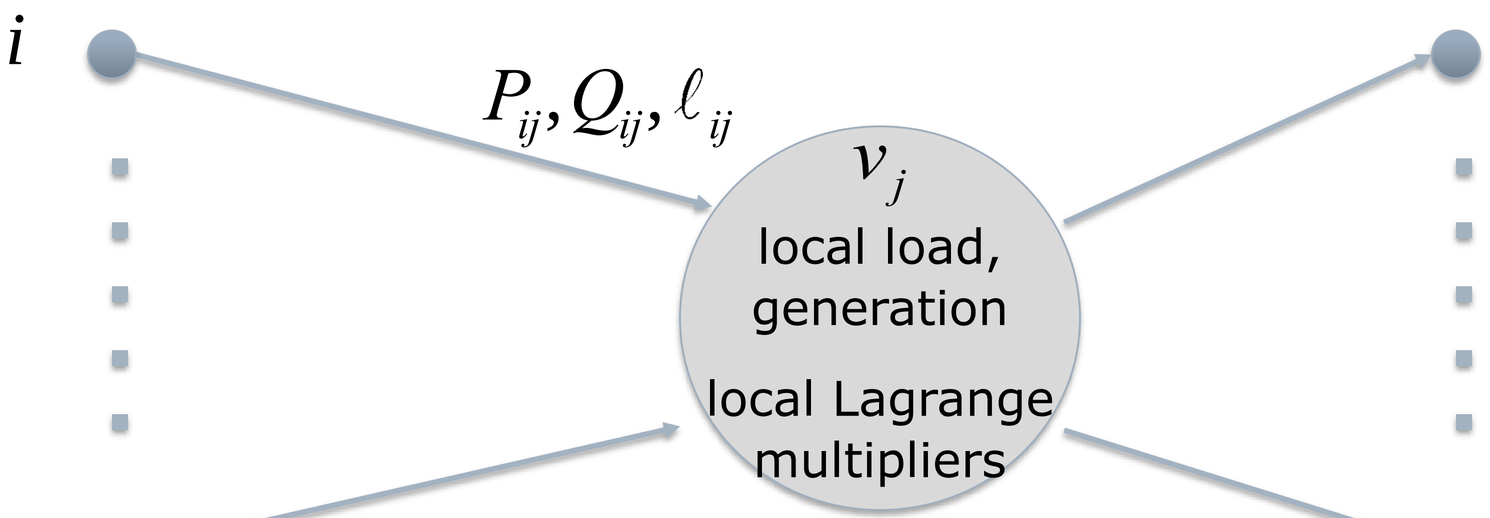


Theorem: radial networks

- $\tilde{y}$ in SOCP $\Longleftrightarrow$ W in SDR
- $\tilde{y}$ satisfies angle cond $\Longleftrightarrow$ W has rank 1

Work in progress with Subhonmesh Bose, Mani Chandy

Extension: distributed solution



highly parallelizable !

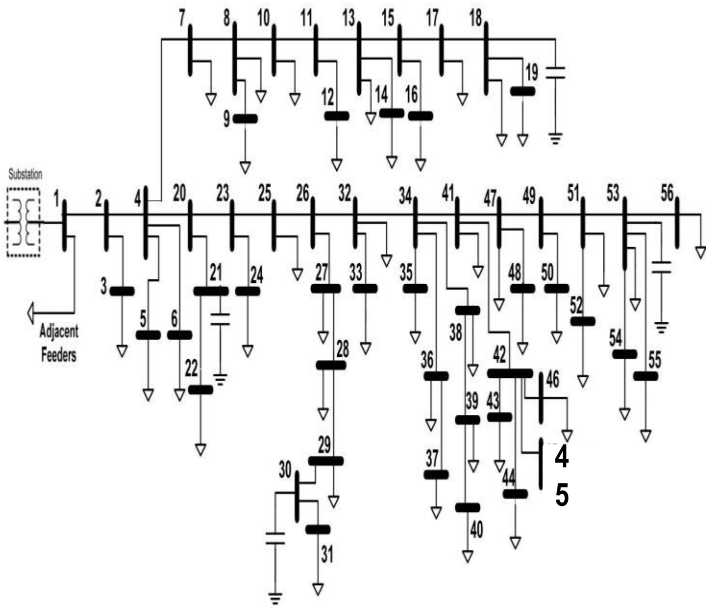
Local algorithm at bus j

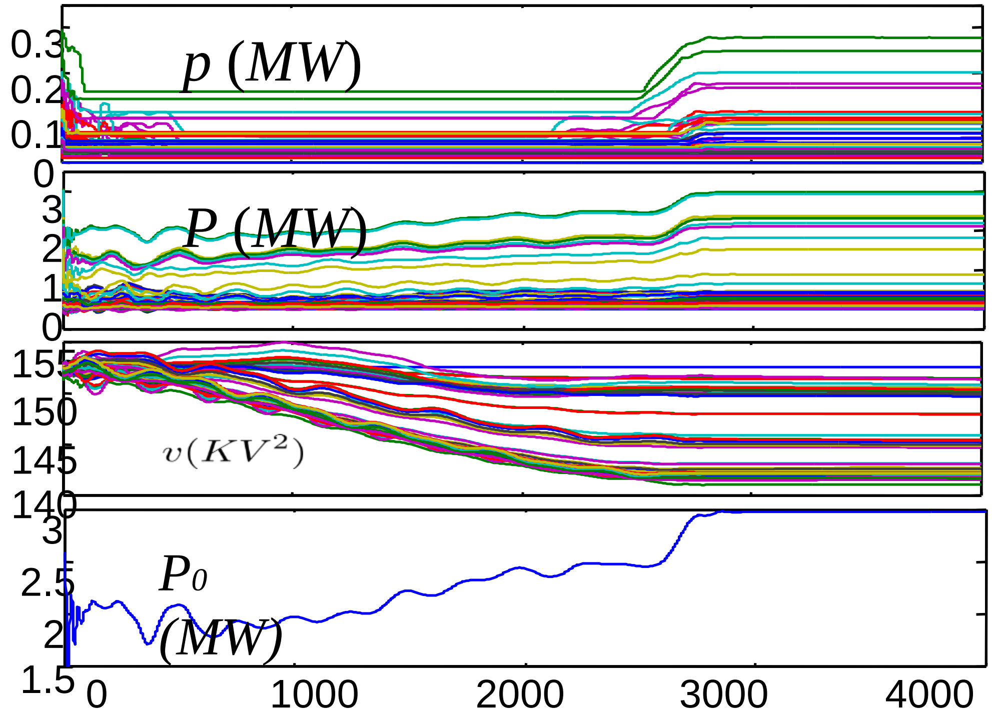
- update local variables based on Lagrange multipliers from children
- send Lagrange multipliers to parents

Work in progress with Lina Li, Lingwen Gan, Caltech

Extension: distributed solution

SCE distribution circuit





Theorem

Distributed algorithm converges

- to global optimal for radial networks
- to global optimal for convexified mesh networks
- to approximate/optimal for general mesh networks

Work in progress with Lina Li, Lingwen Gan, Caltech