Design and Stability of Load-side Frequency Control

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July 2015
Motivation

- All buses synchronized to same nominal frequency (US: 60 Hz; Europe/China: 50 Hz)
- Supply-demand imbalance $\rightarrow$ frequency fluctuation

2011 Southwest blackout
Why load-side participation

Ubiquitous continuous load-side control can supplement generator-side control

- faster (no/low inertia)
- no extra waste or emission
- more reliable (large #)
- better localize disturbances
- reducing generator-side control capacity

[Diagram with: secondary freq control, economic dispatch, primary freq control]
What is the potential

- Residential load accounts for ~1/3 of peak demand
- 61% residential appliances are Grid Friendly

US:
- operating reserve: 13% of peak
- total GFA capacity: 18%

Lu & Hammerstrom (2006), PNNL
How

How to design load-side frequency control?

How does it interact with generator-side control?
Literature: load-side control

Original idea & early analytical work
- Schweppe et al 1980; Bergin, Hill, Qu, Dorsey, Wang, Varaiya ...

Small scale trials around the world

Early simulation studies

Analytical work – load-side control

Recent analysis – generator-side/microgrid control:
Outline

Network model
Load-side frequency control
Simulations
Details

Main references:
Zhao, Topcu, Li, Low, TAC 2014
Mallada, Zhao, Low, Allerton 2014
Zhao, Low, CDC 2014, Zhao et al CISS 2015
Network model

\[ P_i^m \]

\[ d_i + \hat{d}_i \]

loads: controllable + freq-sensitive

\[ i : \text{region/control area/balancing authority} \]

Will include generator-side control later

generation

branch power

\[ P_{ij} \]

\[ x_{ij} \]
Network model

\[ M_i \cdot d_i = P^m_i + \hat{d}_i - \sum_{e} C_{ie} P_e \]

Generator bus: \( M_i > 0 \)
Load bus: \( M_i = 0 \)

Damping/uncontr loads: \( \hat{d}_i = D_i \)
Controllable loads: \( d_i \)
Network model

\[ M_i \cdot d_i = P_i^m - d_i - \hat{d}_i - \sum_{e} C_{ie}P_e \]

\[ \dot{P}_{ij} = b_{ij} \left( i \rightarrow j \right) \]

- swing dynamics
- all variables are deviations from nominal
- extends to nonlinear power flow
Frequency control

\[
M_i \cdot \dot{w}_i = P_i^m - d_i \hat{d}_i \sum_{e} C_{ie} P_e
\]

\[
\dot{P}_{ij} = b_{ij} \left( i \quad j \right)
\]

Suppose the system is in steady state

\[
\dot{w}_i = 0 \quad \dot{P}_{ij} = 0 \quad i = 0
\]

Then: disturbance in gen/load …
Frequency control

\[
M_i \cdot d_i = P_i^m
\]

\[
\hat{d}_i + \sum_{e} C_{ie} P_e
\]

\[
\dot{P}_{ij} = b_{ij} (i \rightarrow j)
\]

Current approach

Load-side control
Outline

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Load-side controller design

\[
M_i \cdot i = P_i^m \quad \hat{d}_i \quad \sum_e C_{ie} P_e
\]

\[
\dot{P}_{ij} = b_{ij} (i \quad j) \quad i \rightarrow j
\]

Control goals
- Rebalance power & stabilize frequency
- Restore nominal frequency
- Restore scheduled inter-area flows
- Respect line limits

Zhao, Topcu, Li, Low
TAC 2014
Mallada, Zhao, Low
Allerton, 2014
Load-side controller design

Control goals (while min disutility)

- Rebalance power & stabilize frequency
- Restore nominal frequency
- Restore scheduled inter-area flows
- Respect line limits

\[ M_i \cdot d_i = P_i^m - d_i \hat{d}_i + \sum_{e} C_{ie} P_e \]

\[ \dot{P}_{ij} = b_{ij} (P_{ij})_i \rightarrow j \]
Load-side controller design

Design control law whose equilibrium solves:

\[
\begin{align*}
\min_{d_i, P} & \quad c_i(d_i) \\
\text{s.t.} & \quad P_i^m \ d_i = C_{ie} P_e \quad \text{node } i \\
& \quad C_{ie} P_e = \hat{P}_k \quad \text{area } k \\
& \quad i \quad N_k \quad e \\
& \quad P_e \quad P_e \quad \bar{P}_e \quad \text{line } e
\end{align*}
\]

Control goals (while min disutility)

- Rebalance power & stabilize frequency
- Restore nominal frequency
- Restore scheduled inter-area flows
- Respect line limits

freq will emerge as Lagrange multiplier for power imbalance
Load-side controller design

Design control \((G, F)\) s.t. closed-loop system

- is stable
- has equilibrium that is optimal

\[
\dot{\omega}_i = M_i \dot{\omega}_i = P_i^m - d_i - \hat{d}_i - \sum_e C_{ie} P_e
\]
\[
\dot{P}_{ij} = b_{ij} (\omega_i - \omega_j)
\]
\[
\dot{\lambda} = G(\omega(t), P(t), \lambda(t))
\]
\[
d_i = F_i(\omega(t), P(t), \lambda(t))
\]

\[
\min_{d_i, P_e} c_i(d_i)
\]
\[
\text{s. t. } P_i^m = d_i = C_{ie} P_e \quad \text{node } i
\]
\[
C_{ie} P_e = \hat{P}_k \quad \text{area } k
\]
\[
P_e \leq \bar{P}_e \quad \text{line } e
\]
Load-side controller design

Idea: exploit system dynamic as part of primal-dual algorithm for modified opt
- Distributed algorithm
- Stability analysis
- Control goals in equilibrium

\[
\begin{align*}
M_i \dot{\omega}_i &= P_i^m - d_i - \hat{d}_i - \sum_{e} C_{ie} P_e \\
\dot{P}_{ij} &= b_{ij} (\omega_i - \omega_j) \\
\dot{\lambda} &= G(\omega(t), P(t), \lambda(t)) \\
d_i &= F_i(\omega(t), P(t), \lambda(t))
\end{align*}
\]

\[
\begin{align*}
\min_{d, P} & \quad c_i(d_i) \\
\text{s. t.} & \quad P_i^m - d_i = C_{ie} P_e \quad \text{node } i \\
 & \quad C_{ie} P_e = \hat{P}_k \quad \text{area } k \\
 & \quad P_{\text{min}} \leq P_e \leq \bar{P}_e \quad \text{line } e
\end{align*}
\]
Summary: control architecture

**Primary** load-side frequency control
- completely decentralized
- **Theorem**: stable dynamic, optimal equilibrium

Zhao, Topcu, Li, Low. TAC 2014
Summary: control architecture

Secondary load-side frequency control
- communication with neighbors
- Theorem: stable dynamic, optimal equilibrium
Summary: control architecture

With generator-side control, nonlinear power flow

- load-side improves both transient & eq
- Theorem: stable dynamic, optimal equilibrium

Zhao, Mallada, Low. CISS 2015
Outline

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Zhao, Low, CDC 2014, Zhao et al CISS 2015
Simulations

Dynamic simulation of IEEE 39-bus system

- Power System Toolbox (RPI)
- Detailed generation model
- Exciter model, power system stabilizer model
- Nonzero resistance lines

Fig. 2: IEEE 39 bus system: New England
Primary control
Secondary control

Fig. 2: IEEE 39 bus system: New England

swing dynamics

with OLC

Area 1

Area 2
Secondary control

Fig. 2: IEEE 39 bus system: New England

Total inter-area flow is the same in both cases

With line limits

No line limits
Conclusion

Forward-engineering design facilitates
- explicit control goals
- distributed algorithms
- stability analysis

Load-side frequency regulation
- primary & secondary control works
- helps generator-side control
Outline

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Mallada, Zhao, Low, Allerton 2014
Zhao, Low, CDC 2014, Zhao et al CISS 2015
Recall: design approach

Idea: exploit system dynamic as part of primal-dual algorithm for modified opt

- closed-loop system is stable
- its equilibria are optimal

\[
\begin{align*}
M_i \ddot{\omega}_i &= P_i^m - d_i - \hat{d}_i - \sum_e C_{ie} P_e \\
\dot{P}_{ij} &= b_{ij} (\omega_i - \omega_j) \\
\dot{\lambda} &= G(\omega(t), P(t), \lambda(t)) \\
d_i &= F_i (\omega(t), P(t), \lambda(t))
\end{align*}
\]

\[
\begin{align*}
\min_{d,P} & & c_i(d_i) \\
\text{s. t.} & & P_i^m - d_i = C_{ie} P_e & \text{node } i \\
& & C_{ie} P_e = \hat{P}_k & \text{area } k \\
& & P_e, \bar{P}_e & \text{line } e
\end{align*}
\]
Load-side frequency control

- Primary control  
  Zhao et al SGC2012, Zhao et al TAC2014
- Secondary control
- Interaction with generator-side control
Optimal load control (OLC)

\[
\begin{align*}
\min_{d, \hat{d}, P} & \quad c_i(d_i) + \frac{\hat{d}_i^2}{2D_i} : \\
\text{s. t.} & \quad P_i^m (d_i + \hat{d}_i) = C_{ie} P_{ie} \quad i \quad \text{demand} = \text{supply} \\
\end{align*}
\]
system dynamics + load control
= primal dual alg

swung dynamics

\[ \dot{\mathbf{d}}_i(t) = \frac{1}{M_i} \left( \mathbf{d}_i(t) + D_i \mathbf{d}_i(t) + P_i^m + \sum_{i \rightarrow j} P_{ij}(t) - \sum_{j \rightarrow i} P_{ji}(t) \right) \]

\[ \dot{P}_{ij} = b_{ij} \left( \mathbf{d}_i(t) - \mathbf{d}_j(t) \right) \]

load control

\[ d_i(t) := \mathbf{c}_i^{-1} \left( \mathbf{d}_i(t) \right) \]

\[ \frac{d_i}{d_i} \]

\[ \text{active control} \]

\[ \text{implicit} \]
Control architecture

Power Network Dynamics

\((\omega, P)\)

\[ d \]

\[ d_i(\cdot) \]

\[ \begin{array}{cc}
\ddots & 0 \\
0 & \ddots
\end{array} \]
Load-side primary control works

Theorem
Starting from any \( \begin{pmatrix} d(0), \hat{d}(0), (0), P(0) \end{pmatrix} \)
system trajectory \( \begin{pmatrix} d(t), \hat{d}(t), (t), P(t) \end{pmatrix} \)
converges to \( \begin{pmatrix} d^*, \hat{d}^*, *, P^* \end{pmatrix} \) as \( t \to \infty \)

- \( \begin{pmatrix} d^*, \hat{d}^* \end{pmatrix} \) is unique optimal of OLC
- * is unique optimal for dual

- completely decentralized
- frequency deviations contain right info for local decisions that are globally optimal
Recap: control goals

Yes  ■  Rebalance power
Yes  ■  Stabilize frequencies

No  ■  Restore nominal frequency $*$ 0
No  ■  Restore scheduled inter-area flows
No  ■  Respect line limits
Load-side frequency control

- Primary control
- Secondary control
- Interaction with generator-side control
OLC for secondary control

\[
\begin{align*}
\min_{d, \hat{d}, P, v} & \quad c_i(d_i) + \frac{1}{2D_i} \hat{d}_i^2 \\
\text{s. t.} & \quad P^m (d + \hat{d}) = CP \\
& \quad P^m d = CBC^T v
\end{align*}
\]

Demand = Supply

Restore nominal freq

\[
\begin{align*}
\min_{d, P} & \quad c_i(d_i) \\
\text{s. t.} & \quad P^m_i d_i = C_{ie} P_e \quad \text{node } i \\
& \quad C_{ie} P_e = \hat{P}_k \quad \text{area } k \\
& \quad P_e \quad \bar{P}_e \quad \text{line } e
\end{align*}
\]
OLC for secondary control

\[
\begin{align*}
\min_{d, \hat{d}, P, v} & \quad c_i(d_i) + \frac{1}{2D_i} \hat{d}_i^2 \\
\text{s. t.} & \quad P^m (d + \hat{d}) = CP \\
& \quad P^m d = CBC^T v
\end{align*}
\]

key idea: “virtual flows”

\[
BC^T v
\]

in steady state:

virtual flow = real flows

\[
BC^T v = P
\]
OLC for secondary control

\[
\min_{d, \hat{d}, P, v} \sum_{i} c_i (d_i) + \frac{1}{2D_i} \hat{d}_i^2 \]

s. t. \[
P^m (d + \hat{d}) = CP \quad \text{demand = supply}
\]
\[
P^m d = CBC^T v \quad \text{restore nominal freq}
\]
\[
\hat{C}BC^T v = \hat{P} \quad \text{restore inter-area flow}
\]
\[
\underline{P} BC^T v \leq \overline{P} \quad \text{respect line limit}
\]

in steady state:
virtual flow = real flows
\[
BC^T v = P
\]
Recall: primary control

swing dynamics:

\[
\dot{\omega}_i = \frac{1}{M_i} \left( d_i(t) + D_i \omega_i(t) + P_i^m + \sum_{e \in E} C_{ie} P_e(t) \right)
\]

\[
\dot{P}_{ij} = b_{ij} \left( \omega_i(t) - \omega_j(t) \right)
\]

load control:

\[
d_i(t) := c_i \dot{1} \left( \omega_i(t) \right)
\]
Control architecture

Power Network Dynamics
$(\omega, P)$

Dynamic Load Control
$(\lambda, \pi, \rho^+, \rho^-, \nu)$
Secondary frequency control

load control: \( d_i(t) := c_i \left( i(t) + \tilde{i}(t) \right) \)

computation & communication:

primal var: \( \dot{\psi} = \chi^v \left( L_B \lambda - CD_B \hat{C}^T \pi - CD_B (\rho^+ - \rho^-) \right) \)

dual vars:
\[
\begin{align*}
\dot{\lambda} &= \zeta^\lambda (P^m - d - L_B \psi) \\
\dot{\pi} &= \zeta^\pi \left( \hat{C} D_B C^T \psi - \hat{P} \right) \\
\dot{\rho}^+ &= \zeta^{\rho^+} \left[ D_B C^T \psi - \hat{P} \right]_{\rho^+}^+ \\
\dot{\rho}^- &= \zeta^{\rho^-} \left[ P - D_B C^T \psi \right]_{\rho^-}^+
\end{align*}
\]
Secondary control works

**Theorem**

starting from any initial point, system trajectory converges s. t.

- \( \left( d^*, \hat{d}^*, P^*, v^* \right) \) is unique optimal of OLC
- nominal frequency is restored \( * = 0 \)
- inter-area flows are restored \( \hat{C}P^* = \hat{P} \)
- line limits are respected \( \underline{P} \leq P^* \leq \overline{P} \)
Recap: key ideas

Design optimal load control (OLC) problem
- Objective function, constraints

Derive control law as primal-dual algorithms
- Lyapunov stability
- Achieve original control goals in equilibrium

Distributed algorithms

primary control: \( d_i(t) := c_i^{-1}(w_i(t)) \)

secondary control: \( d_i(t) := c_i^{-1}(w_i(t) + l_i(t)) \)
Recap: key ideas

Design optimal load control (OLC) problem
- Objective function, constraints

Derive control law as primal-dual algorithms
- Lyapunov stability
- Achieve original control goals in equilibrium

Distributed algorithms

Virtual flows
- Enforce desired properties on line flows

\[ BC^T v = P \]

in steady state: virtual flow = real flows
Recap: control goals

- **Yes** Rebalance power
- **Yes** Resynchronize/stabilize frequency
  
  Secondary control restores nominal frequency but *requires local communication*

  - **Yes** Restore nominal frequency \( \left( \begin{array}{c} * \\ 0 \end{array} \right) \)
  - **Yes** Restore scheduled inter-area flows
  - **Yes** Respect line limits

  Zhao, et al. TAC2014

Load-side frequency control

- Primary control
- Secondary control
- Interaction with generator-side control

Zhao and Low, CDC2014
Zhao, Mallada, Low, CISS 2015
Generator-side control

New model: nonlinear PF, with generator control

\[
\begin{align*}
\dot{\theta}_i &= \omega_i \\
M_i \ddot{\omega}_i &= -D_i \omega_i + \boxed{P_i} - \sum_{e} C_{ie} P_e \\
P_{ij} &= b_{ij} \sin(\theta_i - \theta_j) \quad \forall \ i \rightarrow j
\end{align*}
\]

Recall model: linearized PF, no generator control

\[
\begin{align*}
M_i \ddot{\omega}_i &= -D_i \omega_i + \boxed{P_i^m - d_i} - \sum_{e} C_{ie} P_e \\
\dot{P}_{ij} &= b_{ij} (\omega_i - \omega_j) \quad \forall \ i \rightarrow j
\end{align*}
\]
Generator-side control

New model: nonlinear PF, with generator control

\[ \dot{\theta}_i = \omega_i \]

\[ M_i \dot{\omega}_i = -D_i \omega_i + p_i - \sum_{e} C_{ie} P_e \]

\[ P_{ij} = b_{ij} \sin(\theta_i - \theta_j) \quad \forall \ i \rightarrow j \]

generator bus: real power injection
load bus: controllable load
Generator-side control

New model: nonlinear PF, with generator control

\[ \dot{\theta}_i = \omega_i \]
\[ M_i \ddot{\omega}_i = -D_i \omega_i + p_i - \sum_e C_{ie} P_e \]
\[ P_{ij} = b_{ij} \sin(\theta_i - \theta_j) \quad \forall \ i \rightarrow j \]

**generator buses:**

primary control \( p^c_i(t) = p^c_i(\theta_i(t)) \)
e.g. freq droop \( p^c_i = p^c_i(\theta_i) \)

\[ \dot{p}_i = -\frac{1}{\tau_{bi}} (p_i + a_i) \]
\[ \dot{a}_i = -\frac{1}{\tau_{gi}} (a_i + p^c_i) \]
Load-side control

Power Network Dynamics
\((q, w, p, a)\)

Dynamic Load Control
\((\lambda, \pi, \rho^+, \rho^-, \nu)\)

physical network

cyber network

\[ d \]

\[ d \]

\[ d \]

\[ \omega \]

\[ \lambda \]
Load-side primary control works

**Theorem**

- Every closed-loop equilibrium solves OLC and its dual

Suppose

\[
\begin{vmatrix}
    p_i^c(\cdot)
    & p_i^c(\ast)
    & L_i
    & \ast
\end{vmatrix}
\]

near \(*\) for some \(L_i < D_i\)

- Any closed-loop equilibrium is (locally) asymptotically stable provided

\[
\left| \begin{array}{cc}
    \ast & \ast \\
    i & j
\end{array} \right| < \frac{1}{2}
\]