Wind power over land (exc. Antarctica)
70 – 170 TW

Solar power over land
340 TW

**Worldwide**

energy demand: 16 TW
electricity demand: 2.2 TW

wind capacity (2009): 159 GW
grid-tied PV capacity (2009): 21 GW

Source: Renewable Energy
Global Status Report, 2010
Source: M. Jacobson, 2011
High Levels of Wind and Solar PV Will Present an Operating Challenge!

Source: Rosa Yang, EPRI
Implications

Current control paradigm works well today

- Low uncertainty, few active assets to control
- Centralized, open-loop, human-in-loop, worst-case preventive
- Schedule supplies to match loads

Future needs

- **Fast computation** to cope with rapid, random, large fluctuations in supply, demand, voltage, freq
- **Simple algorithms** to scale to large networks of active DER
- **Real-time data** for adaptive control, e.g. real-time DR
Outline

Optimal demand response
- With L. Chen, L. Jiang, N. Li

Optimal power flow
- With S. Bose, M. Chandy, C. Clarke, M. Farivar, D. Gayme, J. Lavaei
Automated Demand Response Saves Capacity and Energy

Electric load profile for PG&E participants on 8/30/2007

Source: Steven Chu, GridWeek 2009
Optimal demand response

Model

Results

- Uncorrelated demand: distributed alg
- Correlated demand: distributed alg
- Impact of uncertainty

Some refs:
- Caramanis & Foster 2010, 2011
- Varaiya et al 2011
- Ilic et al 2011
Optimal demand response

Model

Results

- Uncorrelated demand: distributed alg
- Correlated demand: distributed alg
- Impact of uncertainty

- L. Jiang and S. H. Low, CDC 2011, Allerton 2011
Features to capture

Wholesale markets
- Day ahead, real-time balancing

Renewable generation
- Non-dispatchable

Demand response
- Real-time control (through pricing)
Model: user

Each user has 1 appliance (wlog)

- Attains utility $u_i(x_i(t))$ when consumes $x_i(t)$

\[
\underline{x}_i(t) \leq x_i(t) \leq \bar{x}_i(t) \leq x_i(t) \leq \bar{X}_i
\]

Demand at $t$: $D(t) := \sum_{i} x_i(t)$
Model: LSE (load serving entity)

Power procurement

- Day-ahead power: $P_d(t), c_d(P_d(t))$
  - Control, decided a day ahead
Model: LSE (load serving entity)

Power procurement

- Day-ahead power: \( P_d(t), \ c_d(P_d(t)) \)
  - Control, decided a day ahead

- Renewable power: \( P_r(t), \ c_r(P_r(t))=0 \)
  - Random variable, realized in real-time
Model: LSE (load serving entity)

Power procurement

- Day-ahead power: $P_d(t)$, $c_d\left(P_d(t)\right)$, $c_o\left(x(t)\right)$
  - Control, decided a day ahead
- Renewable power: $P_r(t)$, $c_r\left(P_r(t)\right)=0$
  - Random variable, realized in real-time
Power procurement

- **Day-ahead power**: $P_d(t), \ c_d(P_d(t)), \ c_o(x(t))$
  - Control, decided a day ahead

- **Renewable power**: $P_r(t), \ c_r(P_r(t)) = 0$
  - Random variable, realized in real-time

- **Real-time balancing power**: $P_b(t), \ c_b(P_b(t))$
  - $P_b(t) = D(t) - P_r(t) - P_d(t)$

- Use as much renewable as possible
- Optimally provision day-ahead power
- Buy sufficient real-time power to balance demand
Simplifying assumption

- No network constraints
Objective

Day-ahead decision
- How much power $P_d$ should LSE buy from day-ahead market?

Real-time decision (at $t-$)
- How much $x_i$ should users consume, given realization of wind power $P_r$ and $P_d$?

How to compute these decisions distributively?
How does closed-loop system behave?

$t - 24hrs$ \hspace{1cm} $t-$

available info: $u_i(\times), F_r(\times)$ \hspace{1cm} $P_r, P_d^*$

decision: $P_d^*$ \hspace{1cm} $x_i^*$
Objective

Real-time (at $t-$)
- Given $P_d$ and realizations of $P_r$, choose optimal $x_i^* = x_i^* (P_d; P_r)$ to max social welfare

Day-ahead
- Choose optimal $P_d^*$ that maximizes expected optimal social welfare

$t - 24hrs$  

available info: $u_i (\times), F_r (\times)$  

decision: $P_d^*, x_i^*$  

$t-$  

$Pr, P_d^*$
Optimal demand response

Model

Results

- Uncorrelated demand: distributed alg
- Correlated demand: distributed alg
- Impact of uncertainty
Uncorrelated demand: T=1

Each user has 1 appliance (wlog)
- Attains utility $u_i(x_i(t))$ when consumes $x_i(t)$

$$x_i(t), \ x_i(t), \ \bar{x}_i(t) \quad \text{for } i \in X$$

Demand at $t$: $D(t) := \sum_{i} x_i(t)$

drop $t$ for this case
Welfare function

Supply cost

\[ c(P_d, x) = c_d(P_d) + c_o((x))^{P_d} + c_b((x) P_d)_+ \]

\[(x) := \sum_{i} x_i P_r \]

excess demand
Welfare function

Supply cost

\[ c(P_d, x) = c_d(P_d) + c_o(x)^{P_d}_0 + c_b(x P_d)_+ \]

\[ (x) := \sum_{i} x_i P_r \]

excess demand

Welfare function (random)

\[ W(P_d, x) = \sum_{i} u_i(x_i) c(P_d, x) \]

user utility  supply cost
Optimal operation

Welfare function (random)

\[ W(P_d, x) = \sum_{i} u_i(x_i) c(P_d, x) \]

Optimal real-time demand response

\[ \max_{x} W(P_d, x) \]

given realization of \( P_r \)
Optimal operation

Welfare function (random)

\[ W(P_d, x) = \sum_{i} u_i(x_i) c(P_d, x) \]

Optimal real-time demand response

\[ x^*(P_d) := \arg \max_{x} W(P_d, x) \]

given realization of \( P_r \)
Optimal operation

Welfare function (random)

\[ W(P_d, x) = \sum_{i} u_i(x_i) - c(P_d, x) \]

Optimal real-time demand response

\[ x^*(P_d) := \arg \max_x W(P_d, x) \]

given realization of \( P_r \)

Optimal day-ahead procurement

\[ P_d^* := \arg \max_{P_d} E W(P_d, x^*(P_d)) \]

Overall problem:

\[ \max_{P_d} E \max_x W(P_d, x) \]
Real-time DR vs scheduling

- Real-time DR: \( \max_{P_d} \mathbb{E} \max_x W(P_d, x) \)
- Scheduling: \( \max_{P_d} \max_x \mathbb{E} W(P_d, x) \)

**Theorem**

Under appropriate assumptions:

\[
W_{real \ time \ DR}^* = W_{scheduling}^* + \frac{N^2}{1+N^2}
\]

benefit increases with
- uncertainty
- marginal real-time cost
Algorithm 1 (real-time DR)

\[
\max_{P_d} \max_{x} E \quad \max_{x} W(P_d, x)
\]

Active user \(i\) computes \(x_i^*\)
- Optimal consumption

LSE computes
- Real-time “price” \(b^*\)
Algorithm 1 (real-time DR)

Active user $i$:

$$x_{i}^{k+1} = \left( x_{i}^{k} + \left( u_{i}'(x_{i}^{k}) \right) \right)_{x_{i}}$$

inc if marginal utility > real-time price

LSE :

$$\frac{b}{k+1} = \left( \frac{k}{b} + \left( \left( x^{k} \right) \frac{y_{o}^{k}}{y_{b}^{k}} \right) \right)_{+}$$

inc if total demand > total supply

- Decentralized
- Iterative computation at $t$-
Algorithm 1 (real-time DR)

Theorem: Algorithm 1

Socially optimal
- Converges to welfare-maximizing DR $x^* = x^*(P_d)$
- Real-time price aligns marginal cost of supply with individual marginal utility
  \[ b = c'(P_d, x^*) = u_i'(x^*_i) \]

Incentive compatible
- $x_i^*$ maximizes $i$'s surplus given price $b$
Algorithm 2 \hspace{0.5cm} (day-ahead procurement)

Optimal day-ahead procurement

$$\max_{P_d} \mathbb{E}W(P_d, x^*(P_d))$$

LSE:

$$P_{d}^{m+1} = \left( P_{d}^{m} + m \left( \sum_{o} c_{d}^{'(P_{d}^{m})} \right) \right)_+$$

calculated from Monte Carlo simulation of Alg 1
(stochastic approximation)
Algorithm 2 (day-ahead procurement)

**Theorem**

Algorithm 2 converges a.s. to optimal $P^*_d$ for appropriate stepsizes $P_d^k$. 
Optimal demand response

Model

Results

- Uncorrelated demand: distributed alg
- Correlated demand: distributed alg
- Impact of uncertainty
Impact of renewable on welfare

Renewable power:

\[ P_r(t; a, b) := a \times m(t) + b \times V(t) \]

Optimal welfare of (1+T)-period DP

\[ W^*(a, b) \]
Impact of renewable on welfare

\[ Pr(t;a,b) := a \times (t) + b \times V(t) \]

**Theorem**

- \( W^*(a,b) \) increases in \( a \), decreases in \( b \)
- \( W^*(s,s) \) increases in \( s \) (plant size)
With ramp rate costs

Day-ahead ramp cost
$$s_d(t) := f_d\left(P_d(t), P_d(t+1)\right)$$

Real-time ramp cost
$$s_b(t) := f_b\left(P_b(t), P_b(t+1)\right)$$

Social welfare
$$W^*(a, b) := \sum_{t=1}^{T} W_t(x(t), P_d(t); P_r(t)) \left( s_d(t) + s_b(t) \right)$$

Theorem

- $W^*(a, b)$ increases in $a$, decreases in $b$
- $W^*(s, s)$ increases in $s$ (plant size)
Outline

Optimal demand response
  - With L. Chen, L. Jiang, N. Li

Optimal power flow
  - With S. Bose, M. Chandy, C. Clarke, M. Farivar, D. Gayme, J. Lavaei
Optimal power flow (OPF)

- OPF is solved routinely to determine:
  - How much power to generate where
  - Market operation & pricing
  - Parameter setting, e.g. taps, VARs

- Non-convex and hard to solve:
  - Huge literature since 1962
  - In practice, operators often use heuristics to find a feasible operating point
  - Or solve DC power flow (LP)
Optimal power flow (OPF)

Problem formulation
- Carpentier 1962

Computational techniques:
- Dommel & Tinney 1968

Bus injection model (SDP formulation):
- Lesieutre et al 2011

Branch flow model
Models

\[ S_{ij} = \begin{pmatrix} r_{ij} \\ x_{ij} \end{pmatrix} \]

\[ z_{ij} = (r_{ij}, x_{ij}) \]

\[ V_i \]

\[ I_{ij} \]

\[ V_j \]

\[ I_j = \begin{pmatrix} I_{jk} \end{pmatrix} \]

\[ S_j = S_{jk} \]

\[ S_j^g \]

\[ S_j^c \]

branch flow \( S_{ij} \)

bus injection
Models: Kirchhoff’s law

\[ S_i = S_{ij} = V_i I_i^* \]

linear relation:

\[ I = YV \]

\[ S_{ij} = V_i I_{ij}^* \]

\[ S_{ij} = \frac{|V_i|^2}{Z_{ij}^*} \quad \frac{V_i V_j^*}{Z_{ij}^*} \]
Outline: OPF

SDP relaxation
- Bus injection model

Conic relaxation
- Branch flow model

Application
Bus injection model

Nodes $i$ and $j$ are linked with an admittance $y_{ij} = g_{ij} - ib_{ij}$

\[
Y_{ij} = \begin{cases} 
  y_{ii} + \sum_{j \sim i} y_{ij}, & \text{if } i = j \\
  -y_{ij}, & \text{if } i \sim j \\
  0 & \text{otherwise}
\end{cases}
\]

• Kirchhoff's Law: $I = YV$
Classical OPF

\[
\begin{align*}
\min \quad & f_k \left( P_k^g \right) \quad \text{Generation cost} \\
\text{over} \quad & \left( S^g_k, V_k \right) \\
\text{subject to} \quad & S^g_k, S^g_k, \bar{S}^g_k \quad \text{Generation power constraints} \\
& V_k, |V_k|, \bar{V}_k \quad \text{Voltage magnitude constraints} \\
& I = YV \quad \text{Kirchhoff law} \\
& V_k I_k^* = S^g_k, S^c_k \quad \text{Power balance}
\end{align*}
\]
Classical OPF

In terms of $V$:

\[ P_k = \text{tr} \ kVV^* \]
\[ Q_k = \text{tr} \ kVV^* \]

\[ k : = \frac{Y_k^* + Y_k}{2} \]

\[ k : = \frac{Y_k^* - Y_k}{2i} \]

\[ \min_k \ \text{tr} \ M_kVV^* \]

over $V$

s.t. \[ \begin{array}{cccc}
  P_k^g & P_k^d & \text{tr} \ kVV^* & \bar{P}_k^g & P_k^d \\
  Q_k^g & Q_k^d & \text{tr} \ kVV^* & \bar{Q}_k^g & Q_k^d \\
  V_k^2 & \text{tr} \ J_kVV^* & V_k^2 \\
\end{array} \]

Key observation [Bai et al 2008]: OPF = rank constrained SDP
Classical OPF

\[
\begin{align*}
\min_{k \in G} & \quad \text{tr} \ M_k W \\
\text{over} & \quad W \text{ positive semidefinite matrix} \\
\text{s.t.} & \quad P_k \quad \text{tr} \quad k W \quad \bar{P}_k \\
& \quad Q_k \quad \text{tr} \quad k W \quad \bar{Q}_k \\
& \quad V_k^2 \quad \text{tr} \quad J_k W \quad \bar{V}_k^2 \\
& \quad W \geq 0, \quad \text{rank} \ W = 1
\end{align*}
\]

convex relaxation: SDP
Semi-definite relaxation

1. Non-convex QCQP
2. Rank-constrained SDP
3. Relax the rank constraint and solve the SDP
4. Does the optimal solution satisfy the rank-constraint?
   - yes: We are done!
   - no: Solution may not be meaningful
SDP relaxation of OPF

\[
\begin{align*}
\min_{k \in G} & \quad \text{tr } M_k W \\
\text{over } & \quad W \text{ positive semidefinite matrix} \\
\text{s.t. } & \quad P_k \text{ tr } W \geq \overline{P}_k \\
& \quad Q_k \text{ tr } W \geq \overline{Q}_k \\
& \quad V_k^2 \text{ tr } J_k W \geq \overline{V}_k^2 \\
& \quad W^T W \geq 0
\end{align*}
\]

\[
A(k, k, k) := M_k + \left( \begin{array}{cc}
\overline{P}_k & \overline{Q}_k \\
\overline{V}_k^2 & 0
\end{array} \right)_{k \in G} + \sum_{k \in G} J_k
\]
Sufficient condition

Theorem

If $A^{opt}$ has rank $n-1$ then

- $W^{opt}$ has rank 1, SDP relaxation is exact
- Duality gap is zero
- A globally optimal $V^{opt}$ can be recovered

All IEEE test systems (essentially) satisfy the condition!

OPF over radial networks

Suppose

- tree (radial) network
- no lower bounds on power injections

**Theorem**

\[ A^{\text{opt}} \text{ always has rank } n-1 \]

- \[ W^{\text{opt}} \text{ always has rank } 1 \text{ (exact relaxation) } \]
- OPF always has zero duality gap
- Globally optimal solvable efficiently

---

OPF over radial networks

Suppose
- tree (radial) network
- no lower bounds on power injections

**Theorem**

\( A^{opt} \) always has rank \( n-1 \)

- \( W^{opt} \) always has rank 1 (exact relaxation)

- OPF always has zero duality gap

- Globally optimal \( x^{opt} \) solvable efficiently

Also: B. Zhang and D. Tse, Allerton 2011
S. Sojoudi and J. Lavaei, submitted 2011
QCQP over tree

\[
\begin{align*}
\text{QCQP} \quad (C,C_k) \\
\min & \quad x^* C x \\
\text{over} & \quad x \in C^n \\
\text{s.t.} & \quad x^* C_k x \leq b_k \quad k \in K
\end{align*}
\]

graph of QCQP

\[ G(C,C_k) \text{ has edge } (i,j) \iff \]

\[ C_{ij} \neq 0 \text{ or } [C_k]_{ij} \neq 0 \text{ for some } k \]

QCQP over tree

\[ G(C,C_k) \text{ is a tree} \]
QCQP over tree

QCQP \((C, C_k)\) 
\[
\begin{align*}
\text{min} & \quad x^* C x \\
\text{over} & \quad x \quad C^n \\
\text{s.t.} & \quad x^* C_k x \quad b_k \quad k \quad K
\end{align*}
\]

Semidefinite relaxation
\[
\begin{align*}
\text{min} & \quad \text{tr } CW \\
\text{over} & \quad W \quad 0 \\
\text{s. t.} & \quad \text{tr } C_k W \quad b_k \quad k \quad K
\end{align*}
\]
QCQP over tree

**QCQP** \((C, C_k)\)

\[
\min \quad x^* C x \\
\text{over} \quad x \in \mathbb{C}^n \\
s.t. \quad x^* C_k x \leq b_k \quad k \in K
\]

**Key assumption**

\((i, j) \in G(C, C_k) : \ 0 \not\in \text{int conv hull } (C_{ij}, [C_k]_{ij}, k)\)

**Theorem**

Semidefinite relaxation is exact for QCQP over tree

OPF over radial networks

**Theorem**

- $A^{\text{opt}}$ always has rank $n-1$
- $W^{\text{opt}}$ always has rank 1 (exact relaxation)
- OPF always has zero duality gap
- Globally optimal, solvable efficiently

“no lower bounds” removes these $[C_k]_{ij}$
OPF over radial networks

**Theorem**

- $A^{opt}$ always has rank $n-1$
- $W^{opt}$ always has rank 1 (exact relaxation)
- OPF always has zero duality gap
- Globally optimal solvable efficiently

Bounds on constraints remove these $[C_k]_{ij}$
Outline: OPF

SDP relaxation
  - Bus injection model

Conic relaxation
  - Branch flow model

Application
Kirchhoff’s Law: $S_{ij} = S_{jk} + z_{ij} \left| I_{ij} \right|^2 + s^c_j + s^g_j$

branch power $S_{ij}$

load - gen
line loss
Kirchhoff’s Law: \[ S_{ij} = S_{jk} + z_{ij} \left| I_{ij} \right|^2 + s^c_j + s^g_j \]

Ohm’s Law: \[ V_j = V_i + z_{ij} I_{ij} \]
OPF using branch flow model

\[ \min \quad r_{ij}l_{ij} + i\nu_i \]

- \( l_{ij} := |I_{ij}|^2 \)
- \( \nu_i := |V_i|^2 \)

real power loss

CVR (conservation voltage reduction)
OPF using branch flow model

\[ \min \sum_{i \sim j} r_{ij} l_{ij} + i v_i \]

over \((S, I, V, s^g, s^c)\)

s. t. \[ s_i^g, s_i^g, s_i^g, s_i^c, s_i, s_i \]

Kirchhoff’s Law: \[ S_{ij} = S_{jk} + z_{ij} |I_{ij}|^2 + s_j^c s_j^g \]

Ohm’s Law: \[ V_j = V_i - z_{ij} I_{ij} \]
**OPF using branch flow model**

\[
\begin{align*}
\text{min} & \quad r_{ij}l_{ij} + \sum_{i} v_i \\
\text{over} & \quad (S, I, V, s^g, s^c) \\
\text{s. t.} & \quad s^g_i - s^g_i - S^g_i = 0 \\
& \quad s^c_i - s^c_i - S^c_i = 0 \\
& \quad \bar{v}_i - v_i - \bar{v}_i = 0 \\
\end{align*}
\]

Kirchhoff’s Law: \( S_{ij} = S_{jk} + z_{ij} \left| I_{ij} \right|^2 + s^c_j s^g_j \)

Ohm’s Law: \( V_j = V_i + z_{ij} I_{ij} \)
OPF using branch flow model

\[
\begin{align*}
\text{min} & \quad r_{ij} l_{ij} + \sum_{i} v_i \\
\text{over} & \quad (S, I, V, s^g, s^c) \\
\text{s. t.} & \quad s_i^g s_i^g \quad s_i^g s_i^g \quad s_i^c s_i^c \\
& \quad \bar{v}_i \quad v_i \quad \bar{v}_i \\
\text{Kirchhoff’s Law:} & \quad S_{ij} = \sum_{k: j \neq k} S_{jk} + z_{ij} |I_{ij}|^2 + s_j^c s_j^g \\
\text{Ohm’s Law:} & \quad V_j = V_i - z_{ij} I_{ij}
\end{align*}
\]
Solution strategy

OPF
nonconvex

OPF-ar
nonconvex

OPF-cr
convex

angle relaxation

inverse projection for tree

conic relaxation

exact relaxation
Angle relaxation

Kirchhoff’s Law: \( S_{ij} = S_{jk} + z_{ij} |I_{ij}|^2 + s^c_j s^g_j \)  

Angles of \( I_{ij}, V_i \) eliminated! 
Points relaxed to circles

\[ |V_i|^2 = |V_j|^2 + 2(r_{ij} P_{ij} + x_{ij} Q_{ij}) \left( r_{ij}^2 + x_{ij}^2 \right) |I_{ij}|^2 \]

\[ |I_{ij}|^2 = \frac{P^2_{ij} + Q^2_{ij}}{|V_i|^2} \]

Baran and Wu 1989 for radial networks
Angle relaxation

\[
P_{ij} = P_{jk} + r_{ij} |I_{ij}|^2 + p_j^c p_j^g \\
Q_{ij} = Q_{jk} + x_{ij} |I_{ij}|^2 + q_j^c q_j^g \\
|V_i|^2 = |V_j|^2 + 2 (r_{ij} P_{ij} + x_{ij} Q_{ij}) \left( r_{ij}^2 + x_{ij}^2 \right) |I_{ij}|^2 \\
|I_{ij}|^2 = \frac{P_{ij}^2 + Q_{ij}^2}{|V_i|^2}
\]

Baran and Wu 1989 for radial networks
min \[ r_{ij} l_{ij} + \sum_{i \sim j} v_i \]

over \((S, l, v, s^g, s^c)\)

s. t. \[ P_{ij} = P_{jk} + r_{ij} l_{ij} + p_j^c \]

\[ Q_{ij} = Q_{jk} + x_{ij} l_{ij} + q_j^c \]

\[ v_i = v_j + 2\left(r_{ij} P_{ij} + x_{ij} Q_{ij}\right) \left(r_{ij}^2 + x_{ij}^2\right) l_{ij} \]

\[ l_{ij} = \frac{P_{ij}^2 + Q_{ij}^2}{v_i} \]

- Linear objective
- Linear constraints
- Quadratic equality
\[
\begin{align*}
\text{min} & \quad r_{ij} l_{ij} + v_i \\
\text{over} & \quad (S, l, v, s^g, s^c) \\
\text{s. t.} & \quad P_{ij} = P_{jk} + r_{ij} l_{ij} + p_j^c - p_j^g \\
& \quad Q_{ij} = Q_{jk} + x_{ij} l_{ij} + q_j^c - q_j^g \\
& \quad v_i = v_j + 2(r_{ij} P_{ij} + x_{ij} Q_{ij}) \left( r_{ij}^2 + x_{ij}^2 \right) l_{ij}
\end{align*}
\]

**Quadratic inequality**

\[
\frac{P_{ij}^2 + Q_{ij}^2}{\nu_i} \leq \begin{cases} s_i & s^g_i \quad s_i \\
\nu_i & \nu_i \quad \overline{\nu}_i, \\
\end{cases}
\]
Theorem
Both relaxation steps are exact
- OPF-cr is convex and exact
- Phase angles can be uniquely determined

OPF-ar has zero duality gap

What about mesh networks??
Solution strategy

- **OPF** (nonconvex)
- **OPF-ar** (nonconvex)
- **OPF-cr** (convex)

Relaxation methods:
- **Angle relaxation**
- **Conic relaxation**
- **Exact relaxation**

Wiki page for further reading: ???
**OPF using branch flow model**

\[
\begin{align*}
\text{min} & \quad r_{ij} l_{ij} + i v_i \\
\text{over} & \quad (S, I, V, s^g, s^c) \\
\text{s. t.} & \quad s_{ij}^g, s_i^g, s_{ij}^g, s_{ii}^g, s_{ii}^c \\
& \quad v_i, v_i, v_i
\end{align*}
\]

Kirchoff’s Law: \( S_{ij} = S_{jk} + z_{ij} \left| I_{ij} \right|^2 + s_j^c s_j^g \)

Ohm’s Law: \( V_j = V_i + z_{ij} I_{ij} \)
Convexification of mesh networks

\[
\begin{align*}
\text{OPF} & & \min_x f(\hat{h}(x)) \quad \text{s.t.} \quad x \in X \\
\text{OPF-ar} & & \min_x f(\hat{h}(x)) \quad \text{s.t.} \quad x \in Y \\
\text{OPF-ps} & & \min_{x, f} f(\hat{h}(x)) \quad \text{s.t.} \quad x \in \overline{X}
\end{align*}
\]

**Theorem**

- \( \overline{X} = Y \)
- Need phase shifters only outside spanning tree
Key message

Radial networks computationally simple

- Exploit tree graph & convex relaxation
- Real-time scalable control promising

Mesh networks can be convexified

- Design for simplicity
- Need few phase shifters (sparse topology)
Application: Volt/VAR control

Motivation

- Static capacitor control cannot cope with rapid random fluctuations of PVs on distr circuits

Inverter control

- Much faster & more frequent
- IEEE 1547 does not optimize VAR currently (unity PF)
Load and Solar Variation

Load Profile $p_i^c$

Distribution of System State (Solar vs Load)

Empirical distribution of (load, solar) for Calabash
Improved reliability

\[(p_i^g, p_i^c)\] for which problem is feasible

Implication: reduced likelihood of violating voltage limits or VAR flow constraints
Energy savings

Operation Cost Benefits (MW)

Normalized Load Level (%)

Normalized Solar Output Level (%)
Summary

<table>
<thead>
<tr>
<th>Voltage Drop Tolerance (pu)</th>
<th>Annual Hours Saved Outside Feasibility Region</th>
<th>Average Power Saving (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.03</td>
<td>842.9</td>
<td>3.93%</td>
</tr>
<tr>
<td>0.04</td>
<td>160.7</td>
<td>3.67%</td>
</tr>
<tr>
<td>0.05</td>
<td>14.5</td>
<td>3.62%</td>
</tr>
</tbody>
</table>

- More reliable operation
- Energy savings