The Flow of Power

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Caltech

Simons Institute: Real-time Decision Making Bookcamp
Power Systems, Berkeley, January 2018

Bootcamp: Power systems

The flow of power (S Low)

- Basic concepts and models
- Power flow and optimization

The flow of information (S Meyn)

- Distributed control architectures

The flow of money (K Poolla)

- Market structures and services

from steady state to dynamics
from engineering to economics

R. Karp's instruction

"... the level should be sufficiently elementary that an expert on the topic will be bored."

The flow of power I

Basic concepts and models

Why smart grid? (15 mins)

Three-phase AC transmission: 3 key ideas (30 mins)

- Phasor representation
- Balanced operation
- Per-phase analysis

Device models (30 mins)

- Transmission line
- Transformer
- Generator

The flow of power II

Power flow and optimization

Network models (10mins)

- Admittance matrix
- Power flow models

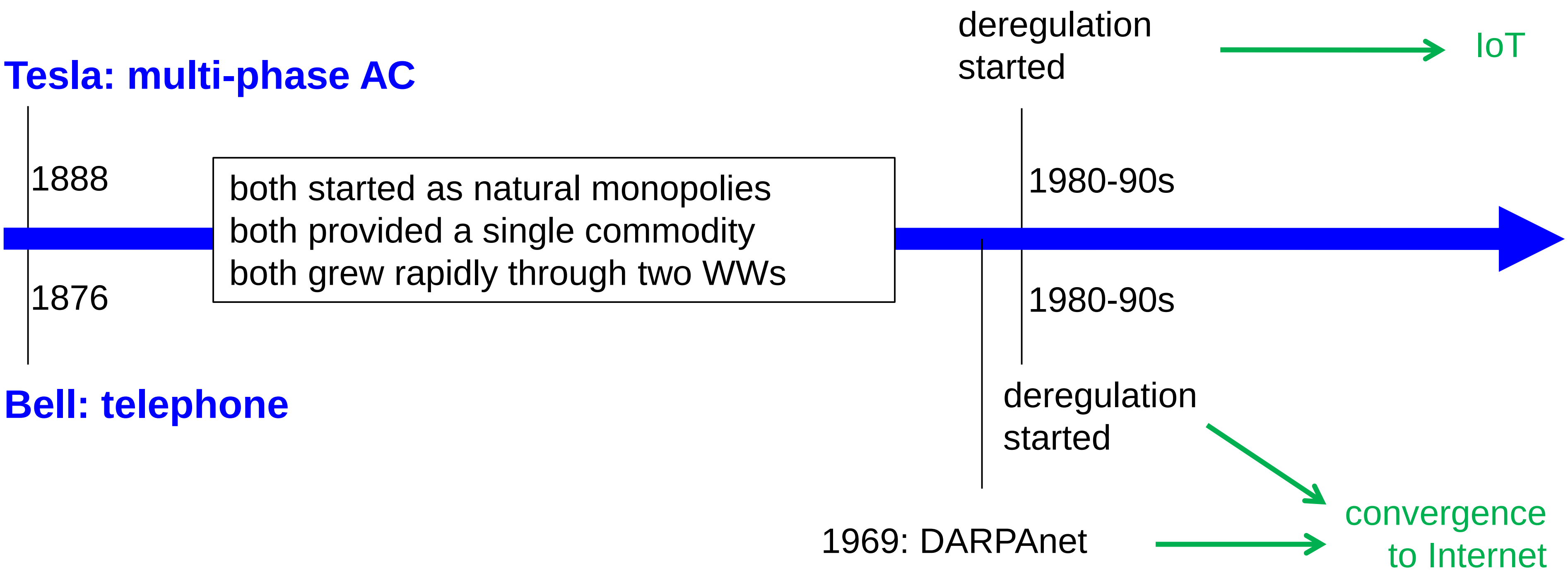
Optimal power flow problems (35mins)

- Formulation and example
- Convex relaxations
- Real-time OPF

Why smart grid?

Watershed moment

Energy network will undergo similar architectural transformation that phone network went through in the last two decades to become the world's largest and most complex IoT



Watershed moment

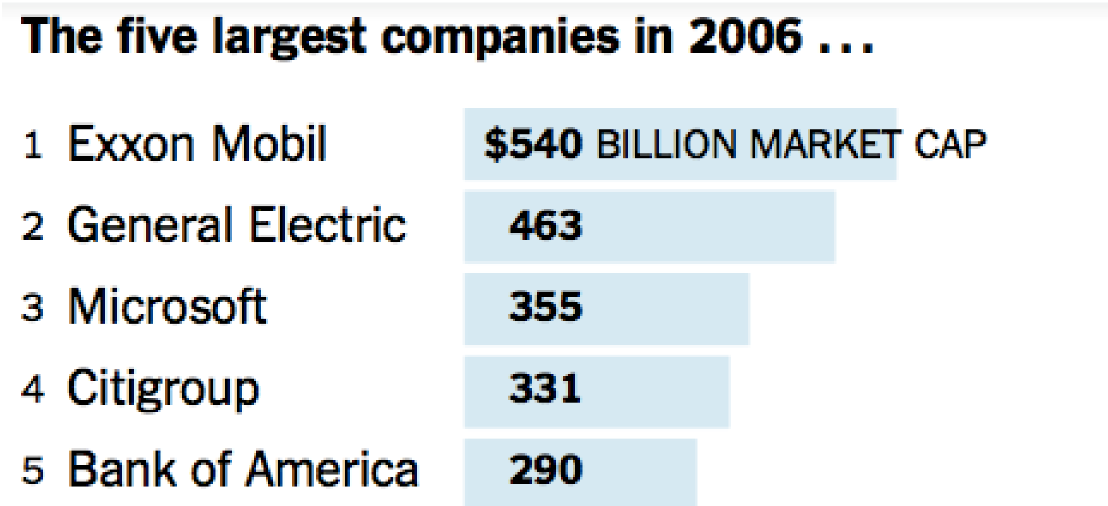
Industries will be restructured

- AT&T, MCI, McCaw Cellular, Qualcom
- Google, Facebook, Twitter, Amazon, eBay, Netflix

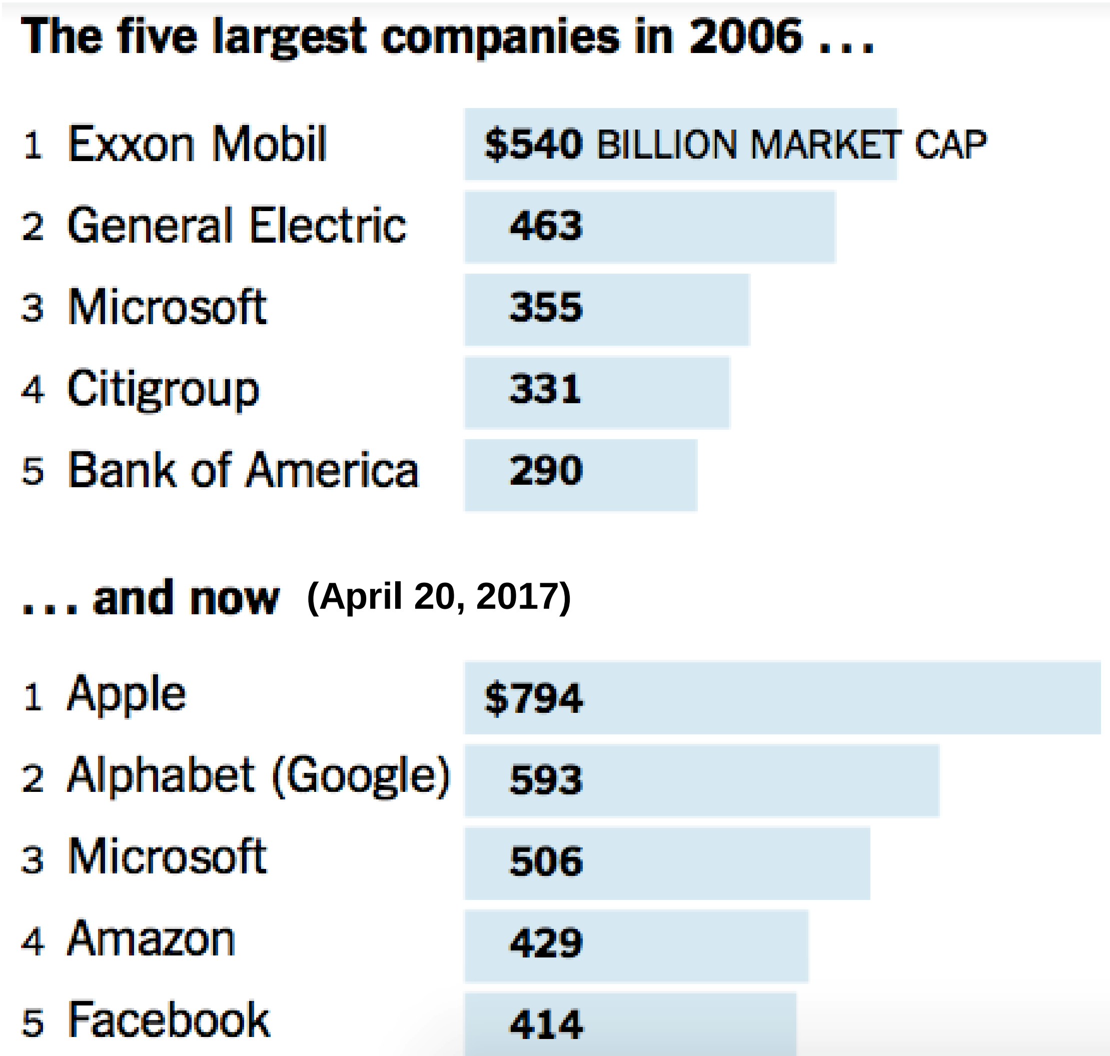
Infrastructure will be reshaped

- Centralized intelligence, vertically optimized
- Distributed intelligence, layered architecture

Watershed moment

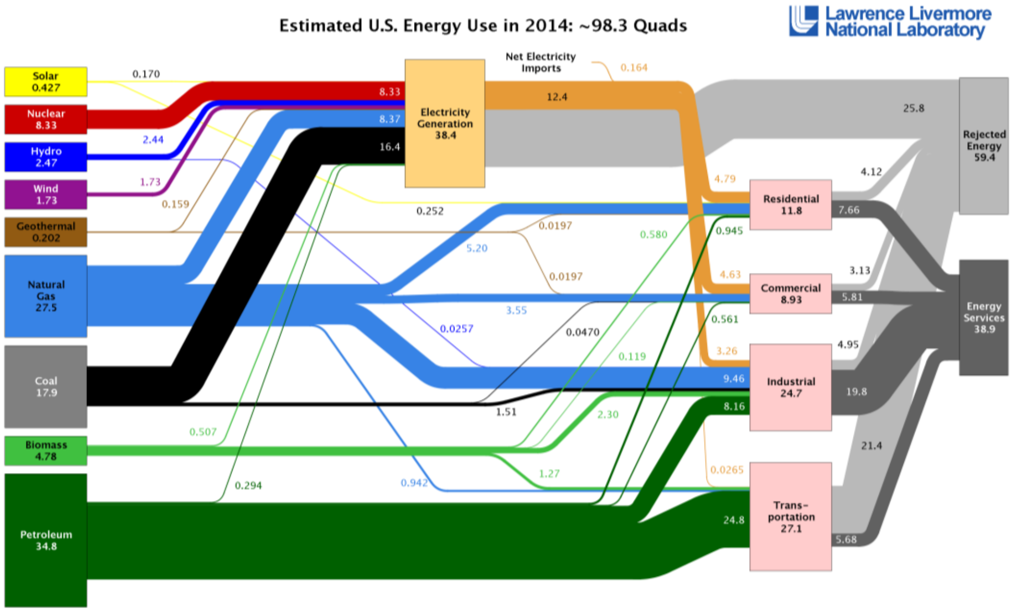


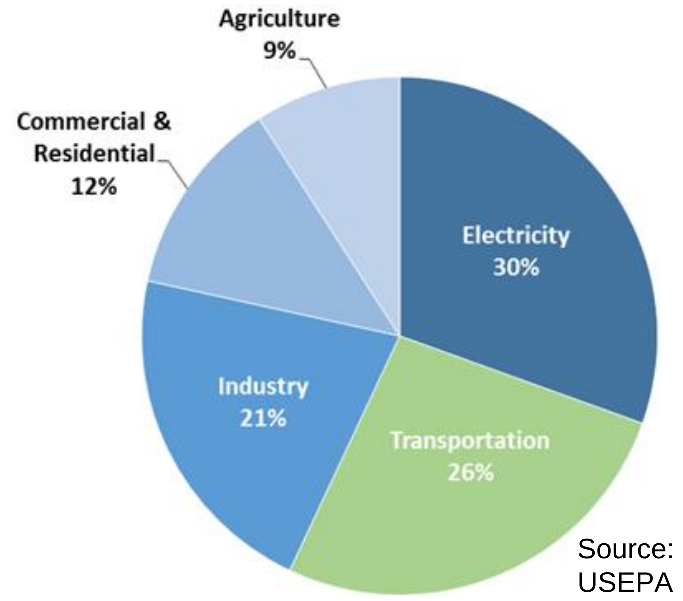
Watershed moment



What will drive power network transformation ?

Electricity gen & transportation





Source: USEPA

They consume the most energy

- Consume 2/3 of all energy in US (2014)

They emit the most greenhouse gases

- Emit >1/2 of all greenhouse gases in US (2014)

To drastically reduce greenhouse gases

- Generate electricity from renewable sources
- Electrify transportation

World energy stats (2011)

Consumption	519 quad BTU
petroleum	34%
coal	29%
gas	23%
renewable (elec)	8%
nuclear	5%

top 5 countries

Consumption	519 (quad BTU)	per capita (mil BTU)
China	20%	78
US	19%	313
Russia	6%	209
India	5%	20
Japan	4%	164
total	**54%**	

Source: EIA

World energy stats (2011)

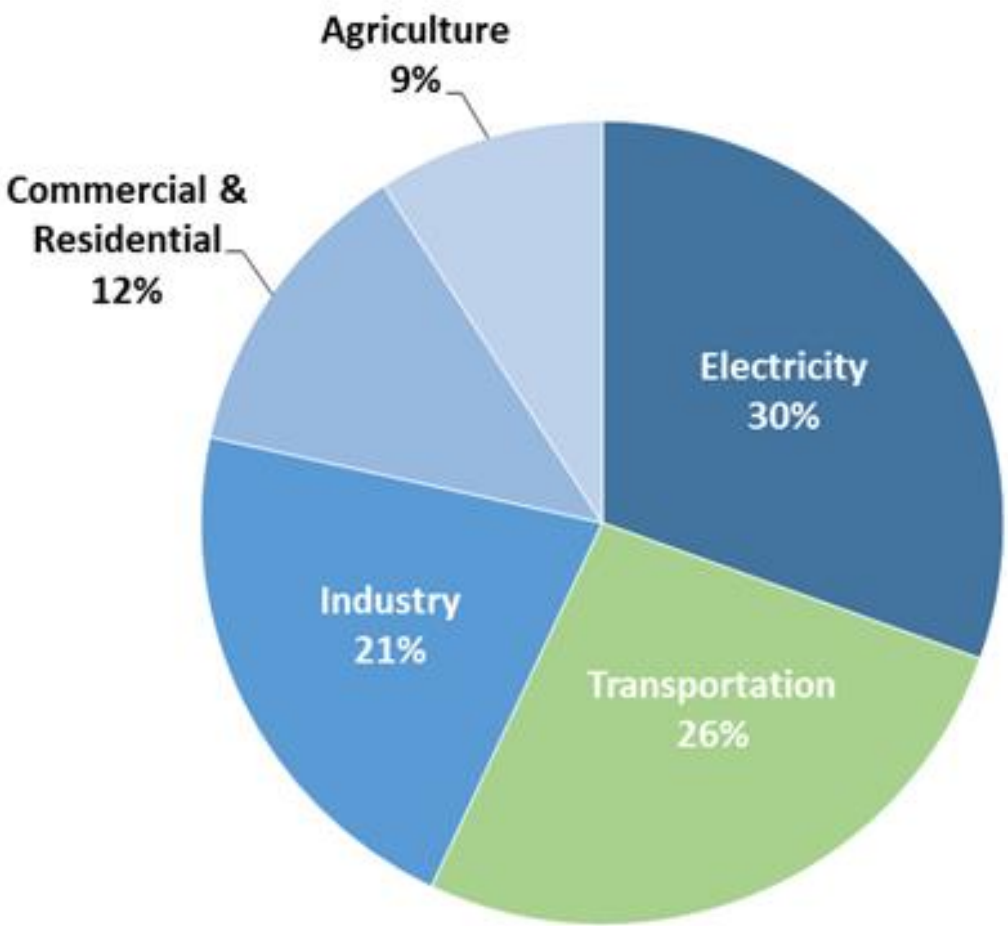
Consumption	519 quad BTU
petroleum	34%
coal	29%
gas	23%
renewable (elec)	8%
nuclear	5%

top 5 countries

Consumption	519 (quad BTU)	CO2 emission
China	20%	27%
US	19%	17%
Russia	6%	5%
India	5%	5%
Japan	4%	4%
total	**54%**	**58%**

Source: EIA

US greenhouse gas emission 2014



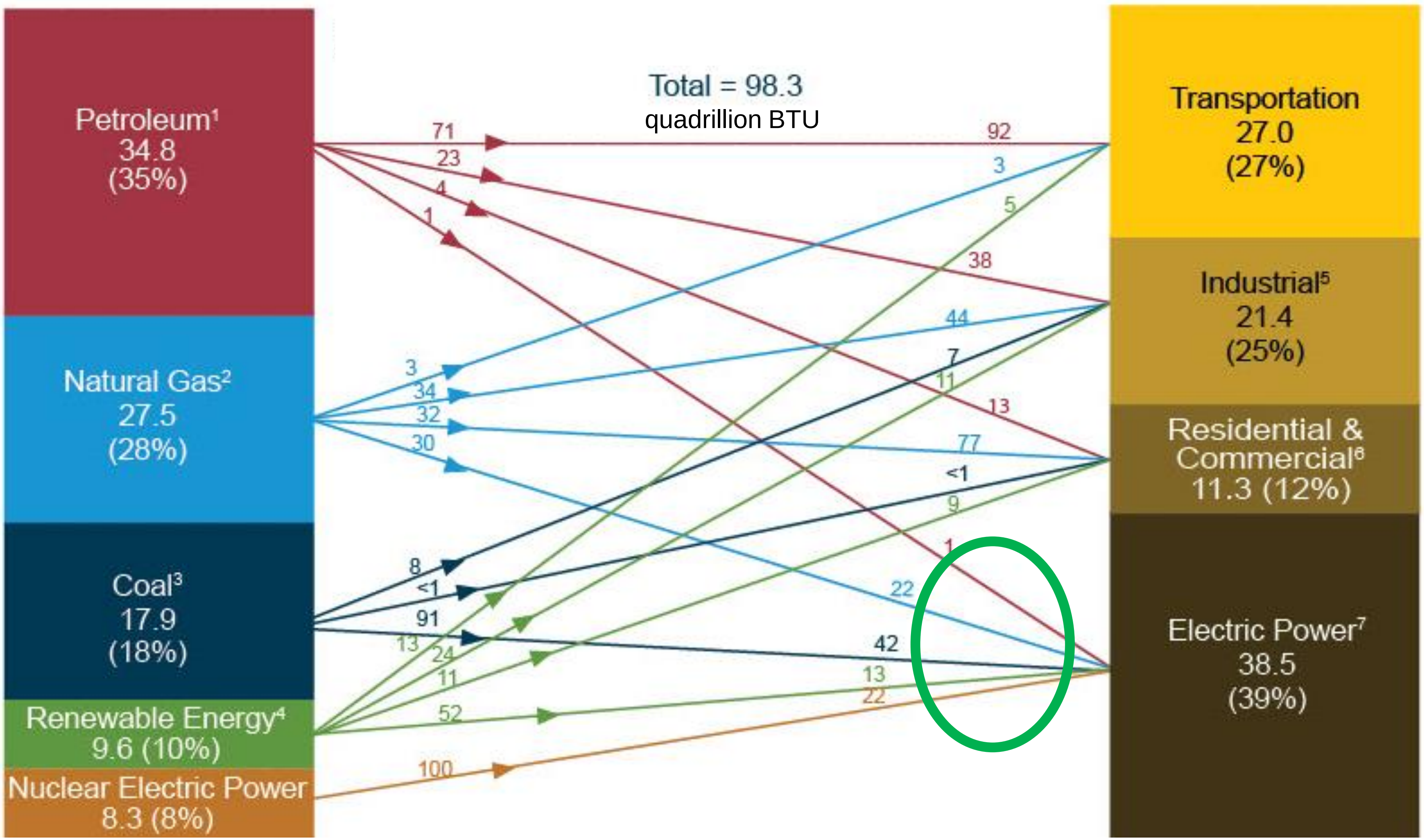
Electricity generation and transportation are top-two GHG emitters (56% total)

… and they consume the most energy (66% total)

Total (2014) = 6,870 Million Metric Tons of CO2 equivalent

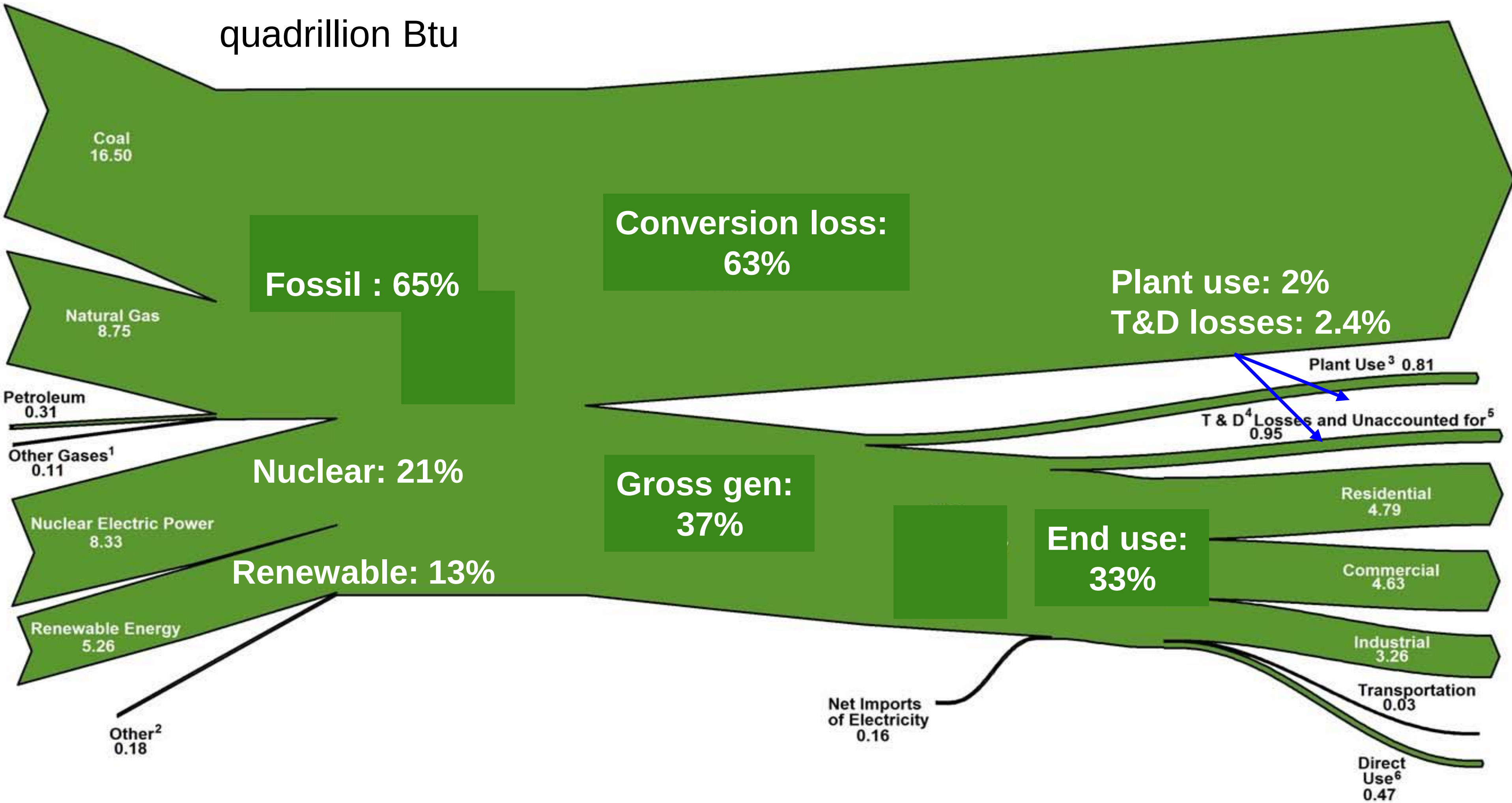
Source: USEPA, https://www3.epa.gov/climatechange/ghgemissions/sources/transportation.html

US energy flow 2014



Source: EIA Monthly Energy Review March 2015

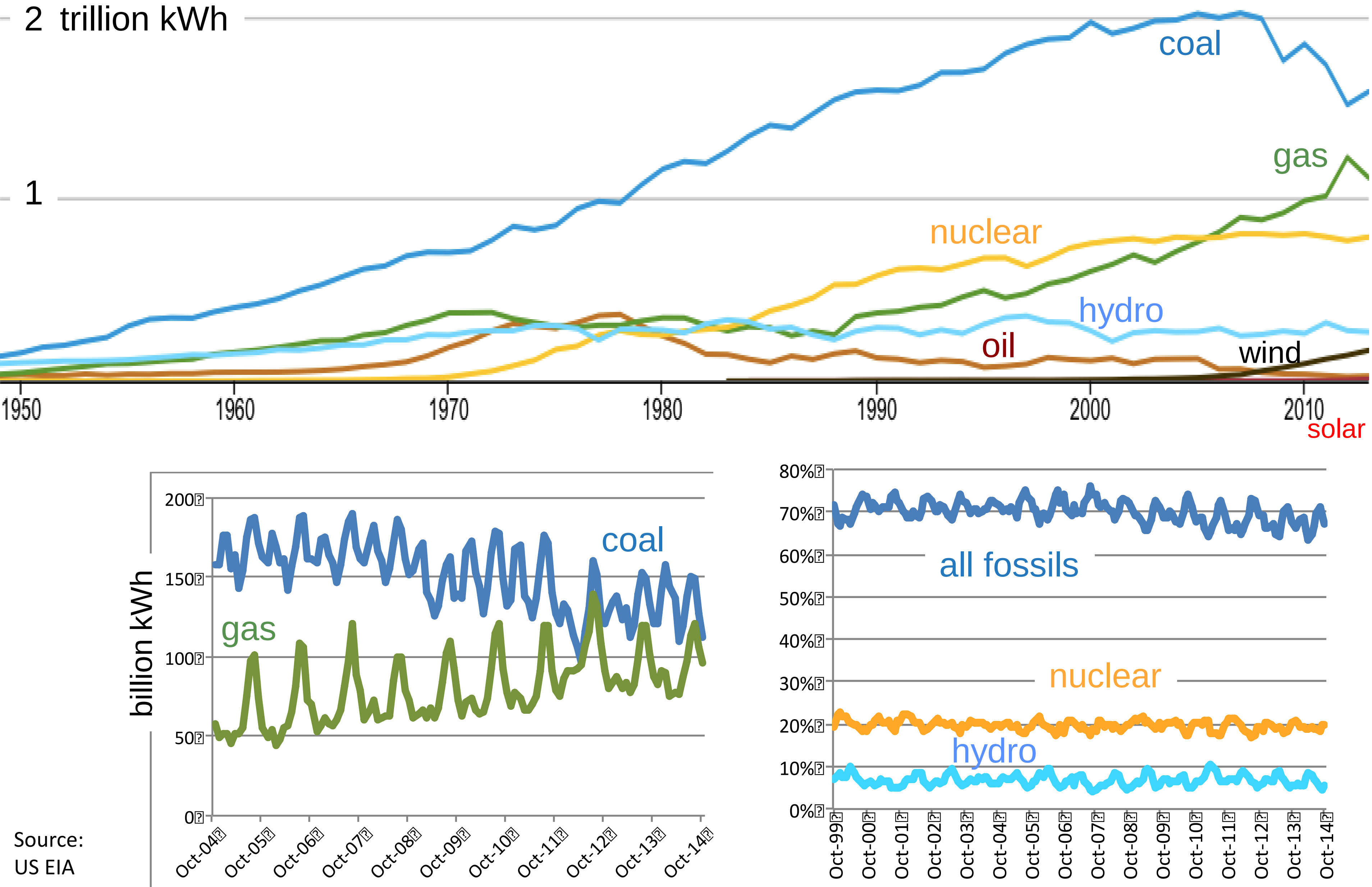
US electricity flow 2014



US total energy use: 98.3 quads
For electricity gen: 39%

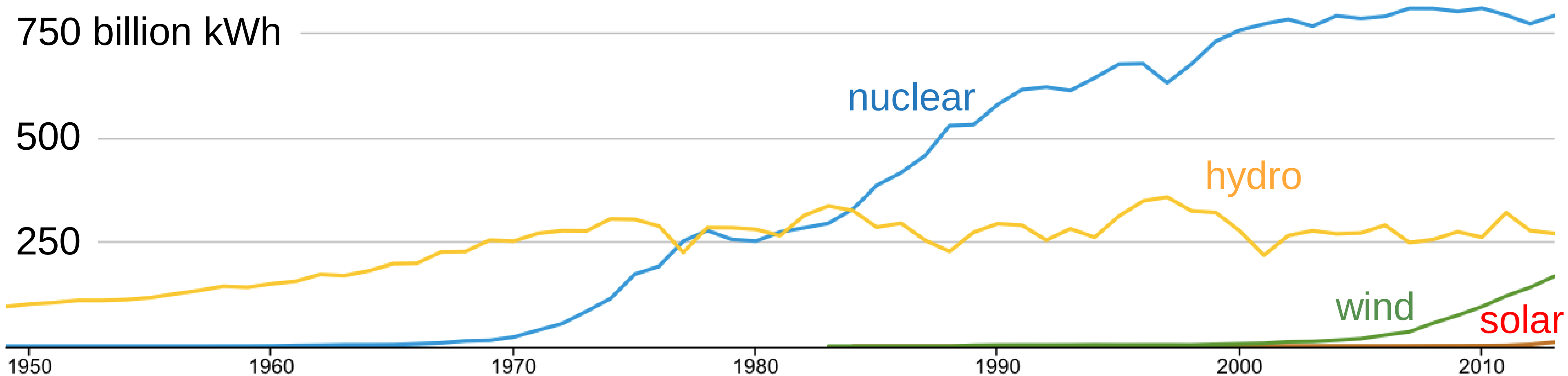
Source: EIA March 2015 Monthly Energy Review

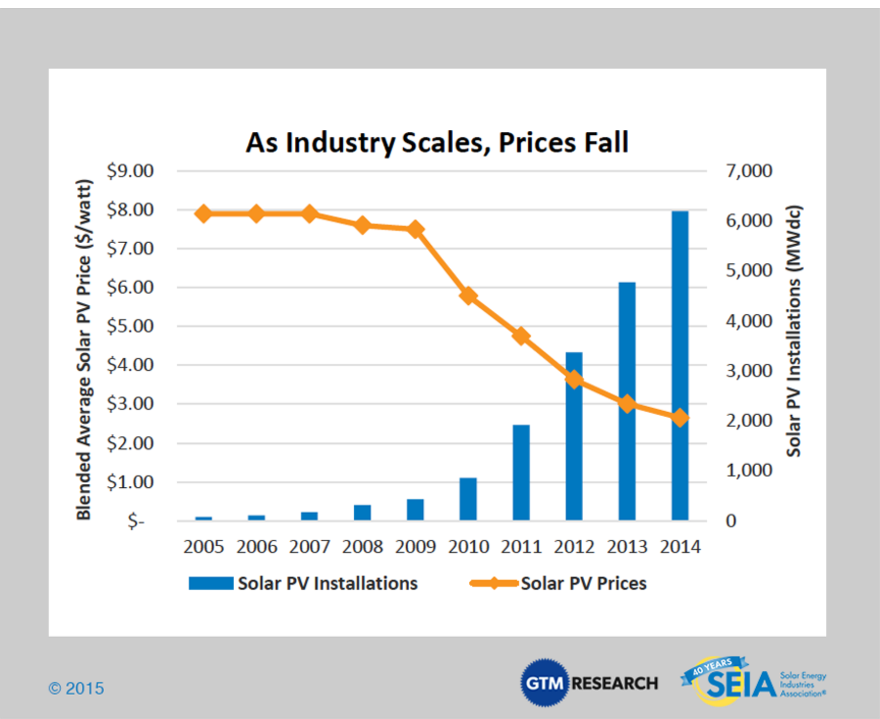
US dirty supply

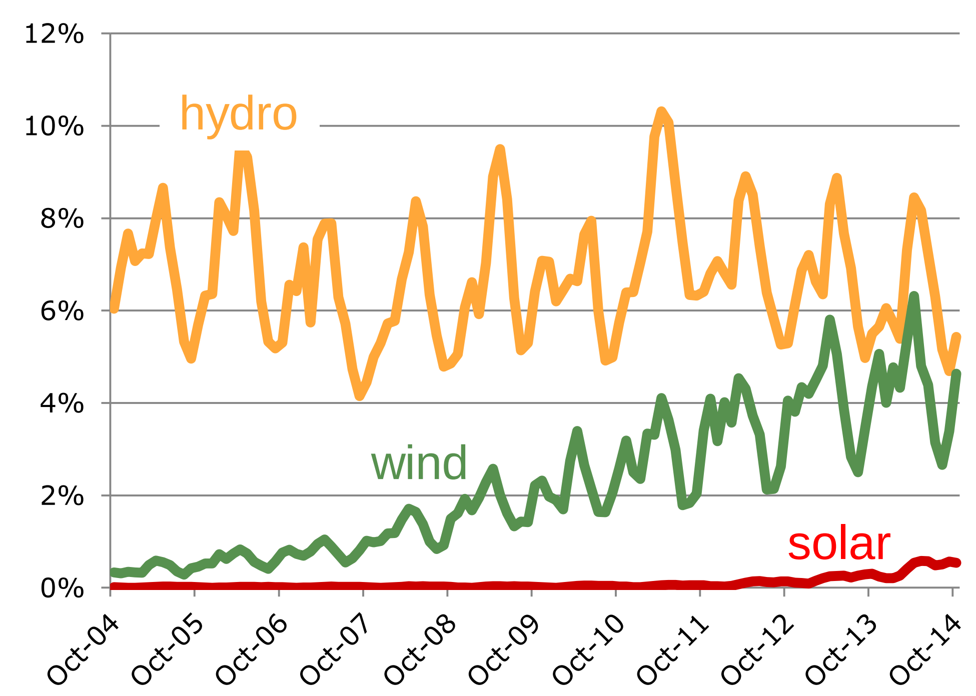


Source:
US EIA

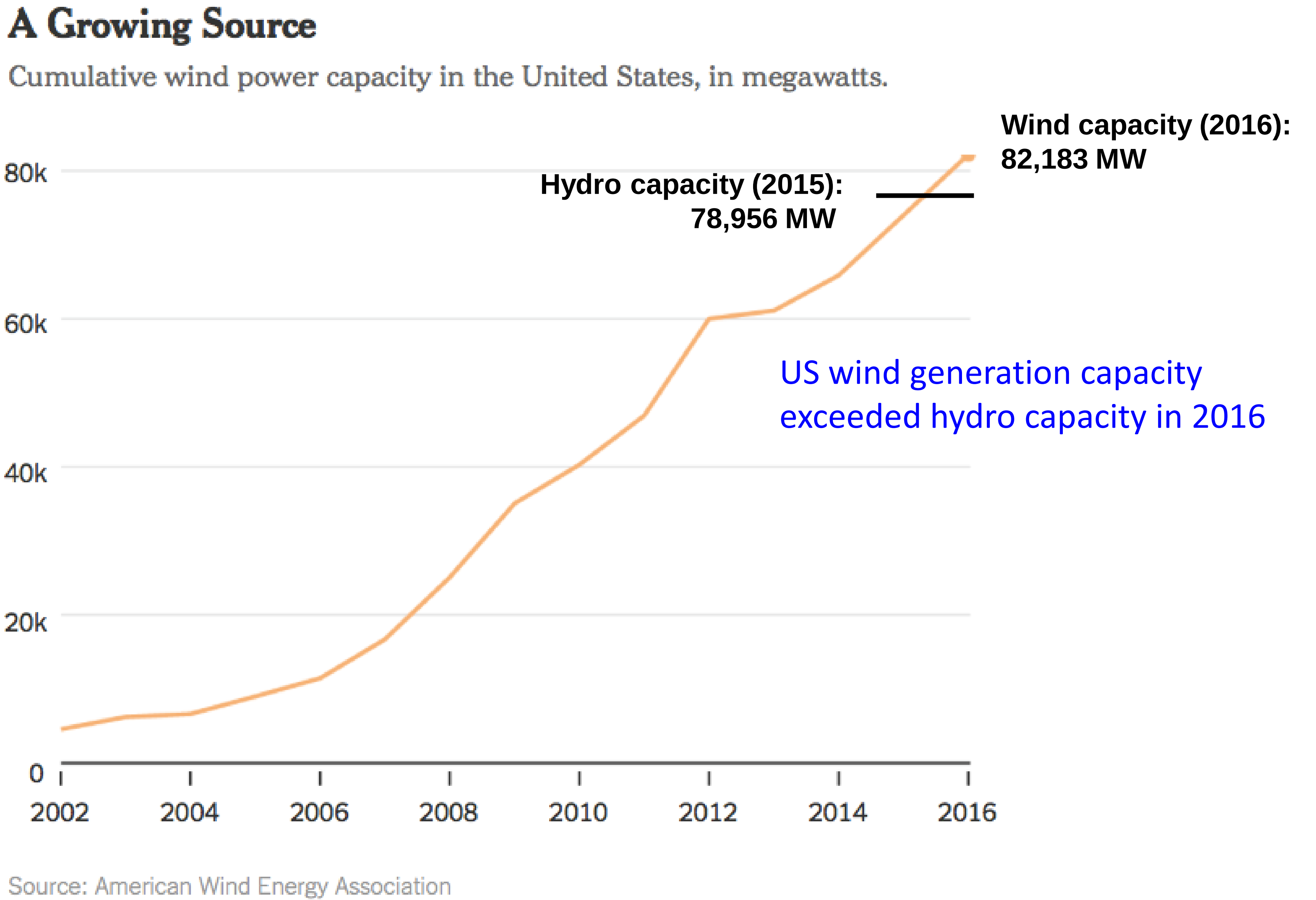
US renewable generations





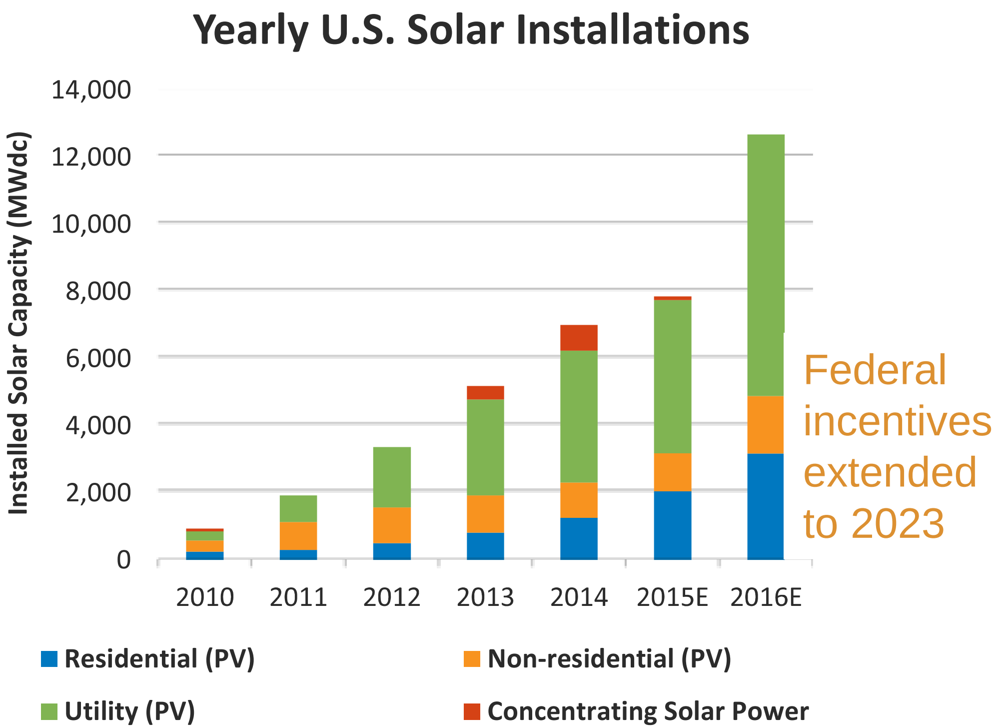


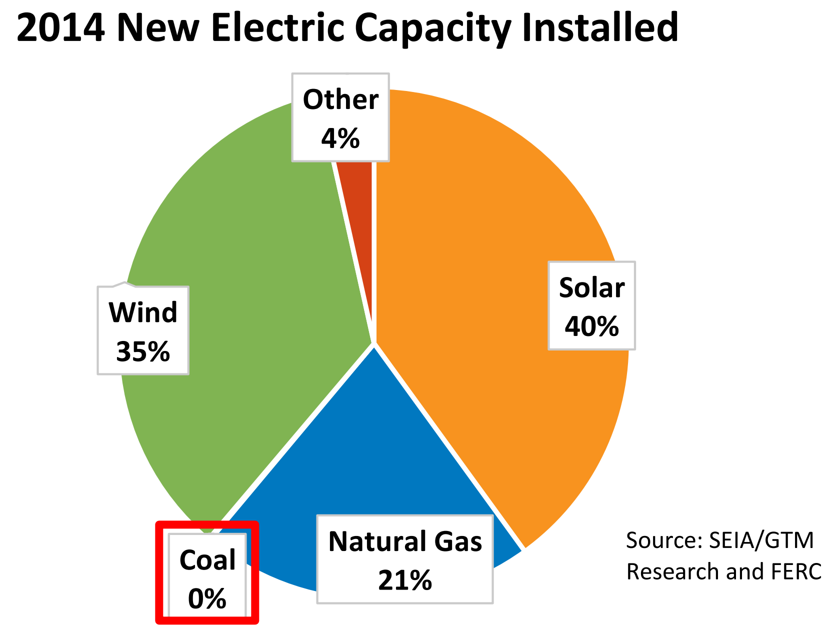
US wind capacity



US wind generation capacity exceeded hydro capacity in 2016

US solar capacity





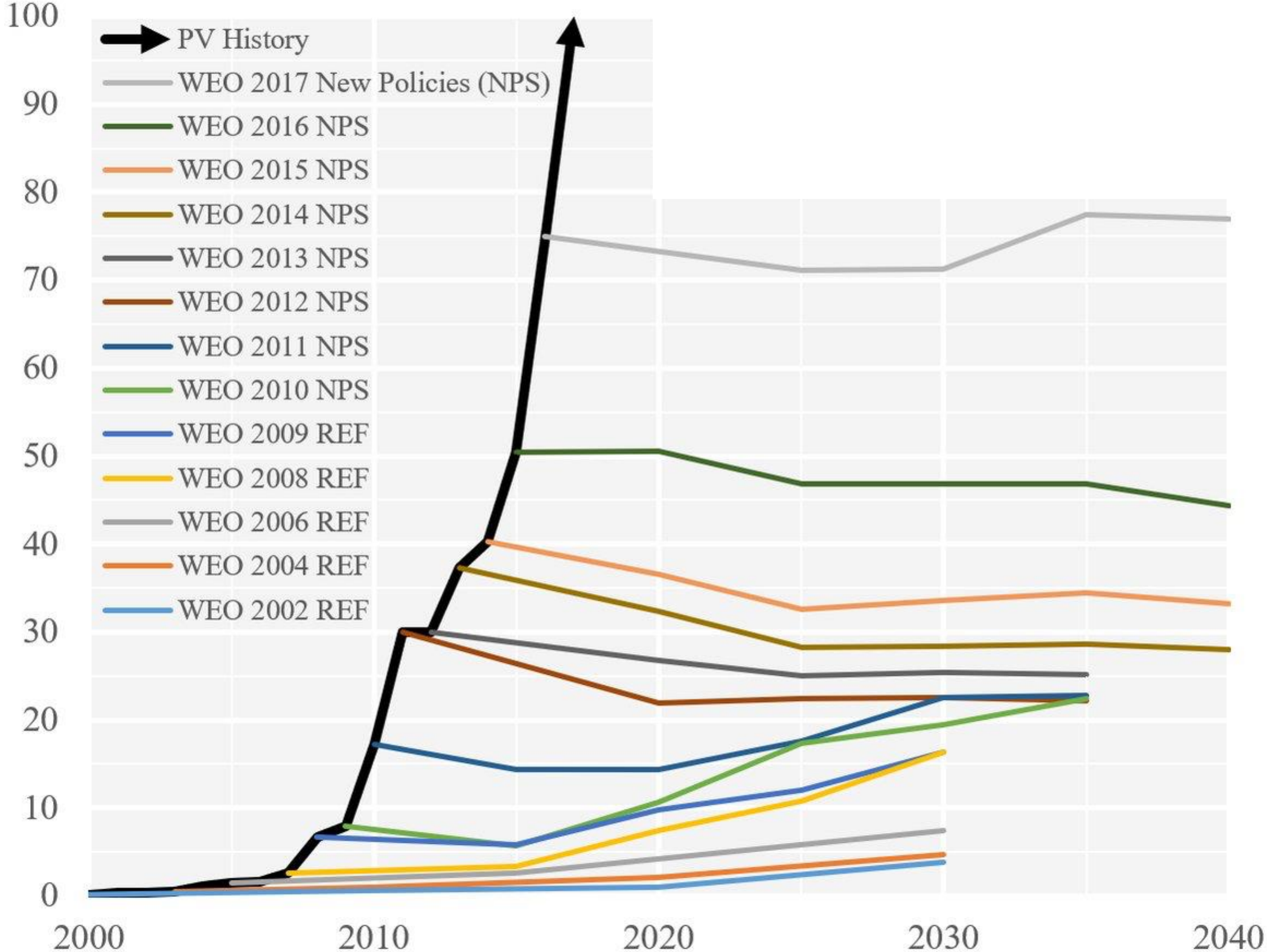
US solar industry snapshot

- US installed solar capacity by mid 2015: ~23 GW
  - 784K homes and businesses
- Q2 2015 solar installation: 1.4 GW
  - Utility: 729 MW
  - Residential: 473 MW (70% growth yr-on-yr)
- H1 2015: a new solar installation / 2 mins

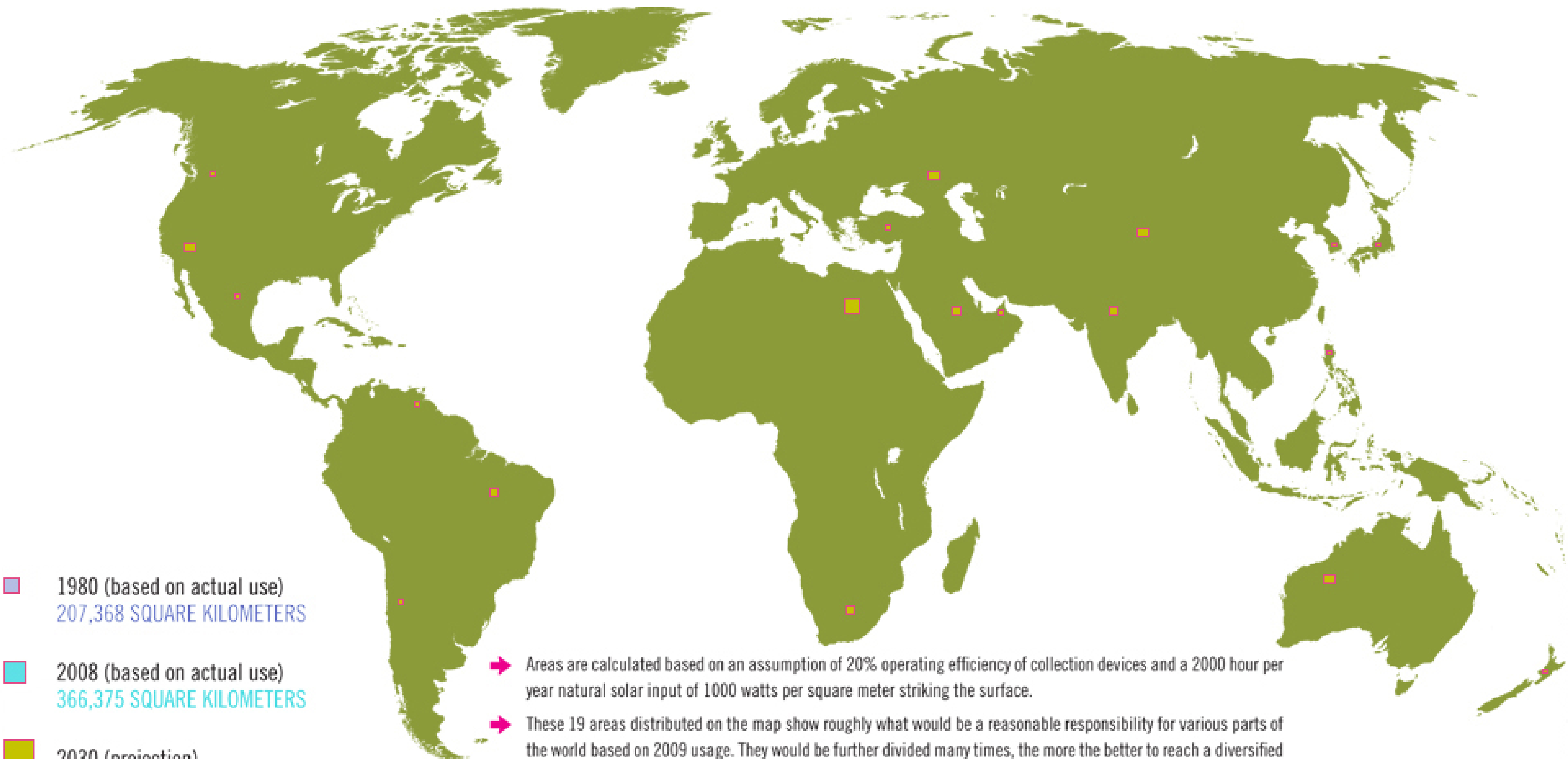
Source: SEIA 2015 (Solar Energy Industries Association)

Annual PV additions: historic data vs IEA WEO predictions

In GW of added capacity per year - source International Energy Agency - World Energy Outlook



Power the world by solar



1980 (based on actual use)
207,368 SQUARE KILOMETERS

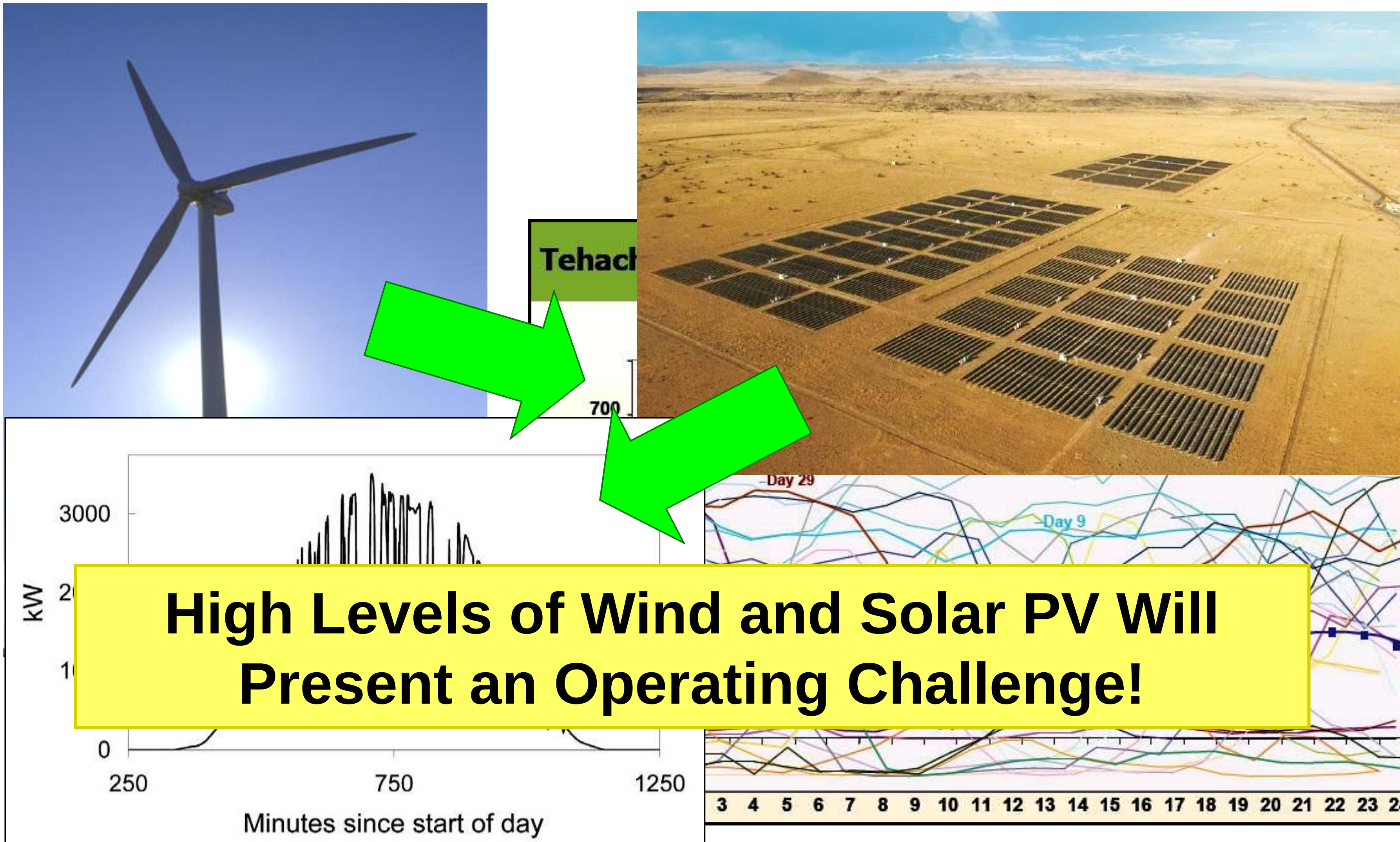
2008 (based on actual use)
366,375 SQUARE KILOMETERS

2030 (projection)
496,805 SQUARE KILOMETERS

- Areas are calculated based on an assumption of 20% operating efficiency of collection devices and a 2000 hour per year natural solar input of 1000 watts per square meter striking the surface.
- These 19 areas distributed on the map show roughly what would be a reasonable responsibility for various parts of the world based on 2009 usage. They would be further divided many times, the more the better to reach a diversified infrastructure that localizes use as much as possible.
- The large square in the Saharan Desert (1/4 of the overall 2030 required area) would power all of Europe and North Africa. Though very large, it is 18 times less than the total area of that desert.
- The definition of "power" covers the fuel required to run all electrical consumption, all machinery, and all forms of transportation. It is based on the US Department of Energy statistics of worldwide Btu consumption and estimates the 2030 usage (678 quadrillion Btu) to be 44% greater than that of 2008.
- Area calculations do not include magenta border lines.

LAND ART GENERATOR INITIATIVE

Uncertainty

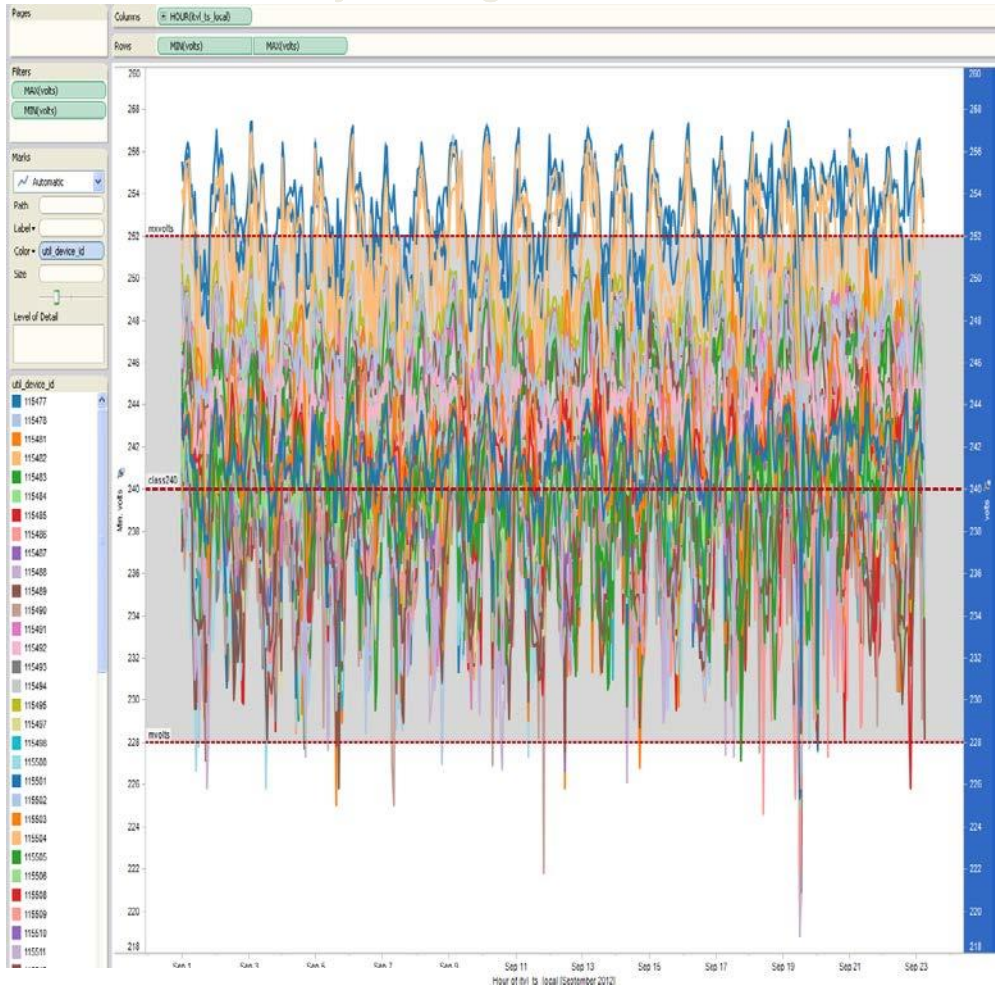


High Levels of Wind and Solar PV Will Present an Operating Challenge!

Source: Rosa Yang, EPRI

- 68 meters (residential)
- Sept 2012 (23 days)
- 240 volts
- +-5% min-228/max-252
- Hourly by meter #
- A few “high” meters
- Larger # of low meters

Hourly Voltage Overview[1]



Voltage violations are quite frequent

SunShot U.S. Department of Energy

GO solar CALIFORNIA

High Penetration

2013
Feb 13-14, San Diego, CA

Source: Leon Roose, University of Hawaii
Development & demo of smart grid inverters for high-penetration PV applications

Hourly Voltage Overview[1]

Hawaii's solar power flare-up: Too much of a good thing?

So many private solar panels are returning power to the grid that utilities ... systems can't handle it all.

November 17, 2013

...imes

Germany's Green Energy Destabilizing Electric Grids

“Energiewende”

JANUARY 23, 2013

Power struggle: Green energy versus a grid that's not ready

Minders of a fragile national power grid say the rush to renewable energy might actually make it harder to keep the lights on.

December 02, 2013 | By Evan Halper

...volution Hiccups: Grid Instability ...or Solutions

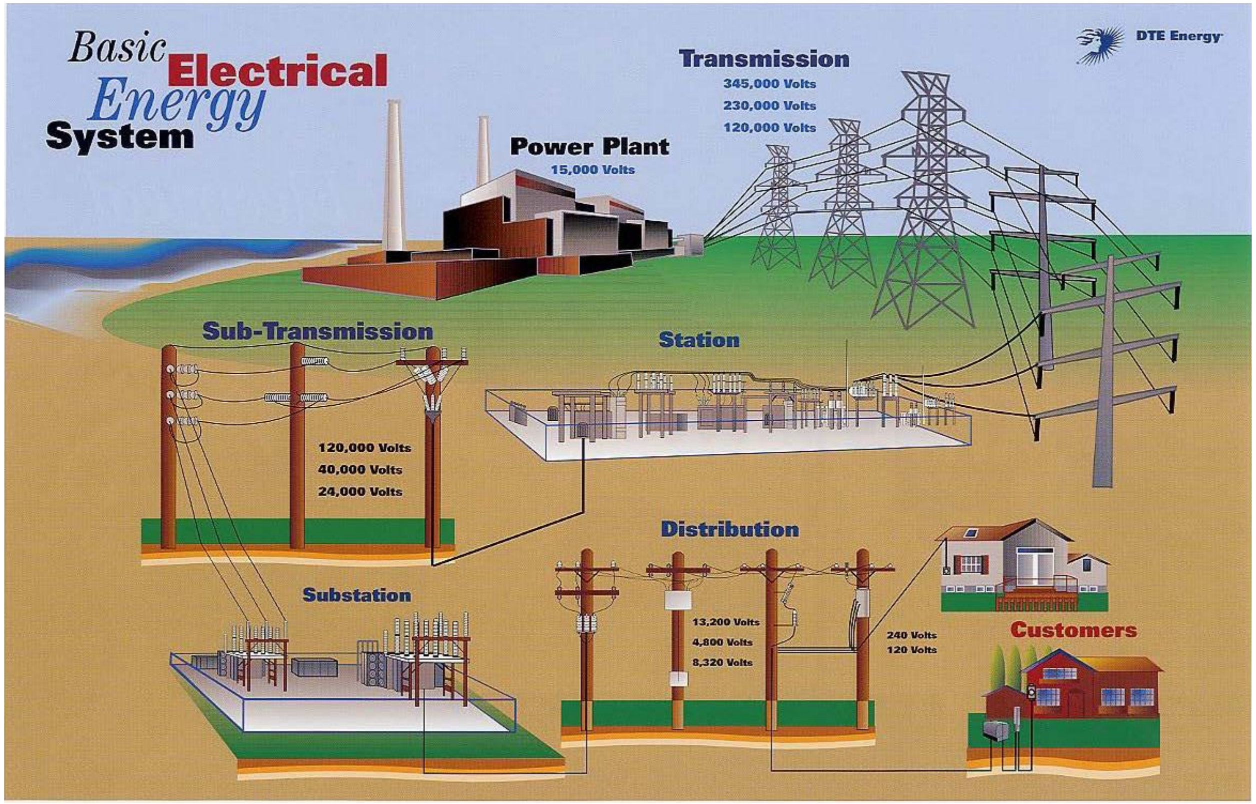
By Catalina Schröder

Sudden fluctuations in Germany's power grid are causing major dam... companies. While many of them have responded by getting their ow... help minimize the risks, they warn that companies might be forced t... with the issues fast.

Power customers opt to go off grid

...WAII

Today's grid



Few large generators

- ~10K bulk generators (>90% capacity), actively controlled

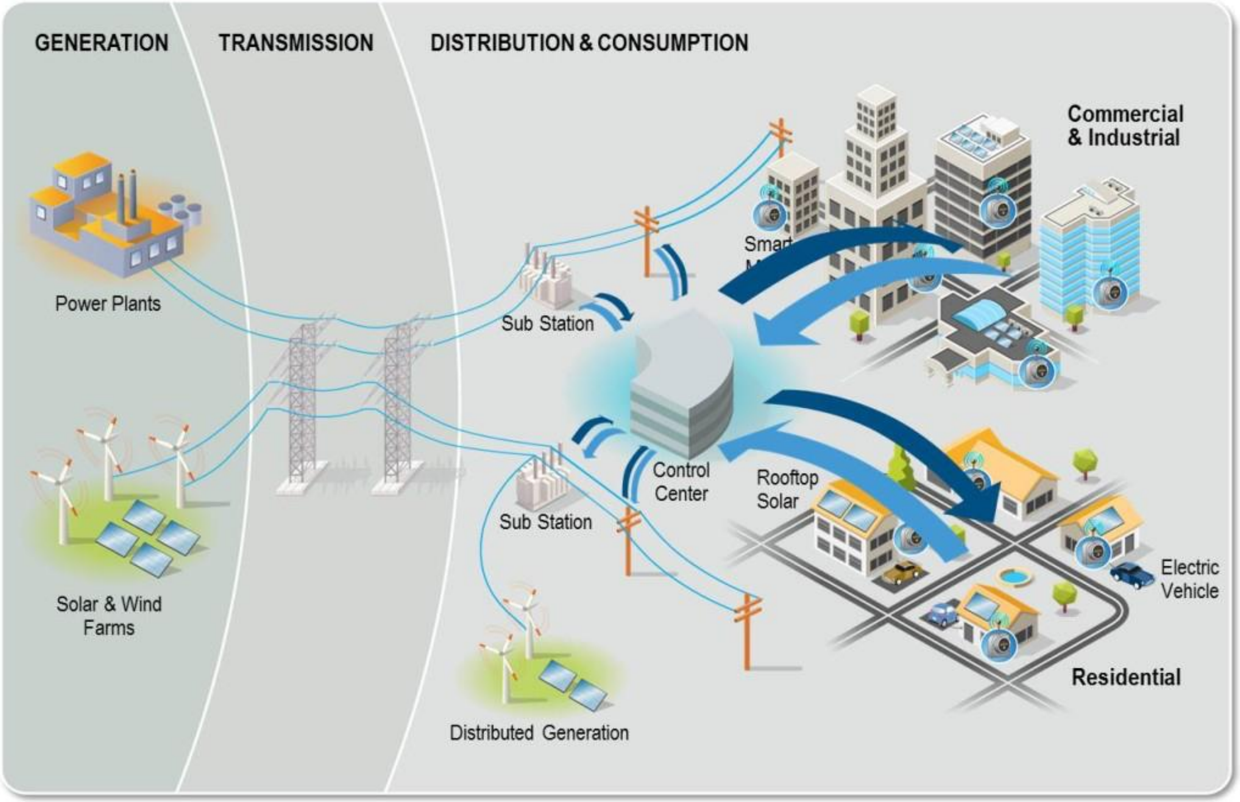
Many dump loads

- 131M customers, 3,100 utilities, ~billion passive loads

Control paradigm: schedule supply to match demand

- Centralized, human-in-the-loop, worst case, deterministic

Future grid



Wind and solar farms are not dispatchable

- Many small distributed generations

Network of distributed energy resources (DERs)

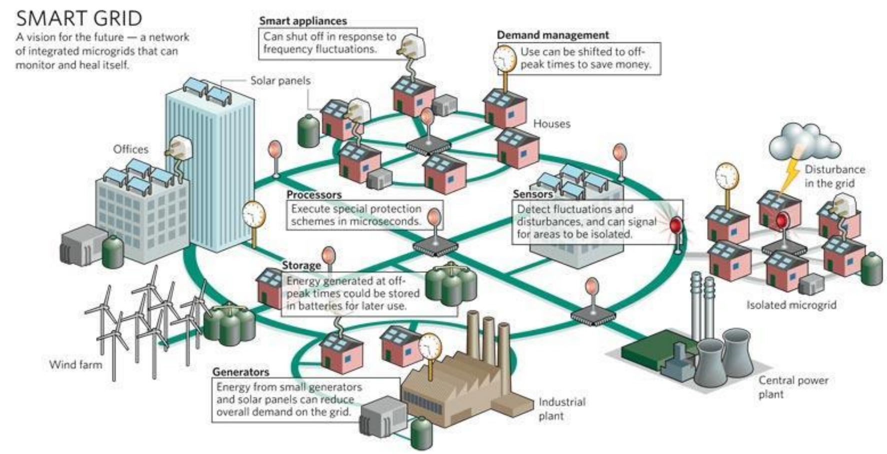
- EVs, smart buildings/appliances/inverters, wind turbines, storage

Control paradigm: match demand to volatile supply

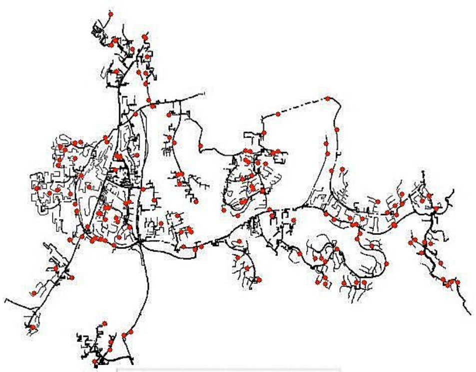
- Distributed, real-time feedback, risk limiting, robust

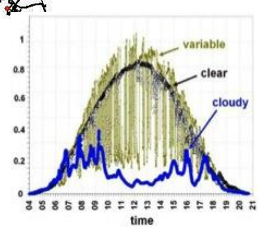
Risk: active DERs introduce rapid random fluctuations in supply, demand, power quality increasing risk of blackouts

Opportunity: active DERs enables realtime dynamic network-wide feedback control, improving robustness, security, efficiency



Caltech research: distributed control of networked DERs





- Foundational theory, practical algorithms, concrete applications
- Integrate engineering and economics
- Active collaboration with industry

Recap

Global energy demand will continue to grow

There is more renewable energy than the world ever needs

- Someone will figure out how to capture and store it

There will be connected intelligence everywhere

- Cost of computing, storage, communication and manufacturing will continue to drop

→ Power system will transform into the largest and most complex Internet of Things

- Generation, transmission, distribution, consumption, storage

Recap

To develop technologies that will enable and guide the historic transformation of our power system

- Materials, devices, systems, theory, algorithms
- Control, optimization, stochastics, data, economics

Motivation: Optimal power flow

min	$\mathrm{tr}\left(CVV^H\right)$	gen cost, power loss
over	$\left(V, s, l\right)$	
subject to	$s_j \;=\; \mathrm{tr}\left(Y_j^H VV^H\right)$	power flow equation
	$l_{jk} \;=\; \mathrm{tr}\left(B_{jk}^H VV^H\right)$	line flow
	$\underline{s}_j \;\text{£}\; s_j \;\text{£}\; \overline{s}_j$	injection limits
	$\underline{l}_{jk} \;\text{£}\; l_{jk} \;\text{£}\; \overline{l}_{jk}$	line limits
	$\underline{V}_j \;\text{£}\; \lvert V_j \rvert \;\text{£}\; \overline{V}_j$	voltage limits

- Y_j^H describes network topology and impedances
- s_j is net power injection (generation) at node j

The flow of power I

Basic concepts and models

Why smart grid? (15 mins)

Three-phase AC transmission: 3 key ideas (30 mins)

- Phasor representation
- Balanced operation
- Per-phase analysis

Device models (30 mins)

- Transmission line
- Transformer
- Generator

Visualizing the grid

adapted from

Electric(Power(Delivery(Systems(

Tutorial(at(U.C.(Berkeley(

September(11,(2009(

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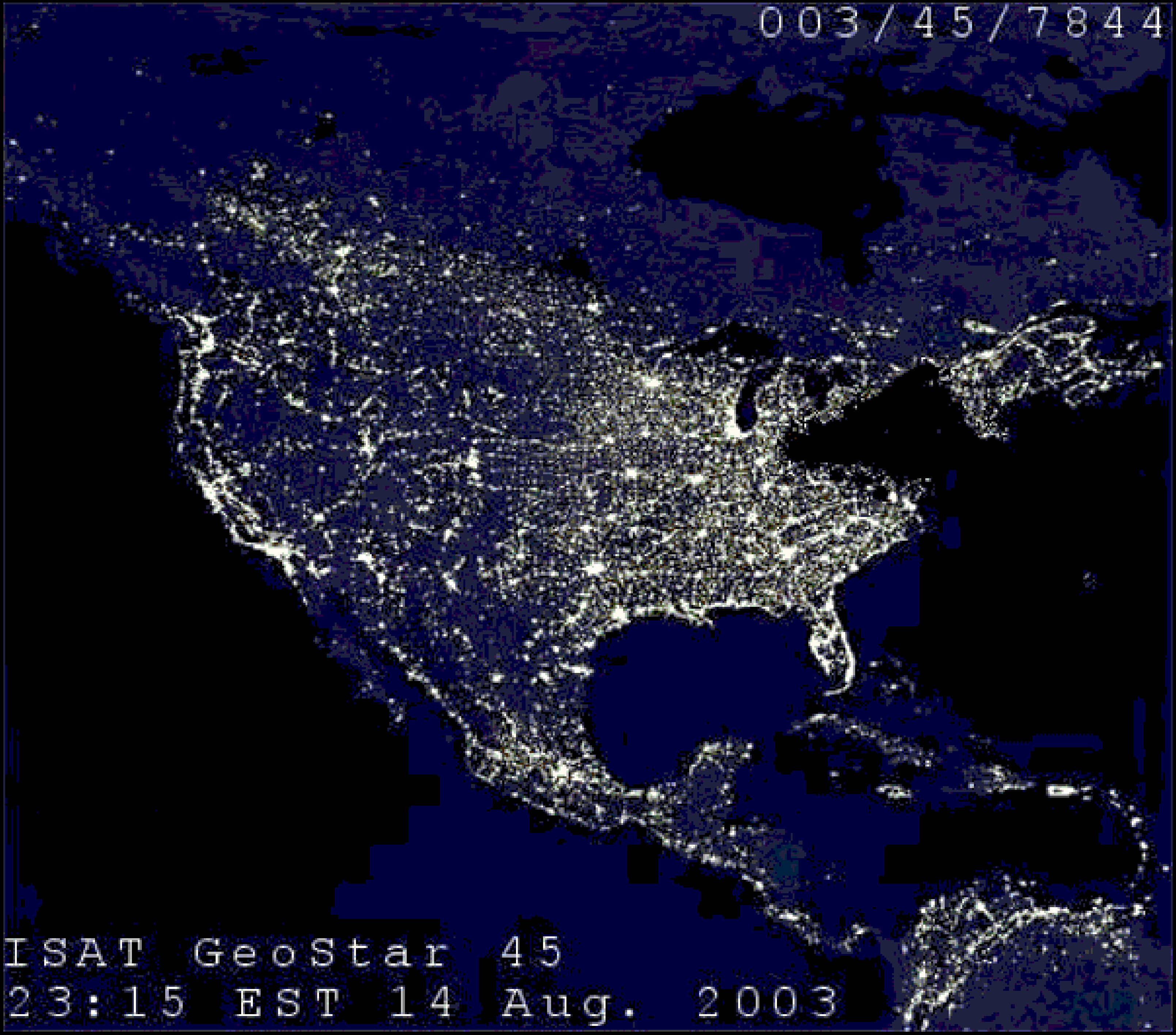
Dr.(Alexandra(“Sascha”(von(Meier(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ISAT GeoStar 45
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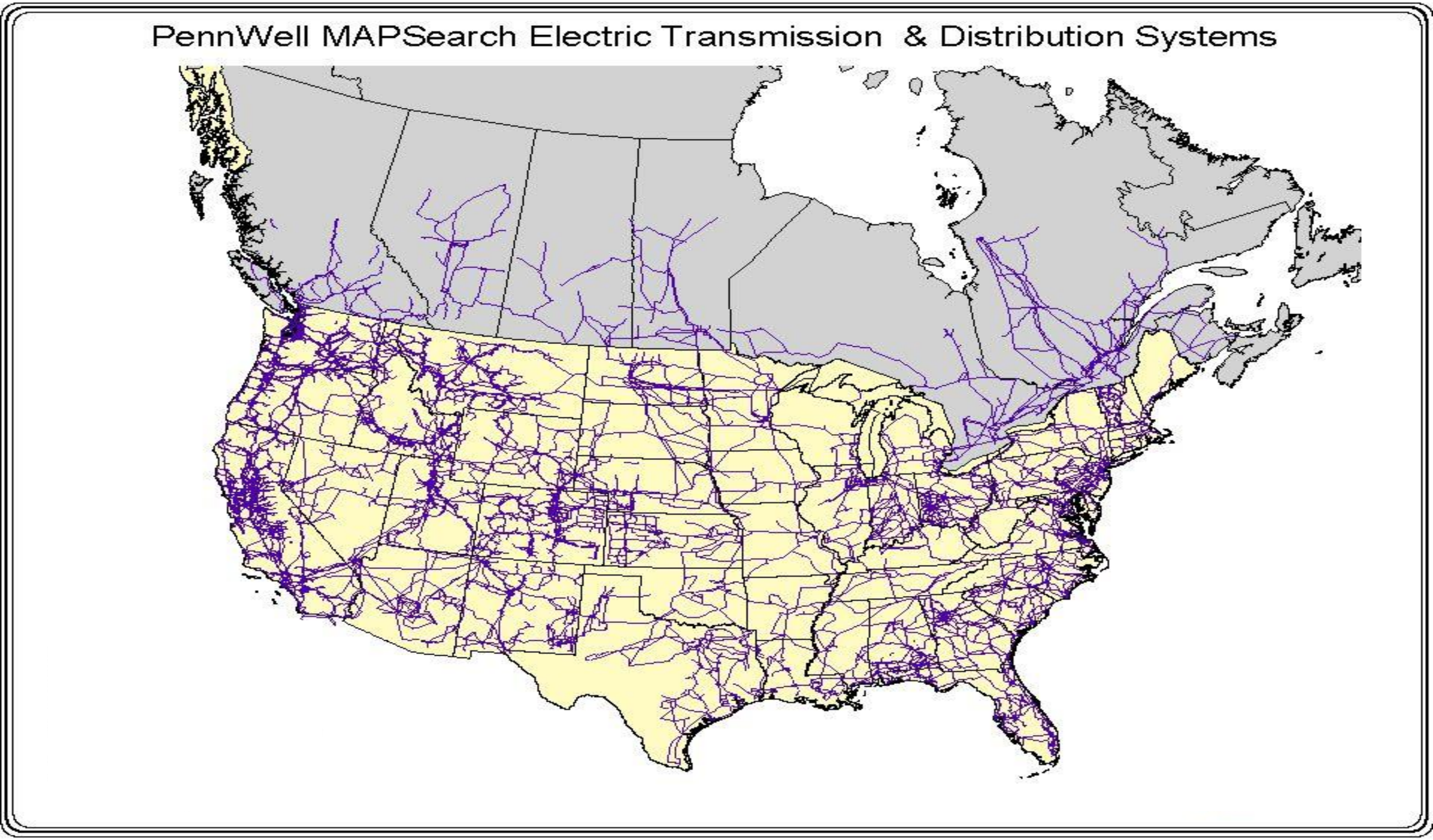
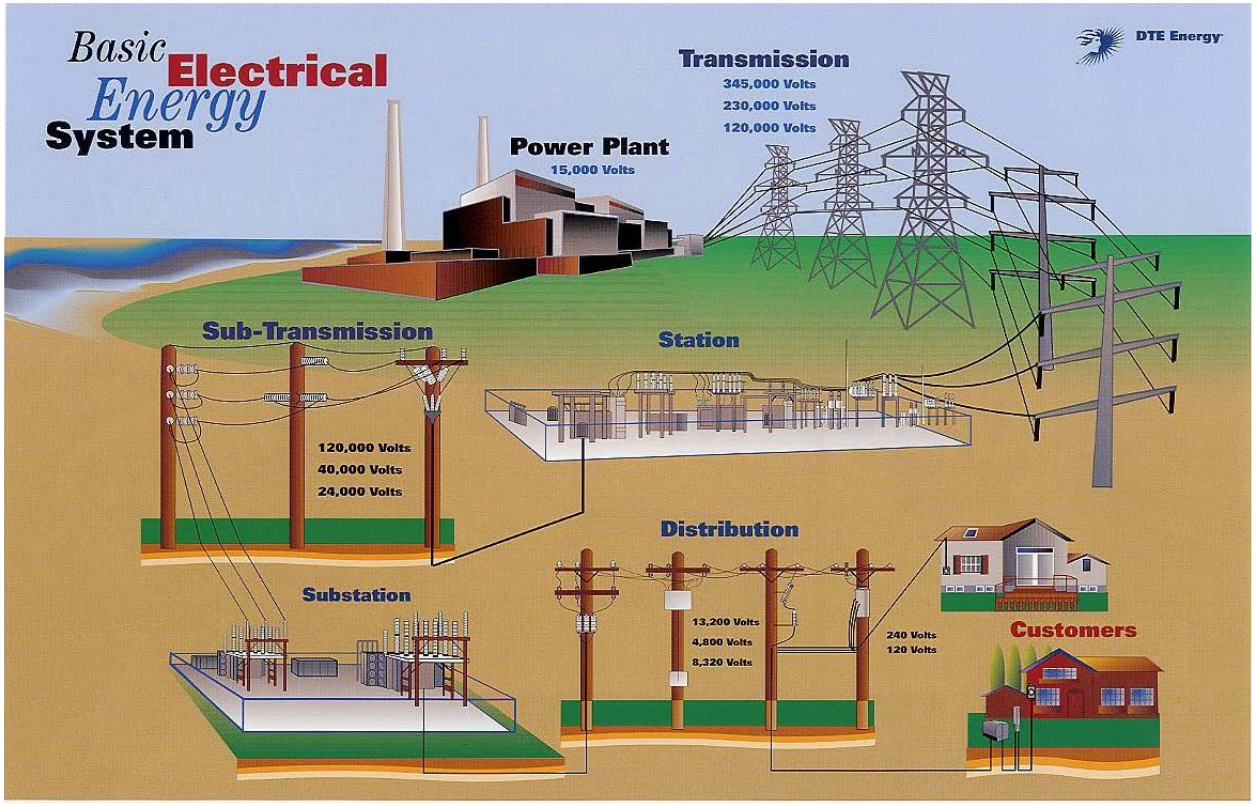


Transmission lines: 190K miles
Distribution lines: 73K miles
(2002)

[Sascha von Meier]

Today's grid



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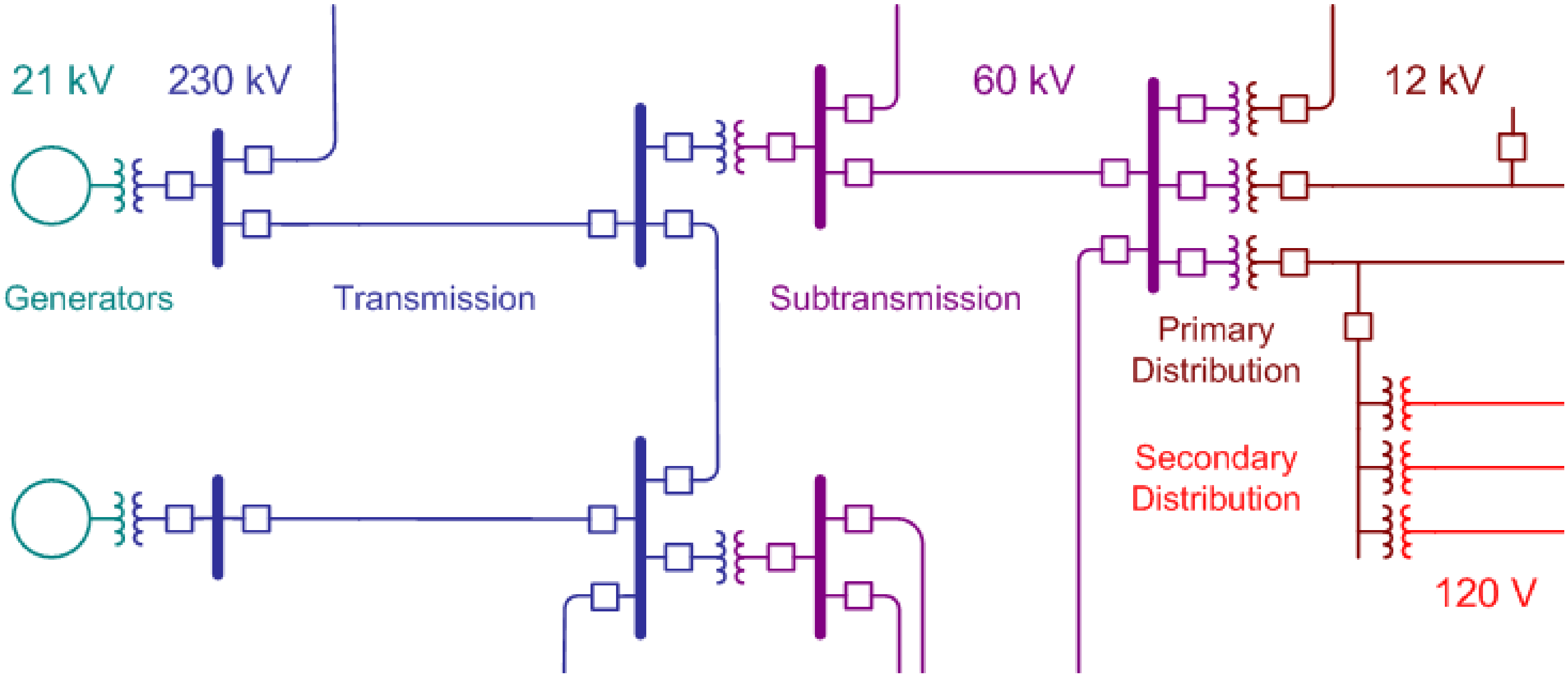
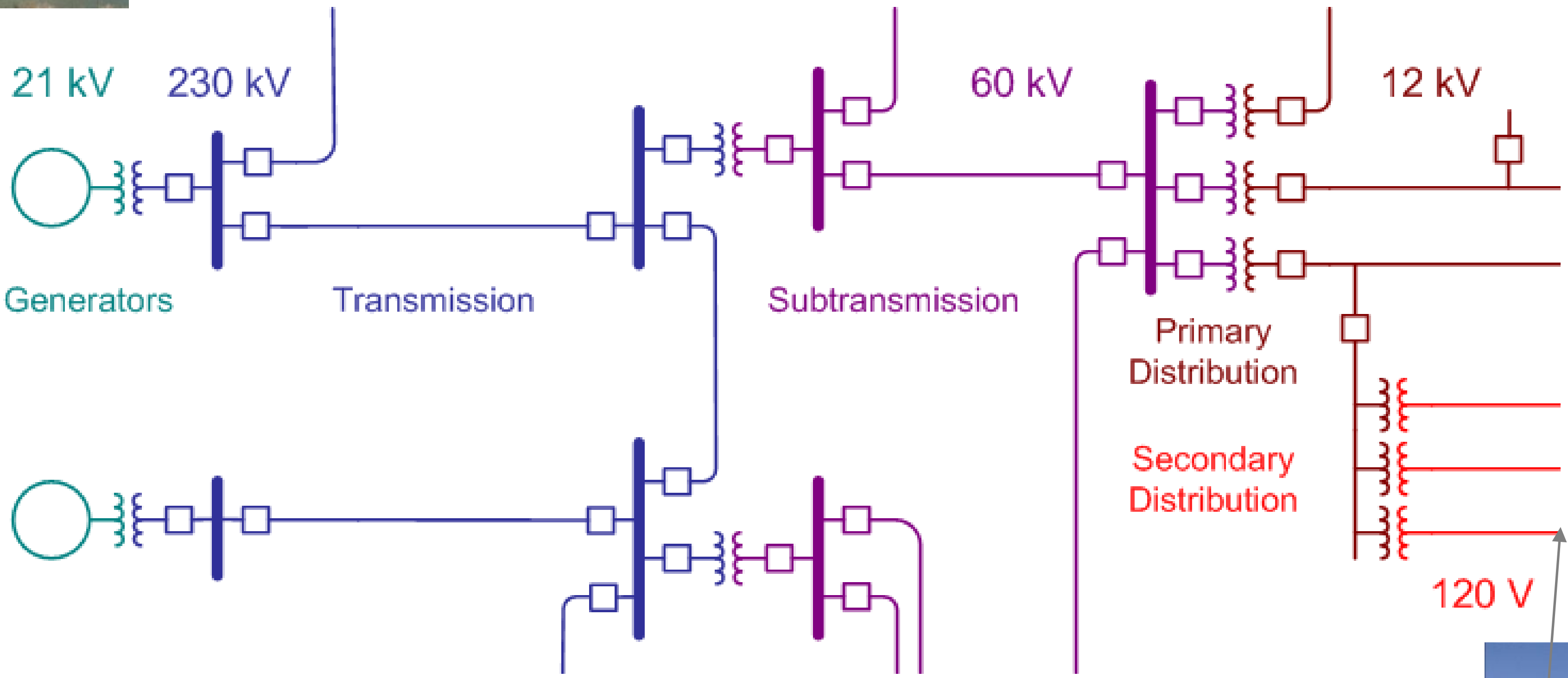
Power System Structure
with typical voltage levels



[Sascha von Meier]

Power System Structure
with typical voltage levels

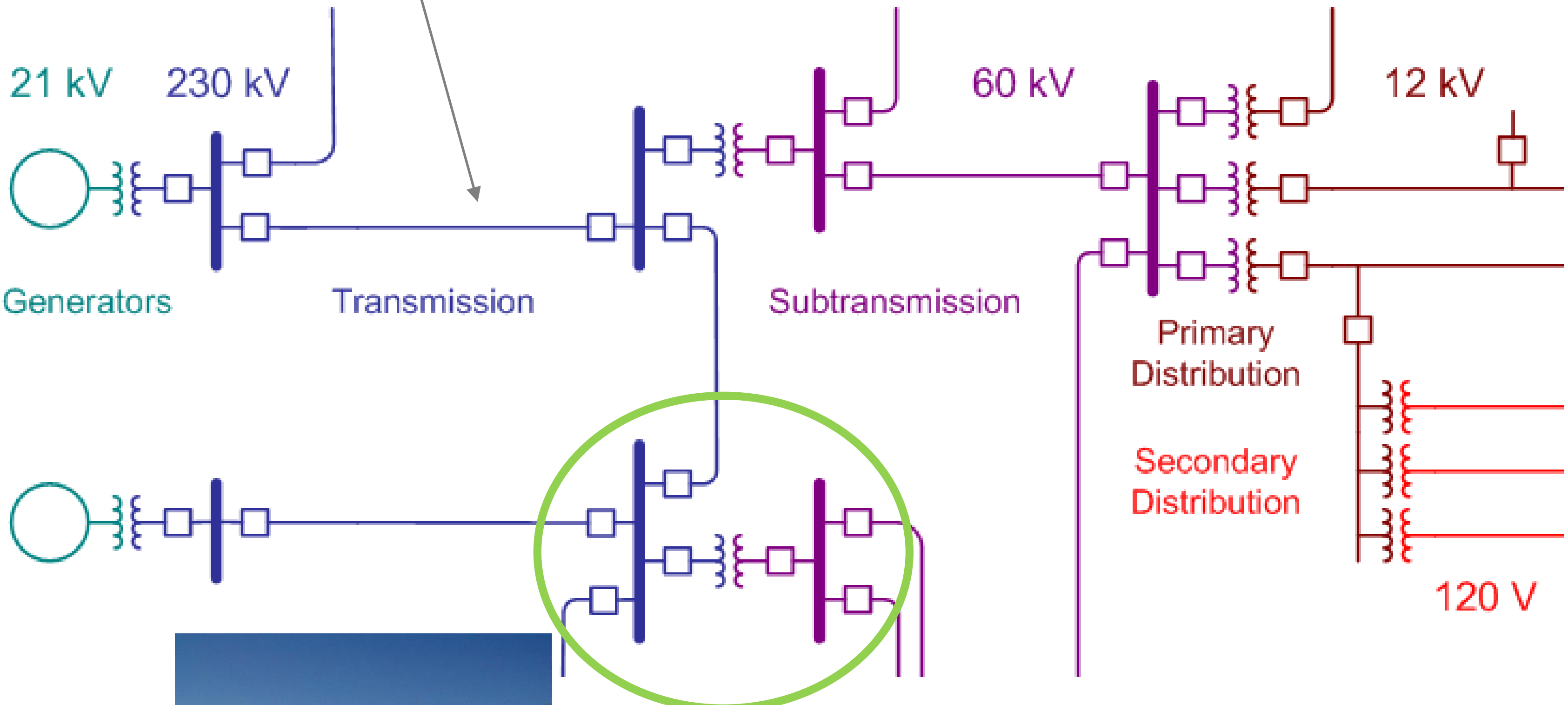


service drop

[Sascha von Meier]

transmission
line

Power System Structure
with typical voltage levels

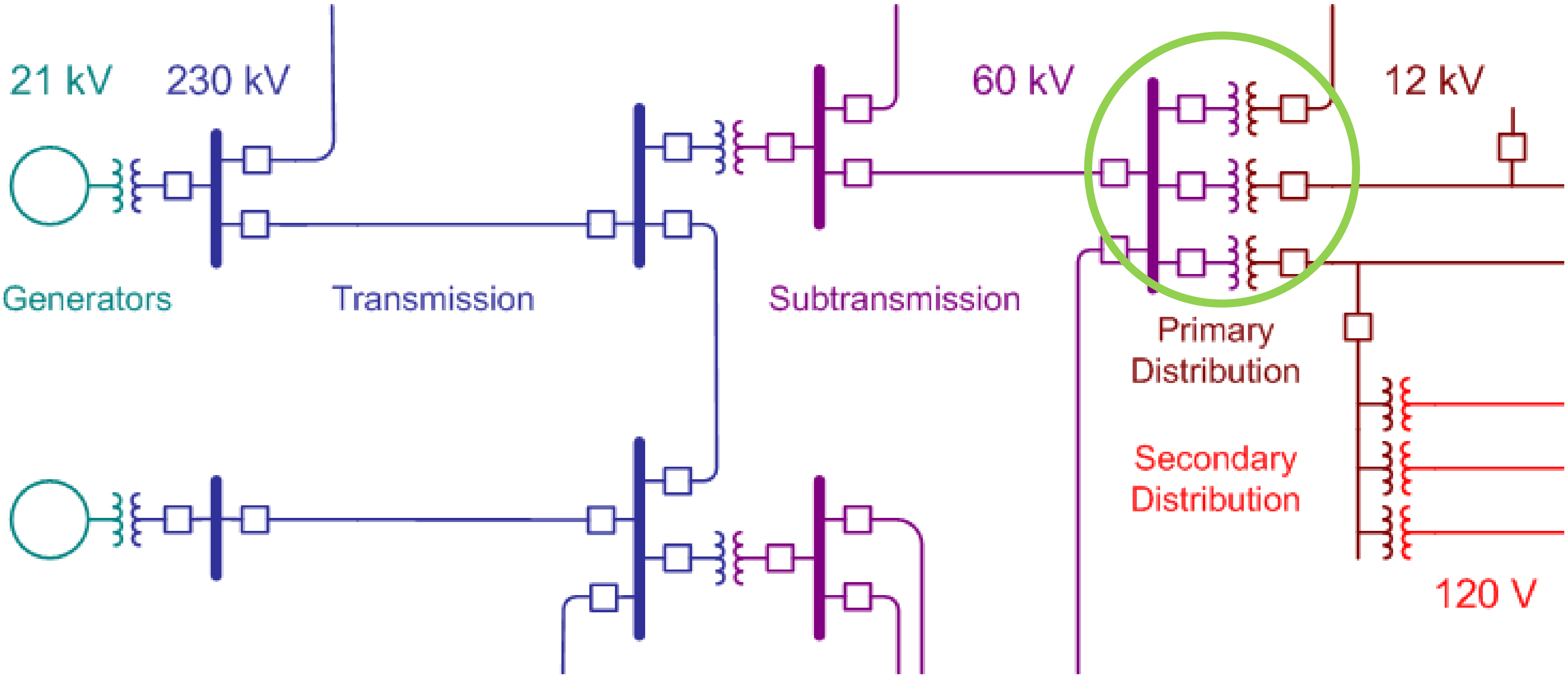


transmission
substation

[Sascha von Meier]

distribution
substation

Power System Structure
with typical voltage levels



[Sascha von Meier]

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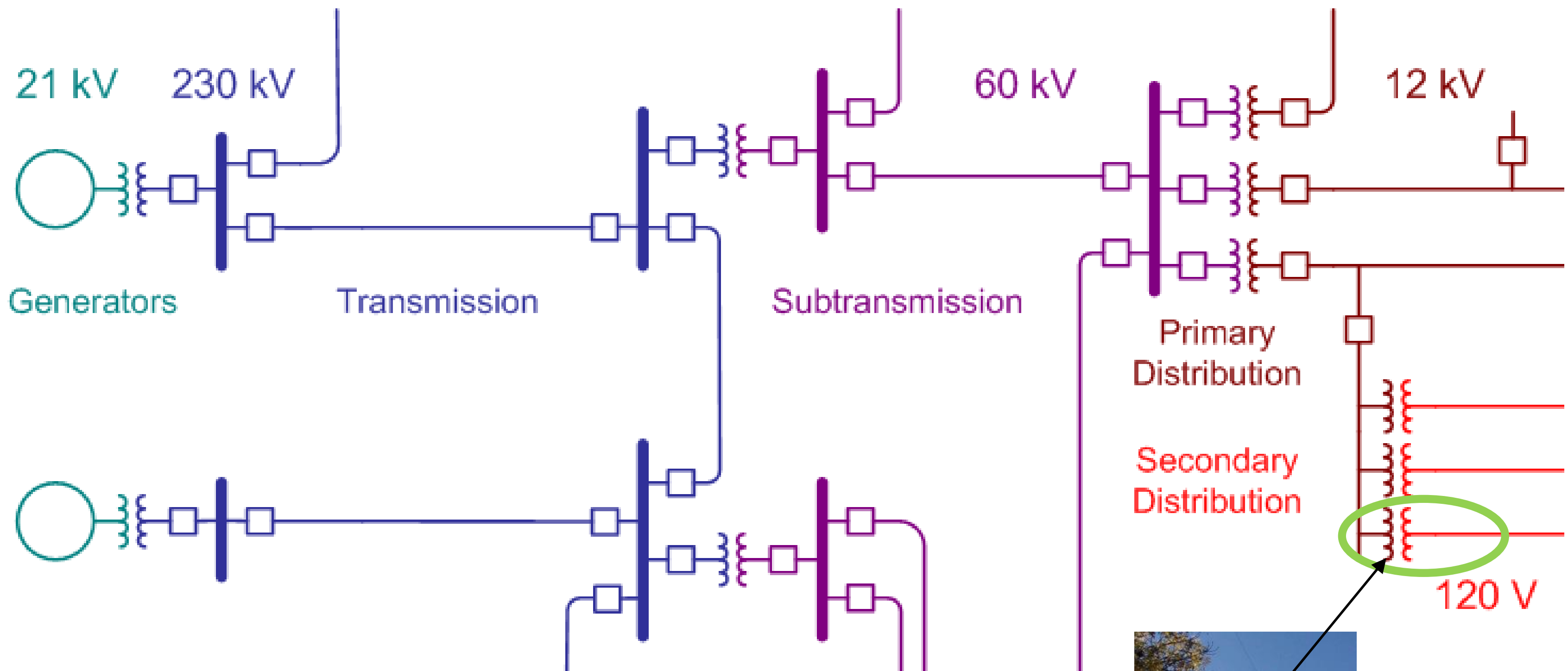
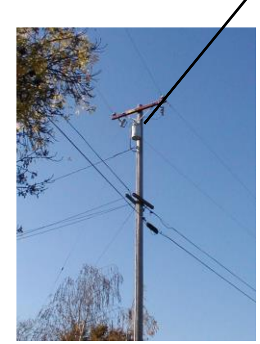
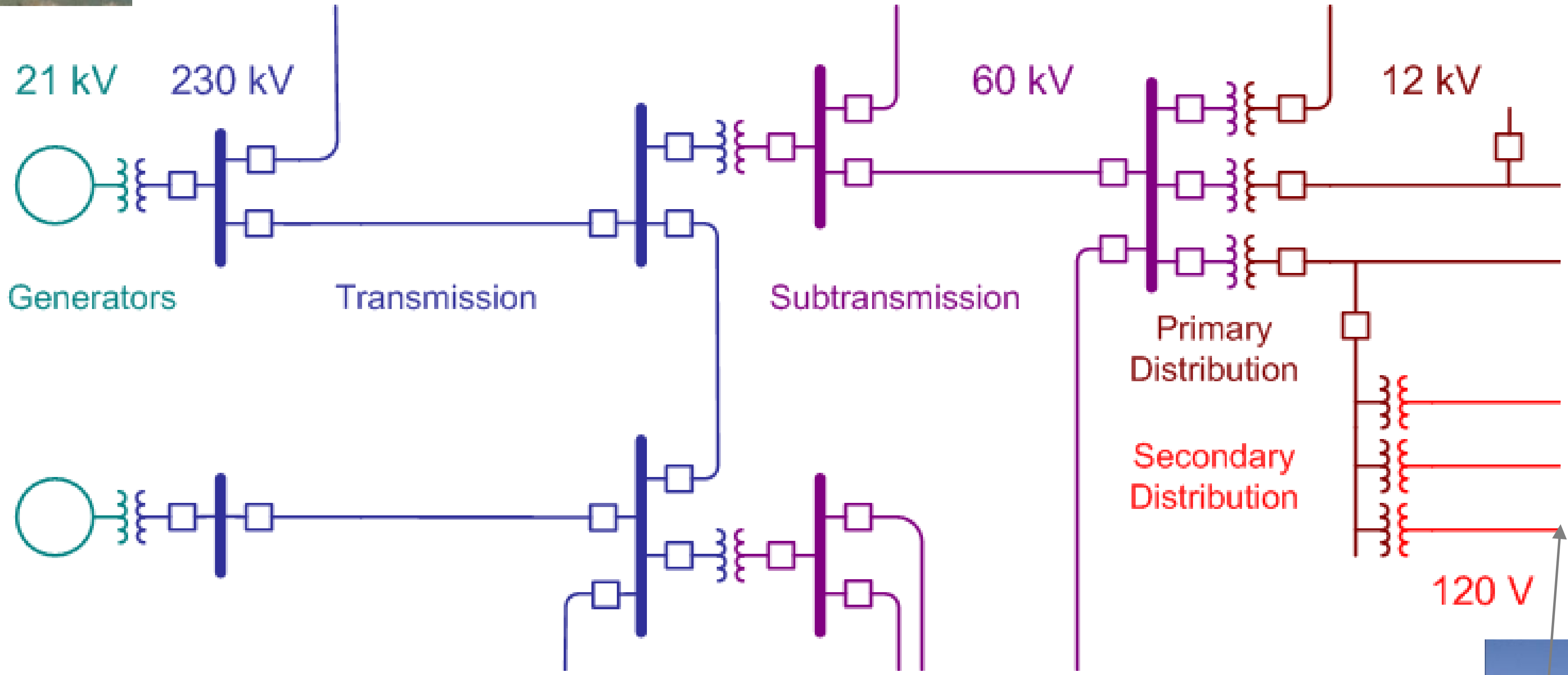




transformer & distribution line

[Sascha von Meier]

Power System Structure
with typical voltage levels



service drop

[Sascha von Meier]

Mathematical model

Quantities of interest

- Voltage, current, power, energy
- All are sinusoidal functions of time

Mathematical model

Quantities of interest

- Voltage, current, power, energy
- All are sinusoidal functions of time

Voltage

$$v(t) = V_{\max} \cos(wt + q_V)$$

nominal frequency
North/Central Americas: 60 Hz
Most other major countries: 50 Hz

- Steady state: frequencies at all points are nominal
- Reasonable model at timescales of minute and up
- Dynamic models at sec-min timescale: S Meyn's tutorial

this part of tutorial is all about steady state

Phasor representation

Quantities of interest

- Voltage, current, power, energy
- All are sinusoidal functions of time

Voltage

$$v(t) = V_{\max} \cos(wt + q_V)$$

amplitude ($V_{\max}$) — phase (q_V)

voltage phasor

$$V = \frac{V_{\max}}{\sqrt{2}} e^{jq_V}$$

Phasor representation

Quantities of interest

- Voltage, current, power, energy
- All are sinusoidal functions of time

Voltage

$$v(t) = V_{\max} \cos(wt + q_V)$$

voltage phasor

$$V = \frac{V_{\max}}{\sqrt{2}} e^{jq_V}$$

$$v(t) = \mathrm{Re}\left\{ \sqrt{2} V e^{jwt} \right\} = \mathrm{Re}\left\{ V_{\max} e^{j(wt + q_V)} \right\}$$

Phasor representation

Quantities of interest

- Voltage, current, power, energy
- All are sinusoidal functions of time

Voltage

$$v(t) = V_{max} \cos(\omega t + \theta_V)$$

voltage phasor

$$V = \frac{V_{max}}{\sqrt{2}} e^{j\theta_V}$$

$$|V| = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt} \qquad \text{RMS}$$

Phasor representation

Voltage

$$v(t) = V_{\max} \cos(wt + q_V)$$

$$V = \frac{V_{\max}}{\sqrt{2}} e^{jq_V}$$

Current

$$i(t) = I_{\max} \cos(wt + q_I)$$

$$I = \frac{I_{\max}}{\sqrt{2}} e^{jq_I}$$

Linear circuit elements

Resistor R $\qquad v(t) = R \times i(t)$

Inductor L $\qquad v(t) = L \times \frac{di}{dt}(t)$

Capacitor C $\qquad i(t) = C \times \frac{dv}{dt}(t)$

these are main circuit elements to model the grid

Linear circuit elements

Resistor R

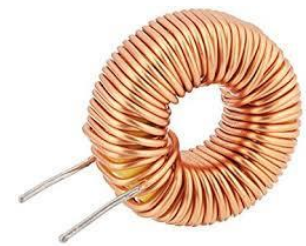
$$v(t) = R \times i(t)$$

$$V = R \times I$$

Inductor L

$$v(t) = L \times \frac{di}{dt}(t)$$

$$V = jWL \times I$$



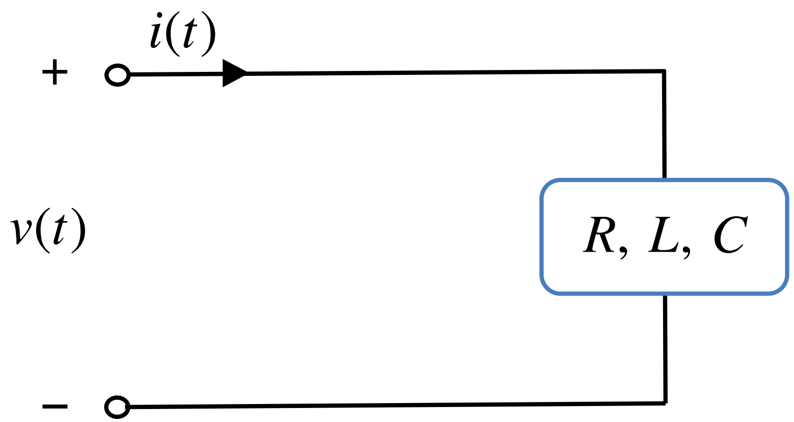
Capacitor C

$$i(t) = C \times \frac{dv}{dt}(t)$$

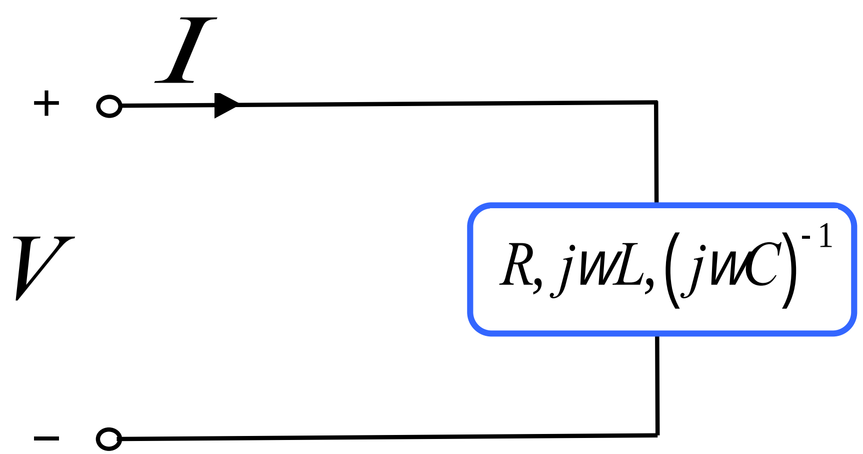
$$V = (j\omega C)^{-1} \cdot I$$

Linear circuit elements

time domain



phasor domain



Complex power

Quantities of interest

- Voltage, current, power, energy
- All are sinusoidal functions of time

Instantaneous power

$$p(t) = v(t)i(t)$$

$$= \frac{V_{max} I_{max}}{2} \left(\cos(\theta_V - \theta_I) + \cos(2\omega t + \theta_V + \theta_I) \right)$$

average power

Complex power

Quantities of interest

- Voltage, current, power, energy
- All are sinusoidal functions of time

Instantaneous power

$$p(t) = v(t)i(t)$$

$$= \frac{V_{max} I_{max}}{2} \left(\cos(\theta_V - \theta_I) + \cos(2\omega t + \theta_V + \theta_I) \right)$$

average power

Complex power

$$S := VI^* = P + jQ$$

real (active) power

reactive power

Phasor analysis

Steady state behavior described by algebraic equations

- Instead of dynamic equations

Circuit analysis

- Voltages and currents are linear

Power flow analysis

- Power flow equations are nonlinear

$$p(t) = v(t)i(t)$$

$$S := VI^*$$

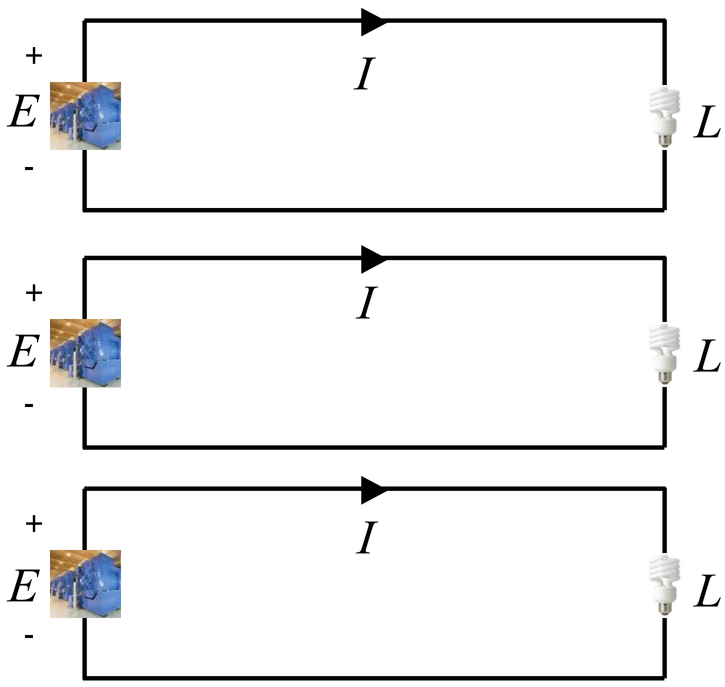
We will describe device and network models, and analyze them, in phasor domain

3-phase AC : 3 key ideas

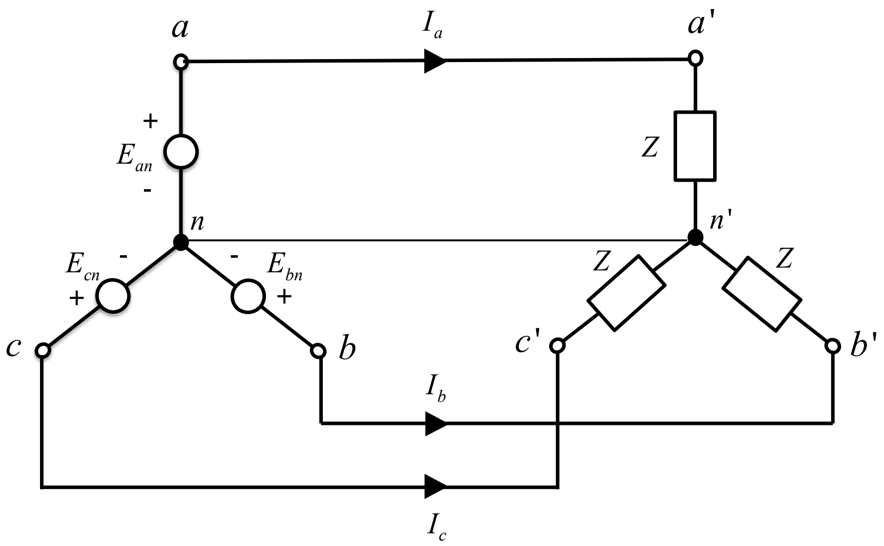
- Phasor representation
- Balanced operation
- Per-phase analysis

3-phase AC system

3 single-phase system:

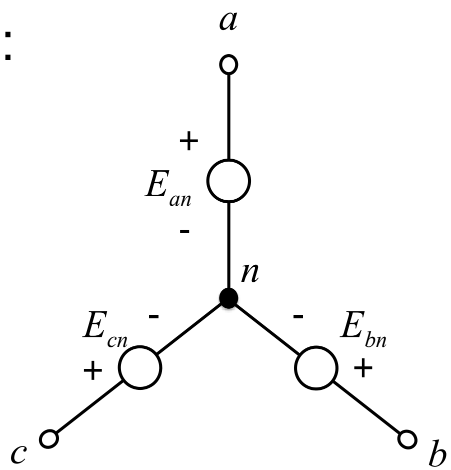


single 3-phase system:

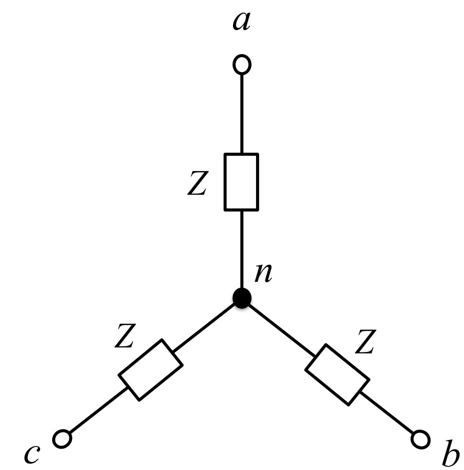


3-phase AC system

Y-configuration:

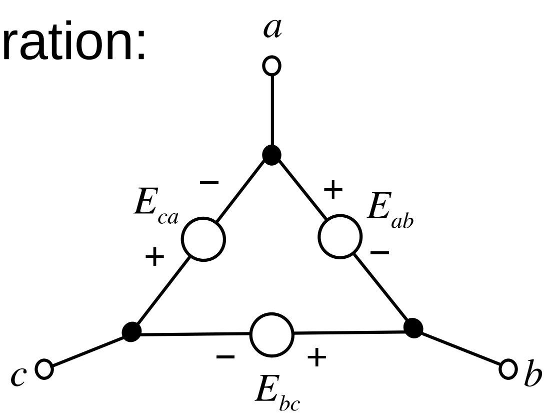


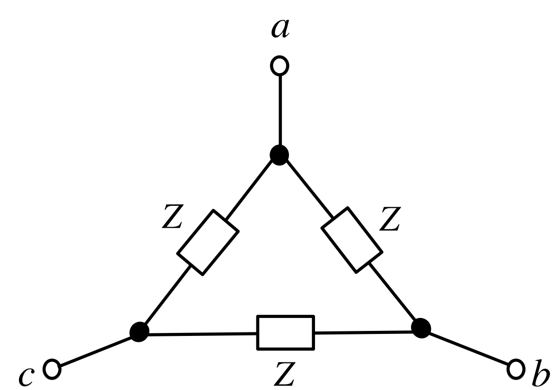
voltage source



impedance load

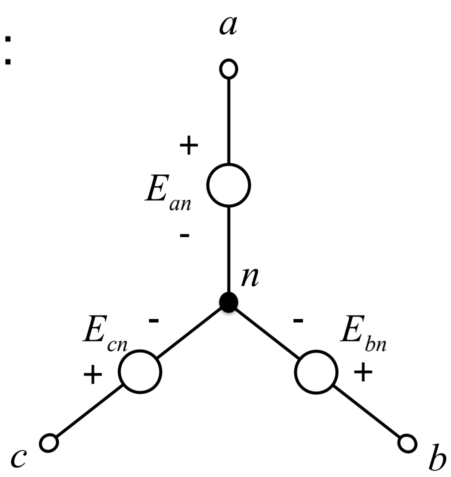
Delta-configuration:



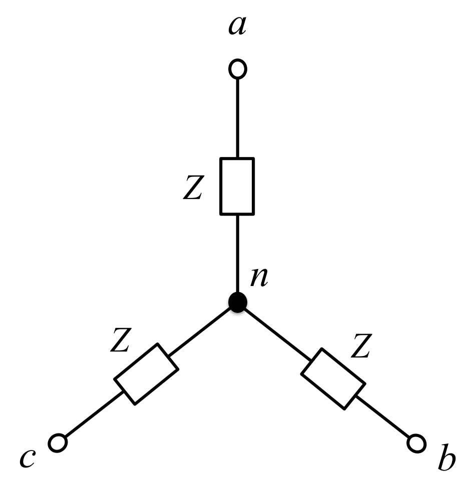


3-phase AC system

Y-configuration:



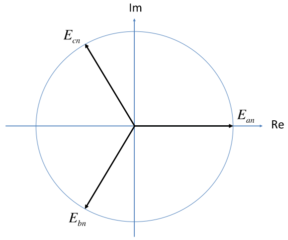
voltage source



impedance load

Balanced 3p source

- Equal in magnitude, 120 deg difference in phase
- $E_{an} = 1\angle\theta, \quad E_{bn} = 1\angle\theta - 120^\circ, \quad E_{cn} = 1\angle\theta + 120^\circ$



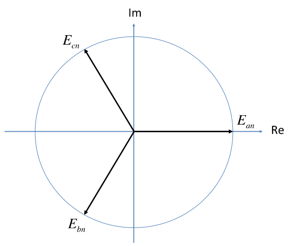
Balanced 3p impedance load

- Identical impedances

3-phase AC system

Balanced 3p source

- Equal in magnitude, 120 deg difference in phase
- $E_{ab} = 1\angle\theta, \quad E_{bc} = 1\angle\theta - 120^{\circ}, \quad E_{ab} = 1\angle\theta + 120^{\circ}$



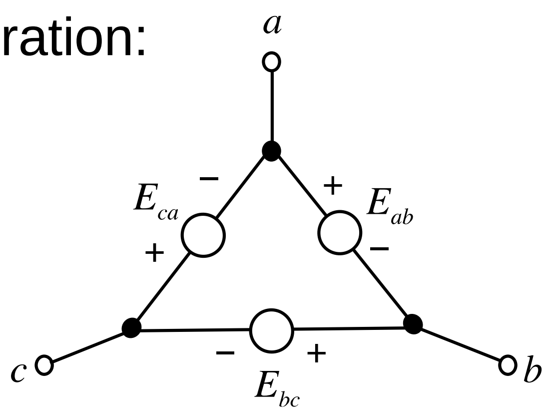
Balanced 3p impedance load

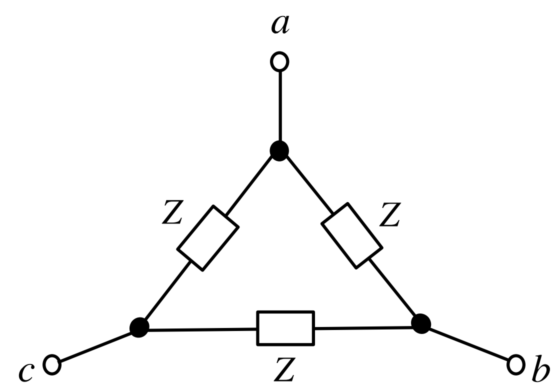
- Identical impedances

voltage source

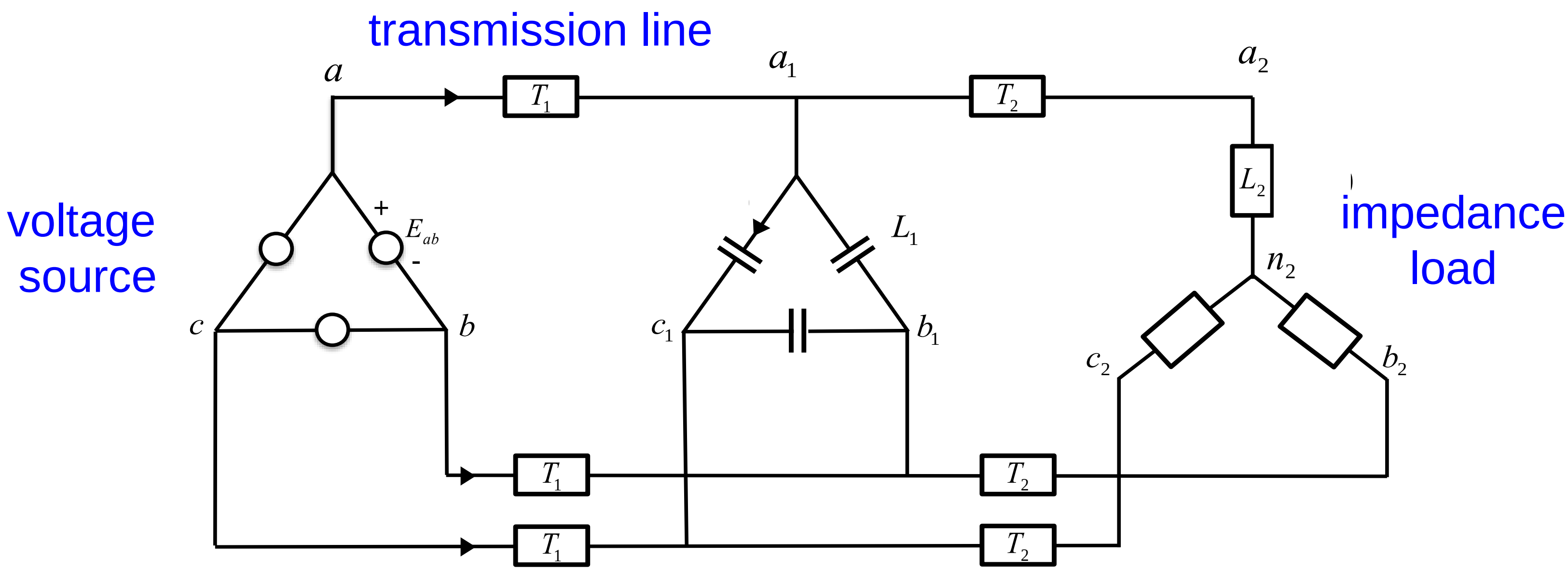
impedance load

Delta-configuration:





Balanced 3-phase system



Balanced 3p operation

- Balanced 3p sources
- Balanced 3p loads
- Balanced (identical) transmission lines

Advantages

1-phase

$$p(t) = v(t)i(t), \qquad S := VI^*$$

3-phase

$$S_{3f} := V_a I_a^* + V_b I_b^* + V_c I_c^*$$

Advantages

1-phase

$$p(t) = v(t)i(t), \qquad S := VI^*$$

3-phase

$$S_{3f} := V_a I_a^* + V_b I_b^* + V_c I_c^* = 3S$$

$$p_{3f}(t) := v_a(t)i_a(t) + v_a(t)i_a(t) + v_a(t)i_a(t)$$

Advantages

1-phase $$p(t) = v(t)i(t), \qquad S := VI^*$$

3-phase $$S_{3f} := V_a I_a^* + V_b I_b^* + V_c I_c^* = 3S$$

$$\begin{aligned} p_{3\phi}(t) &:= v_a(t)i_a(t) + v_b(t)i_b(t) + v_c(t)i_c(t) \\ &= 3|V_a||I_a|\cos(\phi_V - \phi_I) = 3P \end{aligned}$$

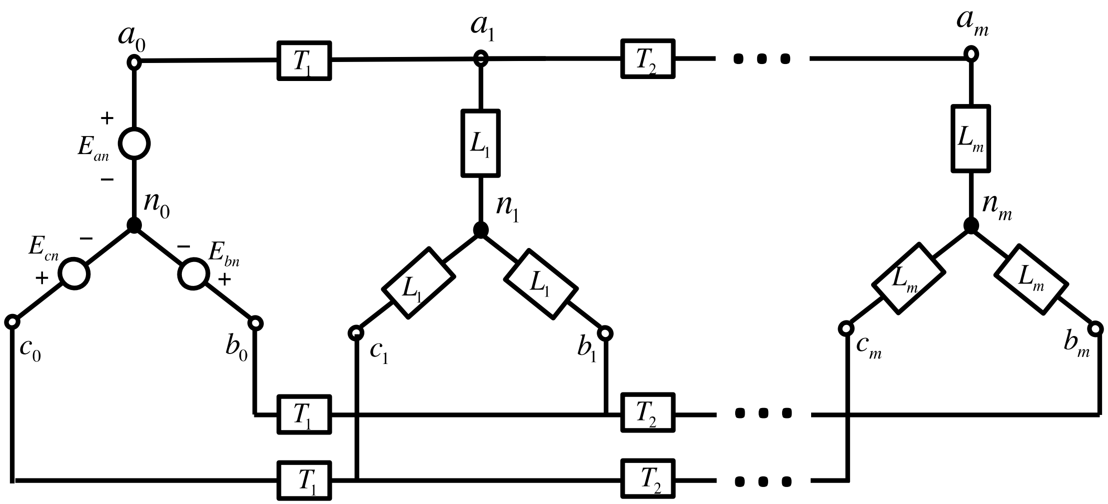
Advantages of balanced 3p operation

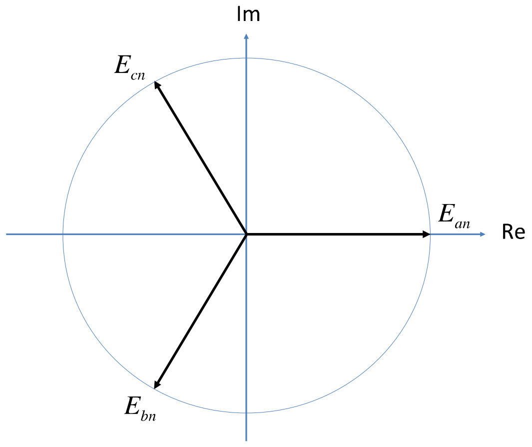
- Instantaneous power is constant in t !
- Uses ~1/2 as much materials (wires) as three 1p system
- Incurs ~1/2 as much active power loss as three 1p system

3-phase AC : 3 key ideas

- Phasor representation
- Balanced operation
- Per-phase analysis

Per-phase analysis: Wye

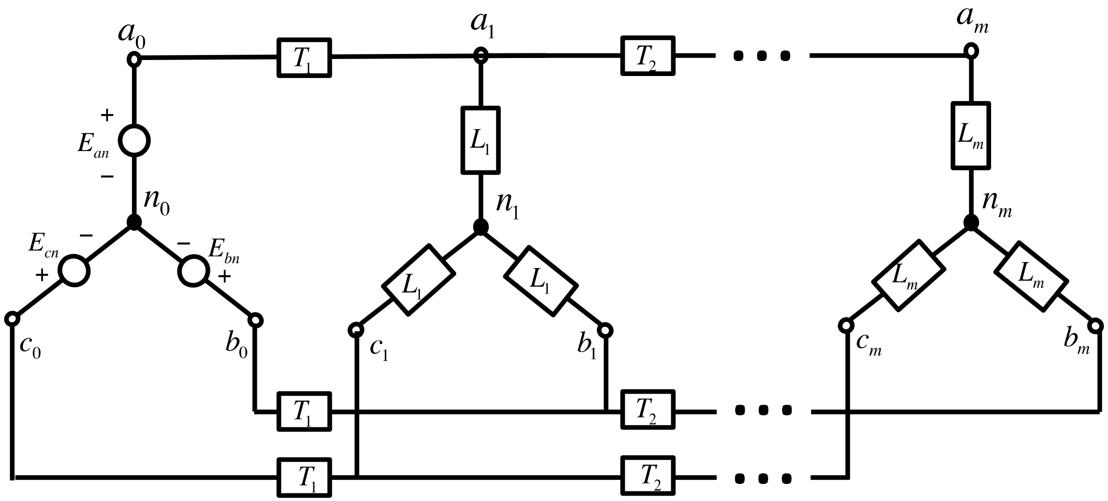


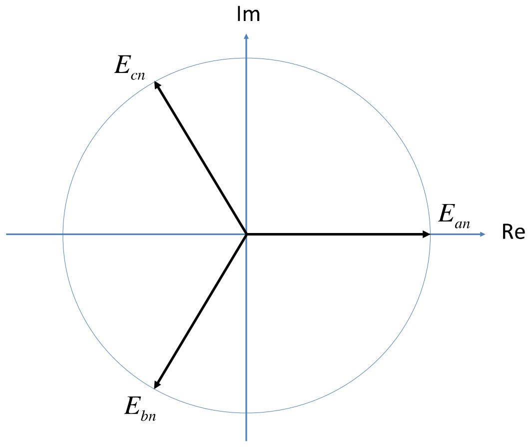


Important properties of balanced 3p system

- All $V_{\text{neutral-neutral}} = 0$

Per-phase analysis: Wye

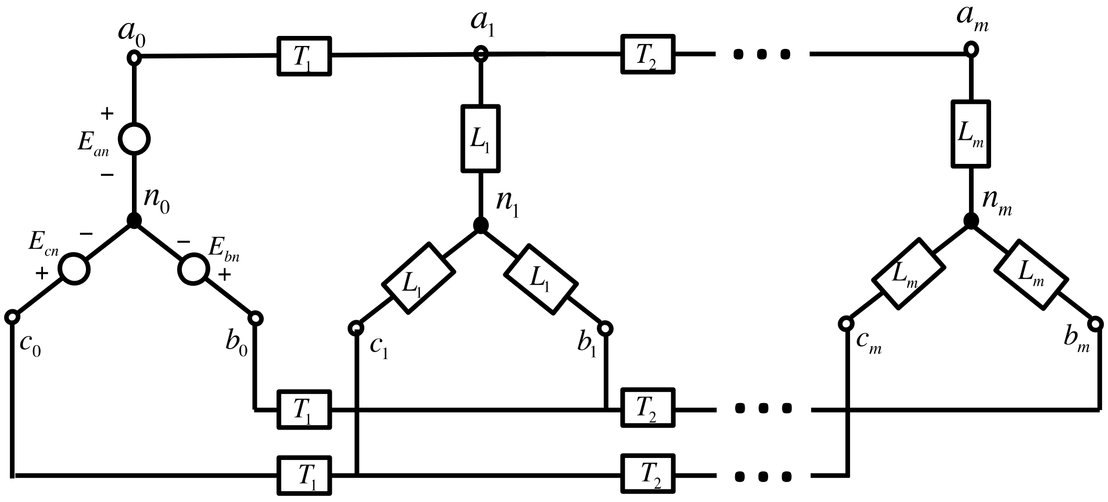




Important properties of balanced 3p system

- All $V_{\text{neutral-neutral}} = 0$
- All voltages and currents are 3-phase balanced
- Phases are decoupled, i.e., variables in each phase depend only on quantities in that phase

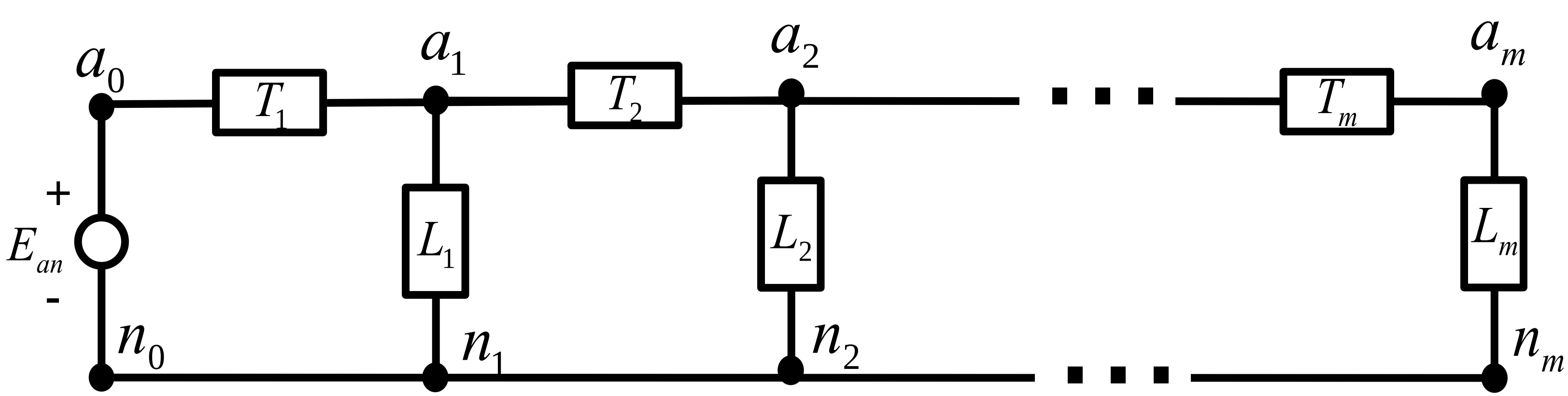
Per-phase analysis: Wye



Properties:

- All $V_{\text{neutral-neutral}} = 0$
- All voltages and currents are 3-phased balanced
- Phases are decoupled

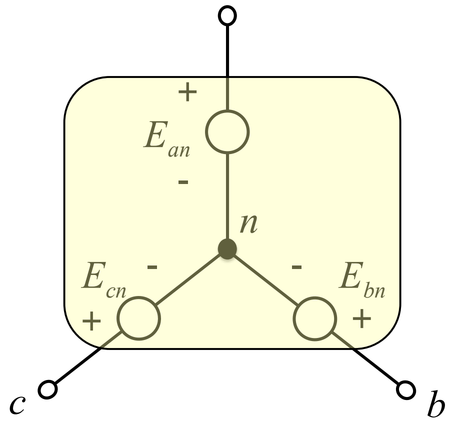
per-phase equivalent circuit



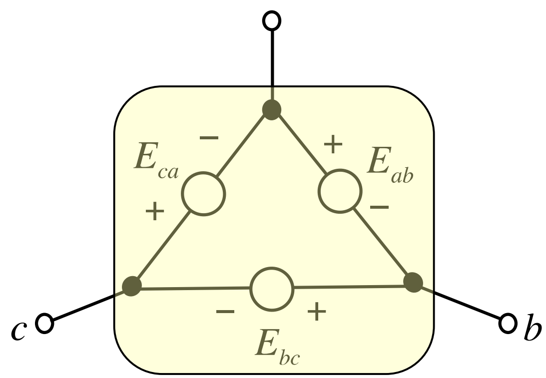
Delta-Wye transformation

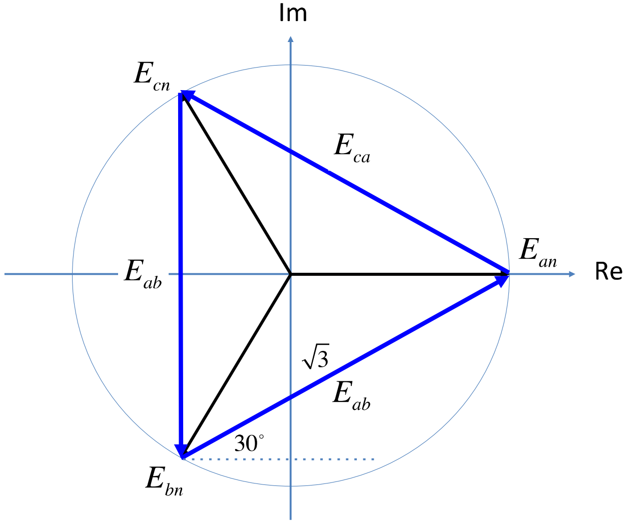
Equivalent 3p sources: same external behavior

line-to-line voltages: $E_{ab}^{Y} = E_{ab}^{\Delta}, \quad E_{bc}^{Y} = E_{bc}^{\Delta}, \quad E_{ca}^{Y} = E_{ca}^{\Delta}$



$$E_{an}^{Y} = \frac{E_{ab}^{\Delta}}{\sqrt{3}\, e^{j\pi/6}}$$

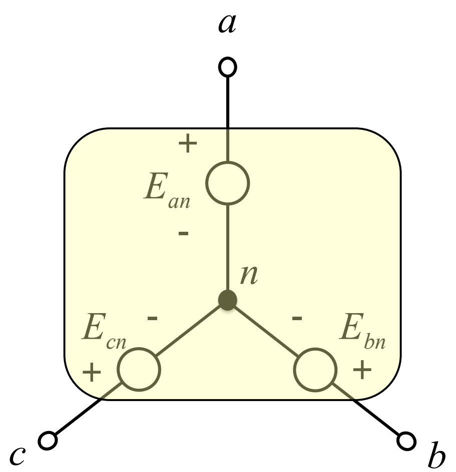




Delta-Wye transformation

Equivalent 3p sources: same external behavior

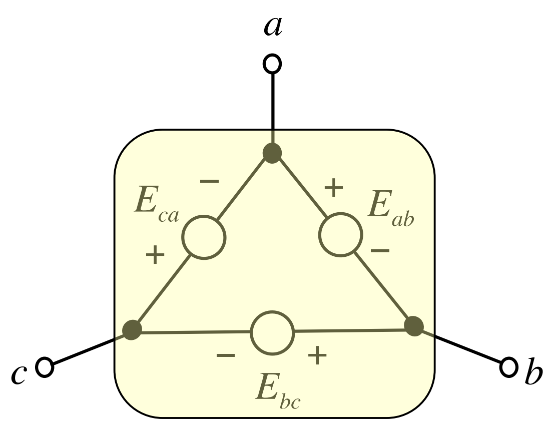
line-to-line voltages: $E^Y_{ab} = E^\Delta_{ab}, \quad E^Y_{bc} = E^\Delta_{bc}, \quad E^Y_{ca} = E^\Delta_{ca}$



$$E^Y_{an} = \frac{E^\Delta_{ab}}{\sqrt{3}\, e^{j\pi/6}}$$

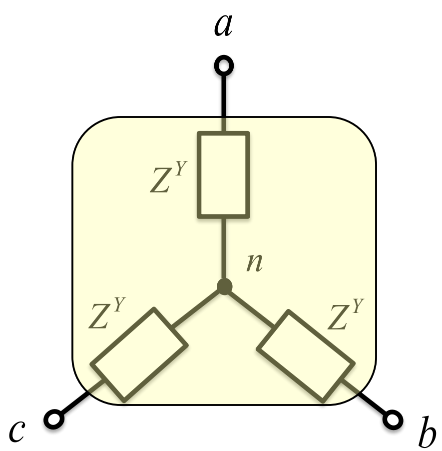
$$E^Y_{bn} = \frac{E^\Delta_{bc}}{\sqrt{3}\, e^{j\pi/6}}$$

$$E^Y_{cn} = \frac{E^\Delta_{ca}}{\sqrt{3}\, e^{j\pi/6}}$$

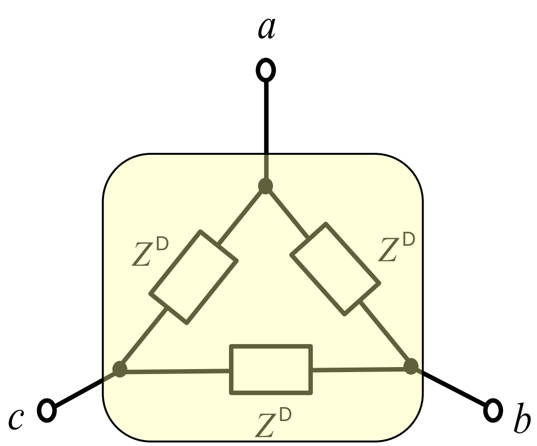


Delta-Wye transformation

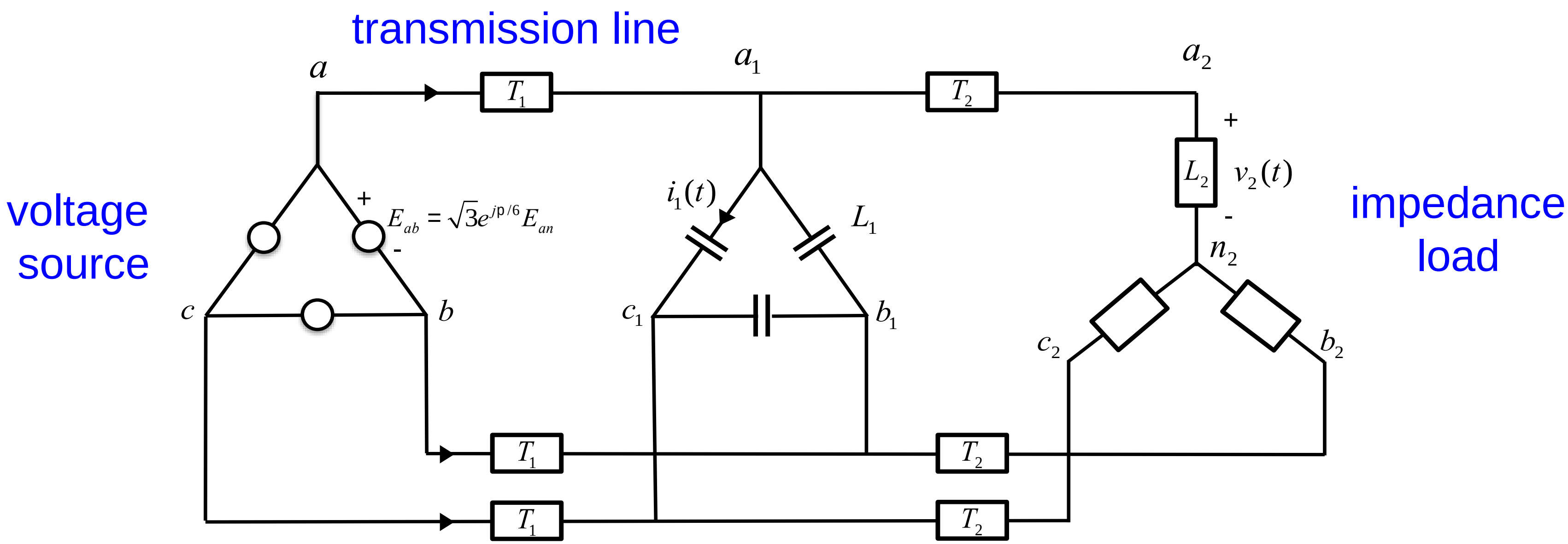
Equivalent 3p sources: same external behavior
same terminal currents on same line-to-line voltages



$$Z^Y = \frac{Z^D}{3}$$



Per-phase analysis

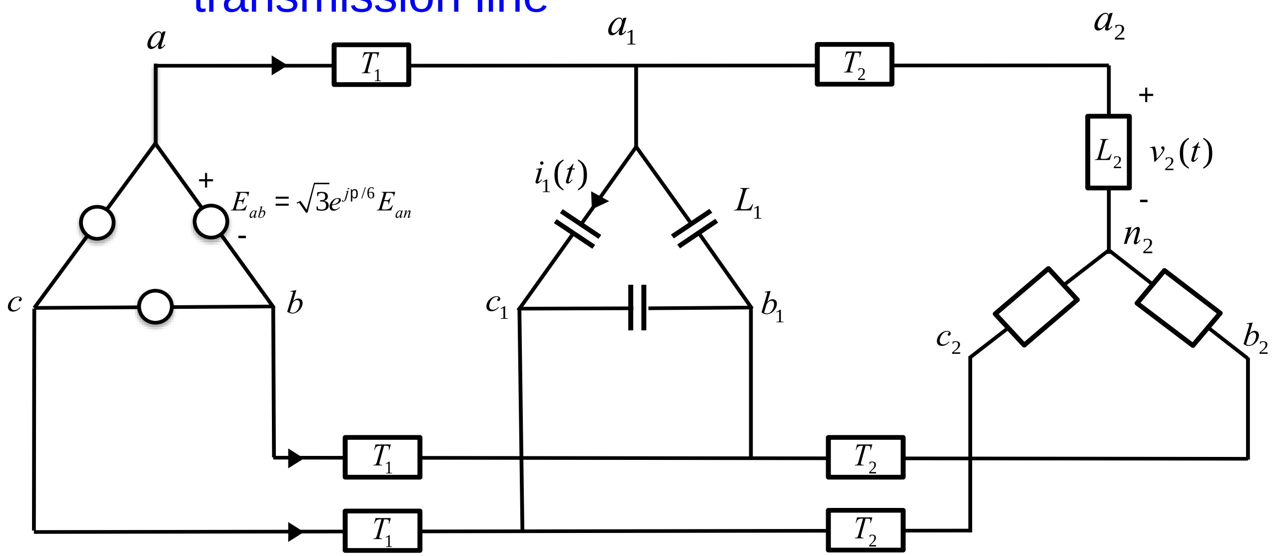


- Convert all Delta sources and loads into Wye
- Solve phase a circuit with all neutrals connected for desired variables
- Phase b / c variables: subtract / add 120deg to phase a variables
- If variables internal to Delta configurations are desired, solve them from original circuit

Per-phase analysis

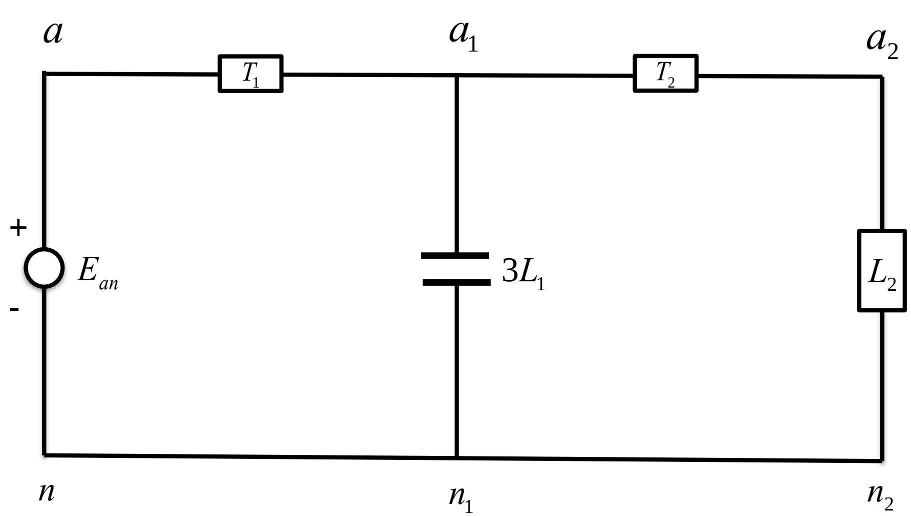
transmission line

voltage source



impedance load

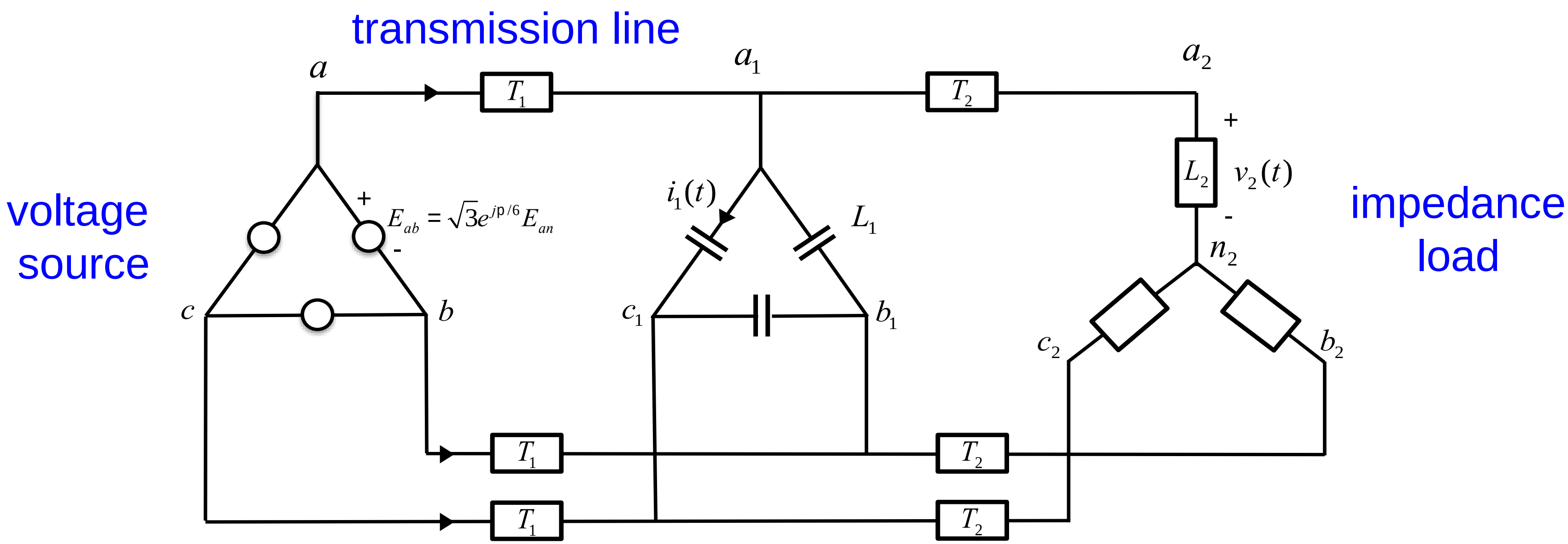
phase-*a* circuit

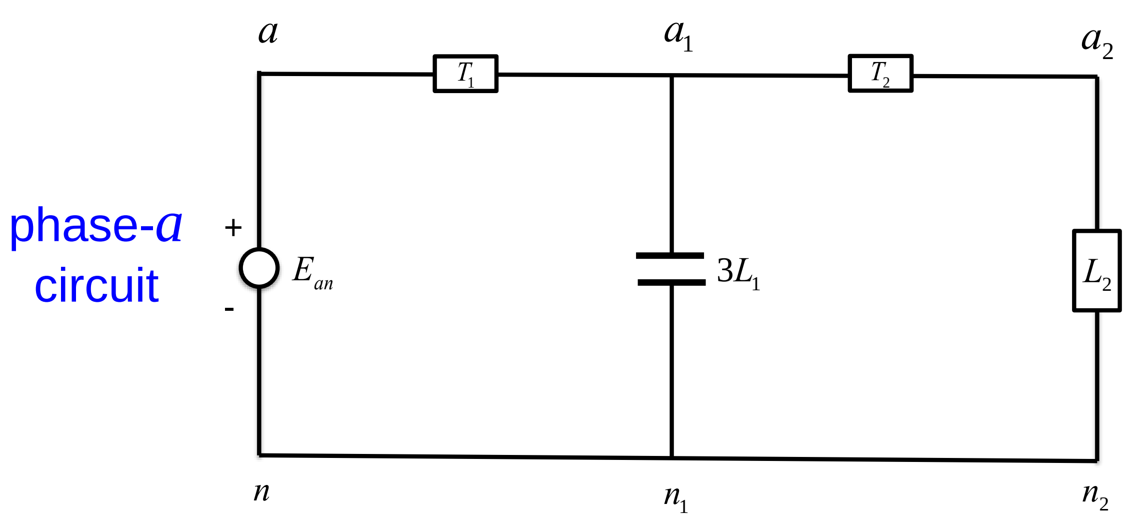


Solve for V_2

Þ $v_2(t) = \mathrm{Re}\left(\sqrt{2}V_2 e^{jWt}\right)$

Per-phase analysis





Solve for V_1

Þ $V_{ca} = \sqrt{3}e^{jp/6}V_1 \times e^{j2p/3}$

Þ $I_{ca} = L_1 V_{ca}$

Þ $i_1(t) = -\ \mathrm{Re}\left(\sqrt{2} I_{ca} e^{jWt}\right)$

Recap: basic concepts

3-phase AC transmission system

- Phasor representation
- Balanced operation
- Per-phase analysis

We will describe device and network models, and analyze them, in phasor domain, using per-phase analysis

The flow of power I

Basic concepts and models

Why smart grid? (15 mins)

Three-phase AC transmission: 3 key ideas (30 mins)

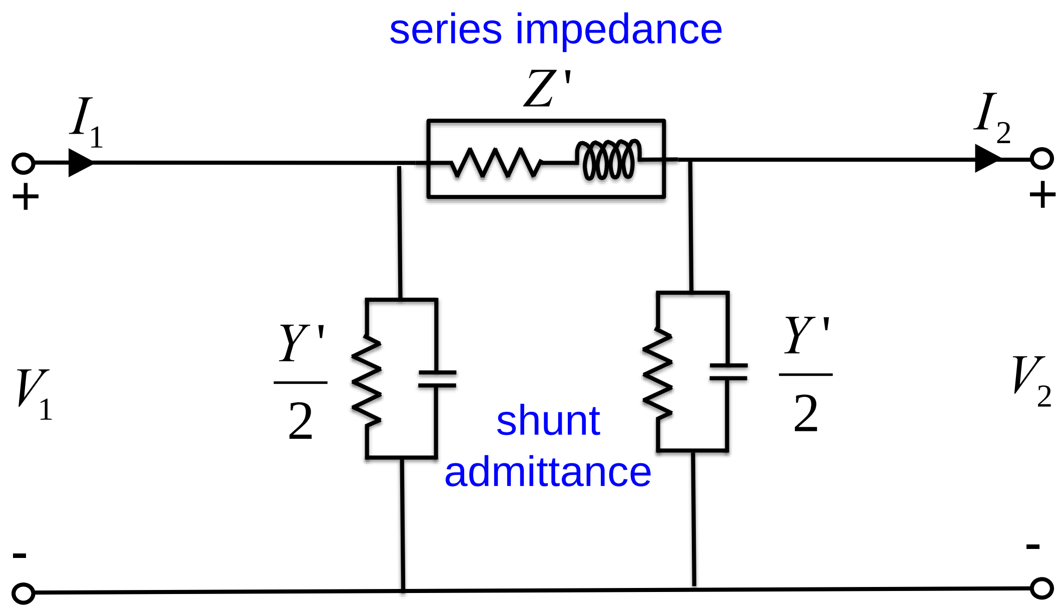
- Phasor representation
- Balanced operation
- Per-phase analysis

Device models (30 mins)

- Transmission line
- Transformer
- Generator

Transmission line model

P model of transmission line



- Terminal behavior $(V_2, I_2) \mapsto (V_1, I_1)$
- What do line parameters (Z', Y') depend on ?
- What about a 3-phase line ?
- What are some implications ?

Transmission line model

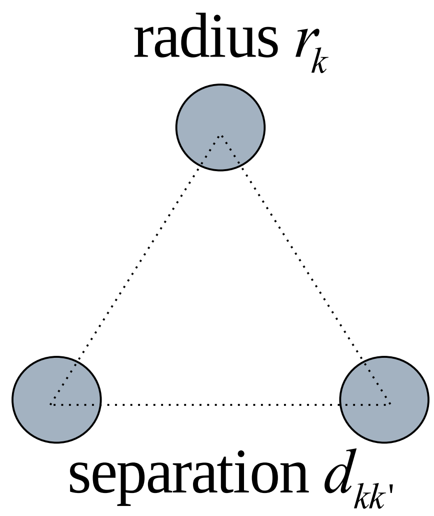
Line inductance l

total flux linkages $\lambda(t) = l \times i(t)$

Multiple conductors

$$\lambda_k = \underbrace{i_k \frac{\mu_0}{2\pi} \ln \frac{1}{r'_k}}_{l_k} + \sum_{k' \neq k} i_{k'} \underbrace{\frac{\mu_0}{2\pi} \ln \frac{1}{d_{kk'}}}_{l_{kk'}}$$

self inductance mutual inductance



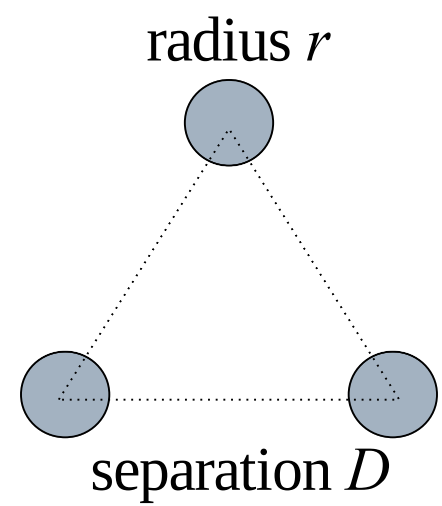
Transmission line model

Conditions

- Symmetric 3-phase line
- $i_a(t) + i_b(t) + i_c(t) = 0$

Multiple conductors

$$\lambda_a(t) = \underbrace{\frac{\mu_0}{2\pi}\ln\frac{D}{r'}}_{\text{“self-inductance” } l \ \mathrm{H/m}} \cdot i_a(t)$$



The phases are decoupled !

Transmission line model

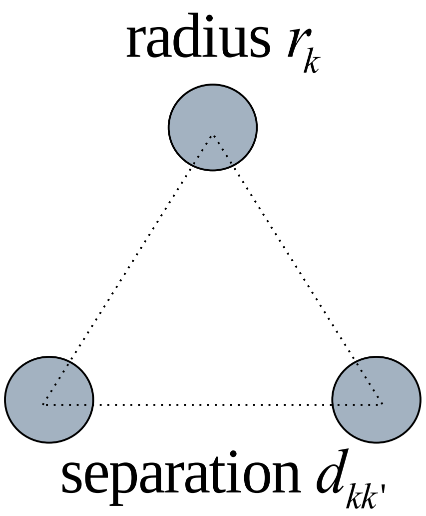
Line capacitance c

total charge / m $\quad q(t) = c \times v(t)$

Multiple conductors

$$v_k = q_k \underbrace{\frac{1}{2\pi\varepsilon} \ln \frac{1}{r_k}}_{1/c_k} + \sum_{k' \neq k} q_{k'} \underbrace{\frac{1}{2\pi\varepsilon} \ln \frac{1}{d_{kk'}}}_{1/c_{kk'}}$$

self inductance $\qquad$ mutual inductance



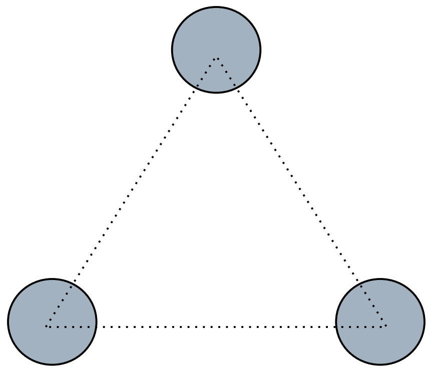
Transmission line model

Conditions

- Symmetric 3-phase line
- $q_a(t) + q_b(t) + q_c(t) = 0$

Multiple conductors

$$v_k(t) = \underbrace{\frac{1}{2\pi\epsilon} \ln \frac{D}{r}}_{\text{“self-capacitance” } 1/c \ \text{F/m}} \cdot q_k(t)$$



The phases are decoupled !

Transmission line model

Line parameters (balanced 3p line)

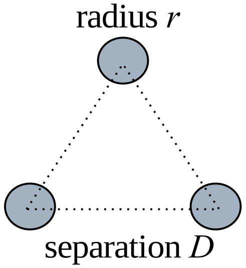
- Phases are decoupled
- series impedance $z = r + jwl$ W/ m

  shunt admittance (to neutral) $y = g + jwc$ W^{1}/m

- Line inductance and capacitance

$$l = \frac{\mu_0}{2\pi} \ln \frac{D}{r'} \qquad \text{H/m}$$

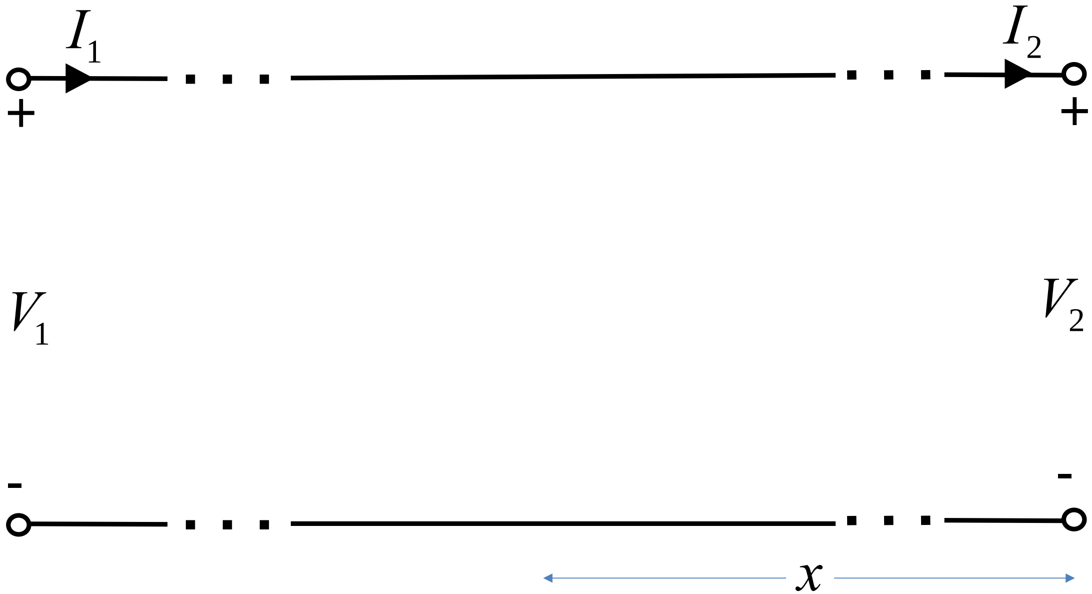
$$c = \frac{2\pi\varepsilon}{\ln(D/r)} \qquad \text{F/m}$$



- Line resistance r / conductance g depend on wire material & size

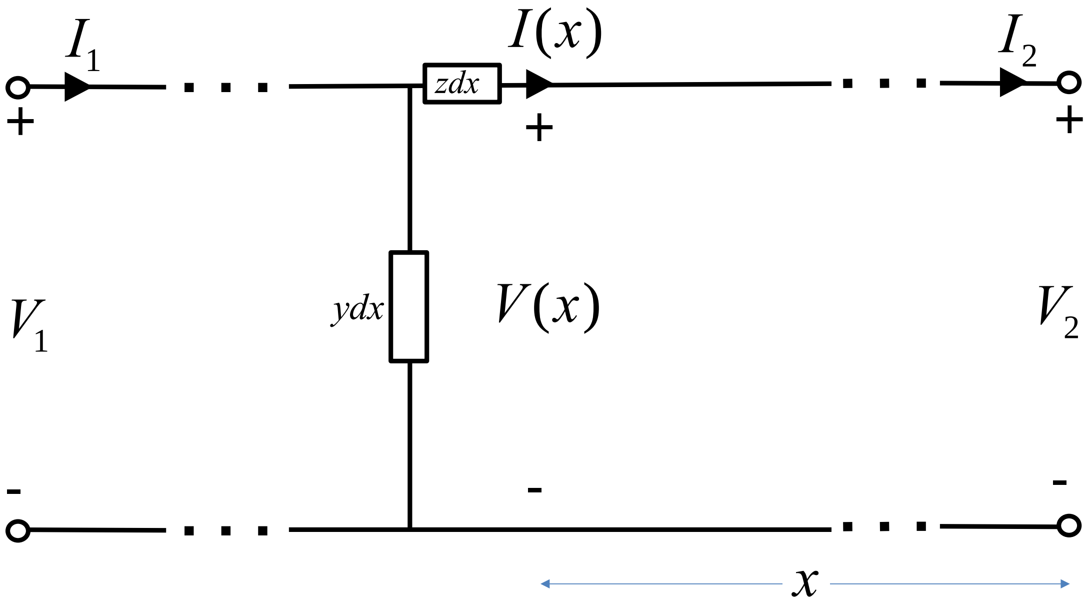
Transmission line model

per-phase model of phase voltage:



Transmission line model

per-phase model of phase voltage:



$$\begin{bmatrix} \frac{dV}{dx} \\ \frac{dI}{dx} \end{bmatrix} = \begin{bmatrix} 0 & z \\ y & 0 \end{bmatrix} \begin{bmatrix} V(x) \\ I(x) \end{bmatrix}$$

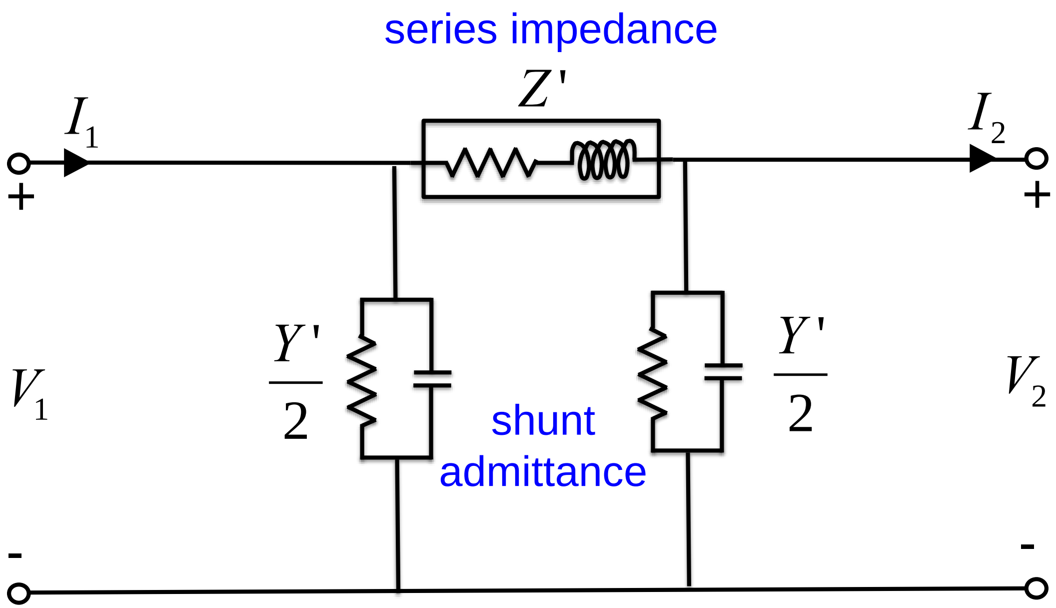
$$(V_2, I_2) \mapsto (V_1, I_1)$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} \cosh(\gamma\ell) & Z_c \sinh(\gamma\ell) \\ Z_c^{-1} \sinh(\gamma\ell) & \cosh(\gamma\ell) \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

$$Z_c := \sqrt{\frac{z}{y}} \quad \text{and} \quad \gamma := \sqrt{zy}$$

Transmission line model

P model of transmission line



$$Z' = Z \times \frac{\sinh(\gamma\ell)}{\gamma\ell}$$

$$Y' = Y \times \frac{\tanh(\gamma\ell/2)}{\gamma\ell/2}$$

$$Z := z\ell$$

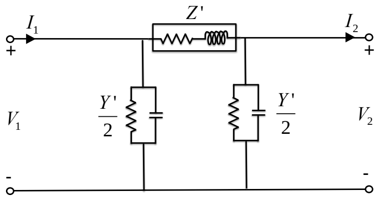
$$Y := y\ell$$

Transmission line model

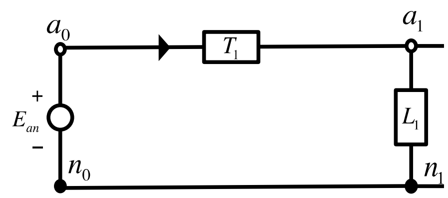
Long line (l>150mi): $$Z' = Z \times \frac{\sinh(\gamma l)}{\gamma l}$$

$$Y' = Y \times \frac{\tanh(\gamma l/2)}{\gamma l/2}$$

Long line (50<l<150mi): $$Z' = Z$$ $$Y' = Y$$



Long line (l<50mi): $$Z' = Z$$ $$Y' = 0$$

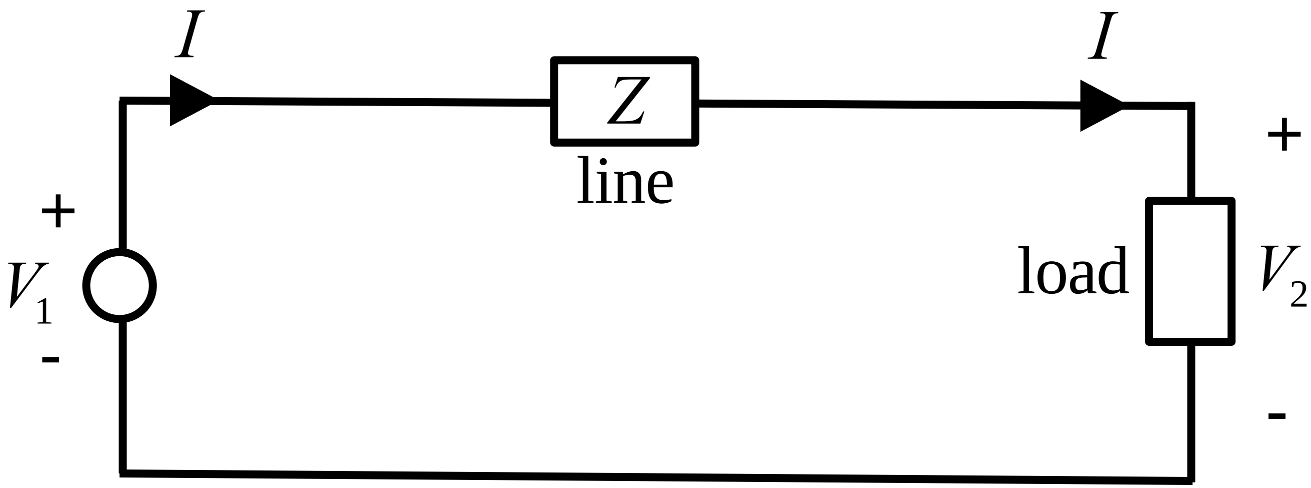


$$Z := z\ell$$

$$Y := y\ell$$

Transmission line model

High voltage min transmission line loss



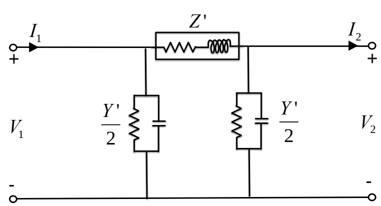
Specified: required load power $|S_2|$ and voltage $|V_2|$

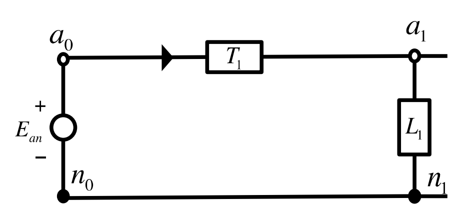
Þ $|I| = \dfrac{|S_2|}{|V_2|}$ $\qquad$ $\text{line loss} = R|I|^2$

Transmission line model

Recap

- Line characteristics depend on materials, size, and geometry of 3-phase line
- Linear per-phase circuit model $(V_2, I_2) \mapsto (V_1, I_1)$
- P circuit model: series impedance + shunt admittance





The flow of power I

Basic concepts and models

Why smart grid? (15 mins)

Three-phase AC transmission: 3 key ideas (30 mins)

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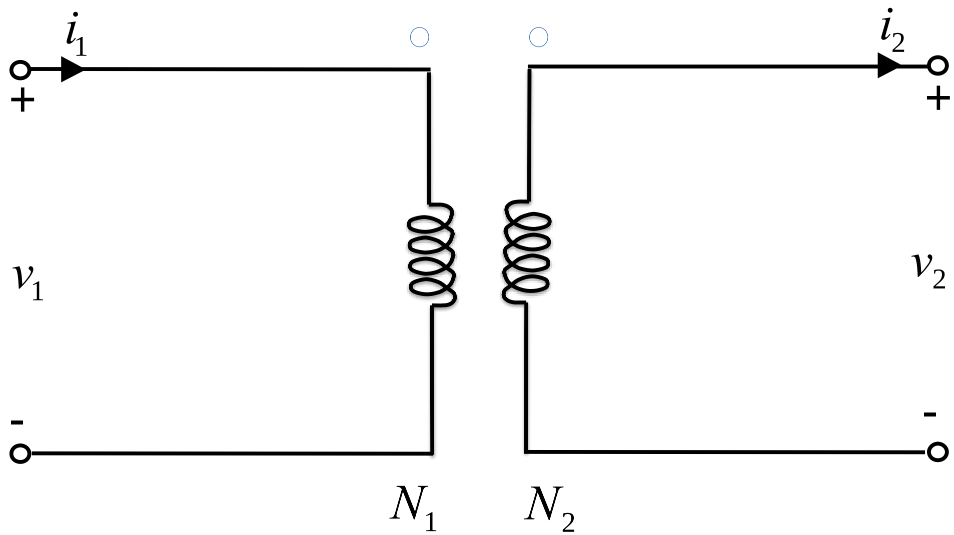
- Per-phase analysis

Device models (30 mins)

- Transmission line
- Transformer
- Generator

Transformer model

Single-phase ideal transformer n

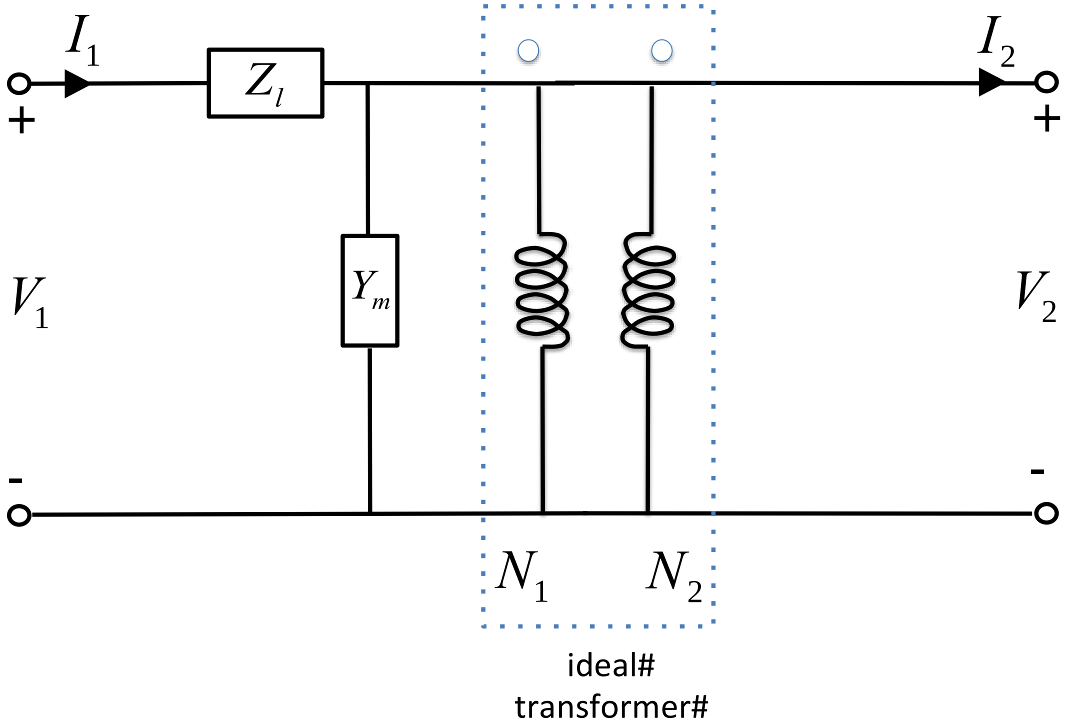


$$n := \frac{N_2}{N_1} \qquad a := \frac{N_1}{N_2}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \underbrace{\begin{bmatrix} a & 0 \\ 0 & n \end{bmatrix}}_{T_{\text{ideal}}} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} \qquad \frac{-S_{21}}{S_{12}} := \frac{V_2 I_2^*}{V_1 I_1^*} = n\,a = 1$$

Transformer model

Single-phase (non-ideal) transformer

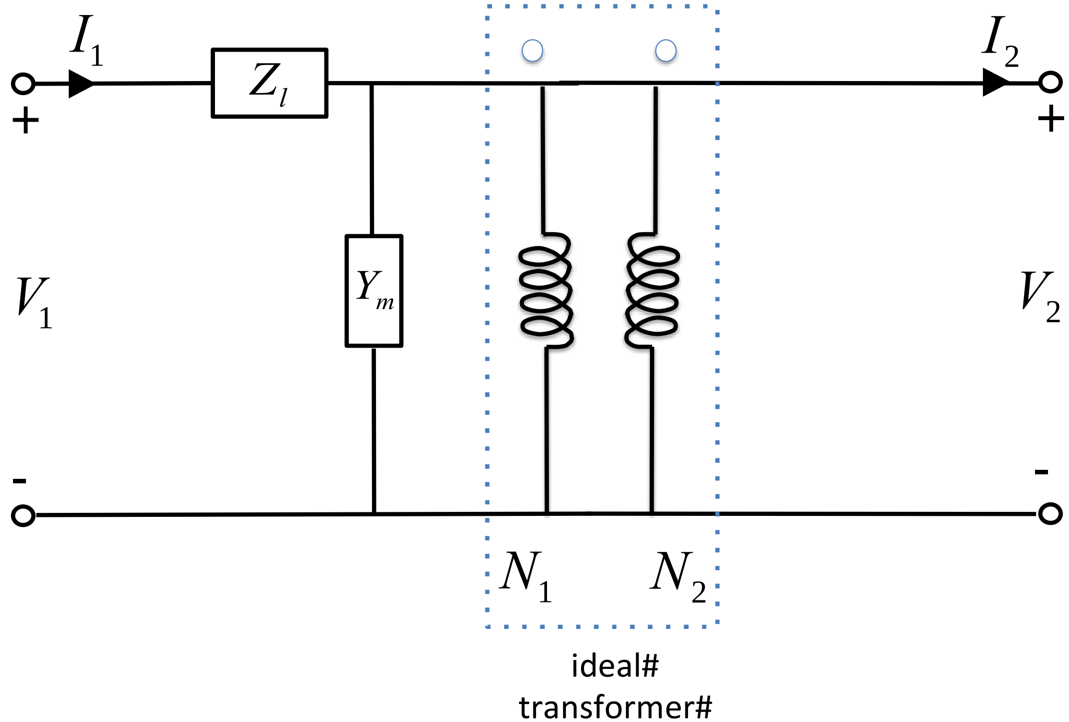


parameters (n, Z_l, Y_m)

(Z_l, Y_m) can be easily measured

Transformer model

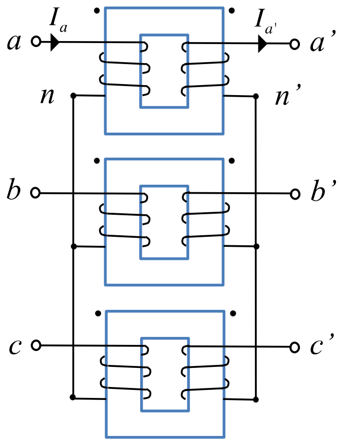
Single-phase (non-ideal) transformer

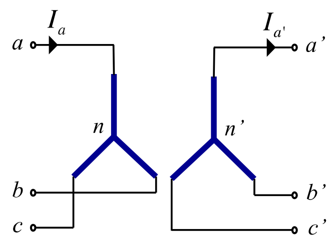


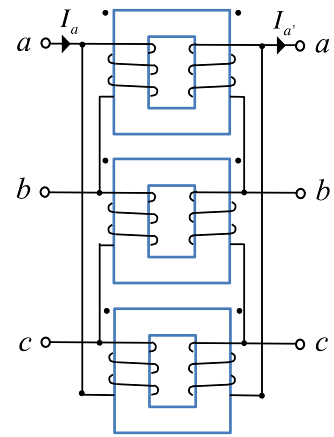
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} a(1+Z_l Y_m) & n Z_l \\ a Y_m & n \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

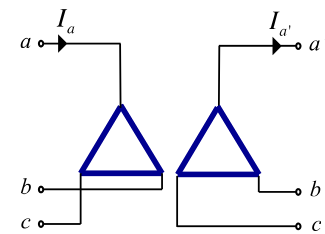
Transformer model

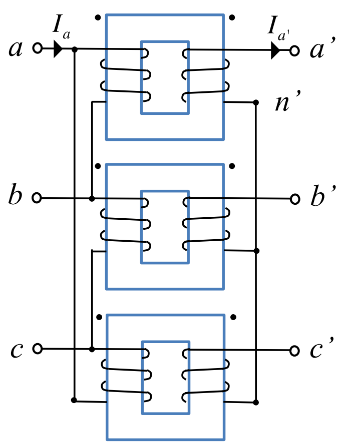
3-phase ideal transformer

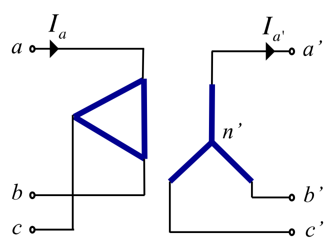












Transformer model

3-phase ideal transformer

Property	Gain
Voltage gain	$K(n)$
Current gain	$\frac{1}{K^*(n)}$
Power gain	1

Configuration	Gain
YY	$K_{YY}(n) := n$

per-phase properties

Transformer model

3-phase ideal transformer

Property	Gain
Voltage gain	$K(n)$
Current gain	$\frac{1}{K^*(n)}$
Power gain	1

Configuration	Gain
YY	$K_{YY}(n) := n$
$\Delta\Delta$	$K_{\Delta\Delta}(n) := n$

per-phase properties

Transformer model

3-phase ideal transformer

Property	Gain
Voltage gain	$K(n)$
Current gain	$\frac{1}{K^*(n)}$
Power gain	1

Configuration	Gain
YY	$K_{YY}(n) := n$
$\Delta\Delta$	$K_{\Delta\Delta}(n) := n$
ΔY	$K_{\Delta Y}(n) := \sqrt{3}n\ e^{j\pi/6}$

per-phase properties

Transformer model

3-phase ideal transformer

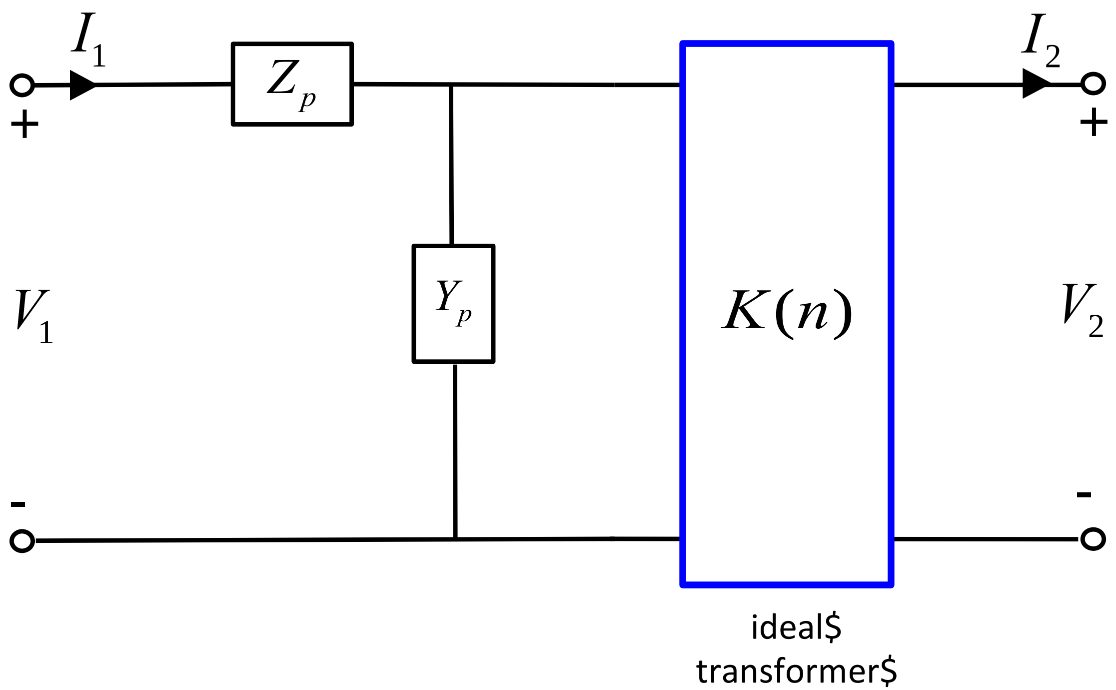
Property	Gain
Voltage gain	$K(n)$
Current gain	$\frac{1}{K^*(n)}$
Power gain	1

Configuration	Gain
YY	$K_{YY}(n) := n$
$\Delta\Delta$	$K_{\Delta\Delta}(n) := n$
ΔY	$K_{\Delta Y}(n) := \sqrt{3}n\ e^{j\pi/6}$
$Y\Delta$	$K_{Y\Delta}(n) := \frac{n}{\sqrt{3}}\ e^{j\pi/6}$

per-phase properties

Transformer model

Per-phase equivalent circuit

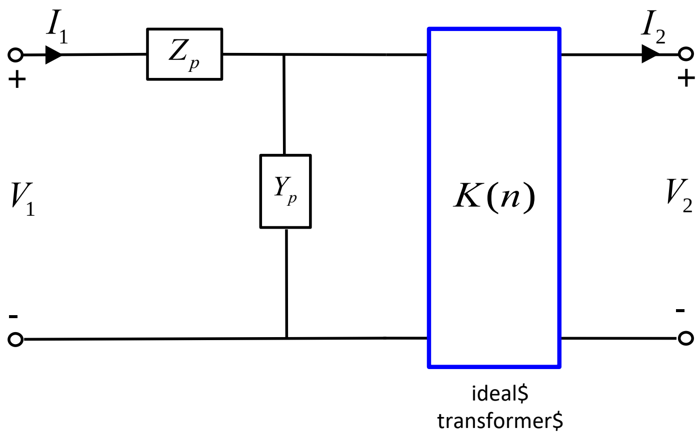


$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 1 + Z_p Y_p & Z_p \\ Y_p & 1 \end{bmatrix} \begin{bmatrix} K^{-1}(n) & 0 \\ 0 & K^*(n) \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

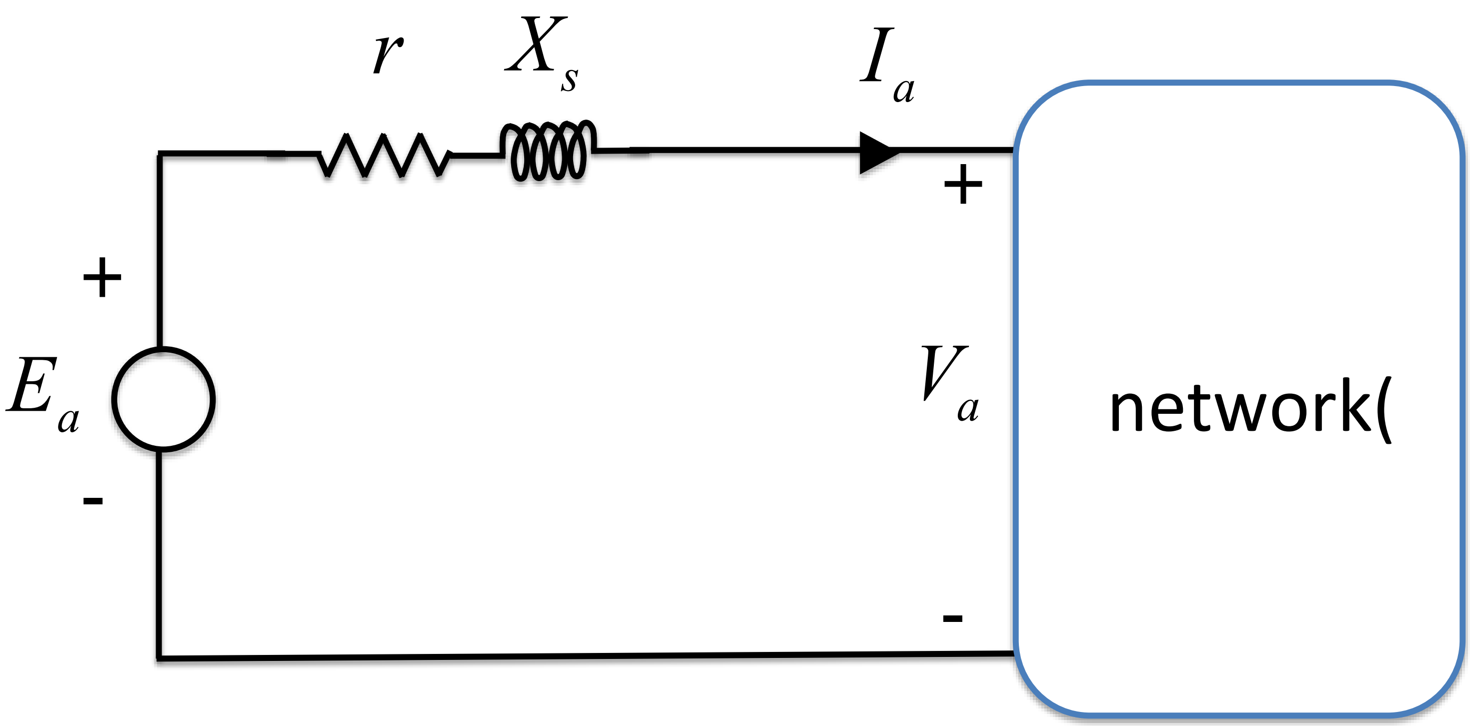
Transformer model

Recap

- Four configurations: YY, DD, DY, YD
- Linear per-phase circuit model $(V_2, I_2) \mapsto (V_1, I_1)$



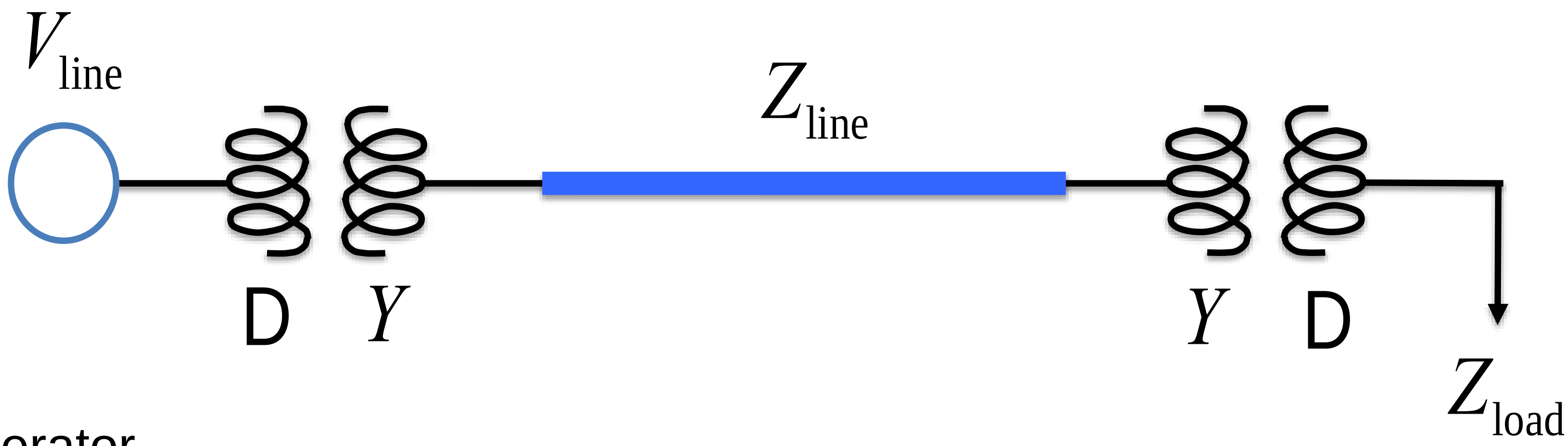
Generator model



V_a : terminal voltage

E_a : open-circuit (internal) voltage

Putting everything together



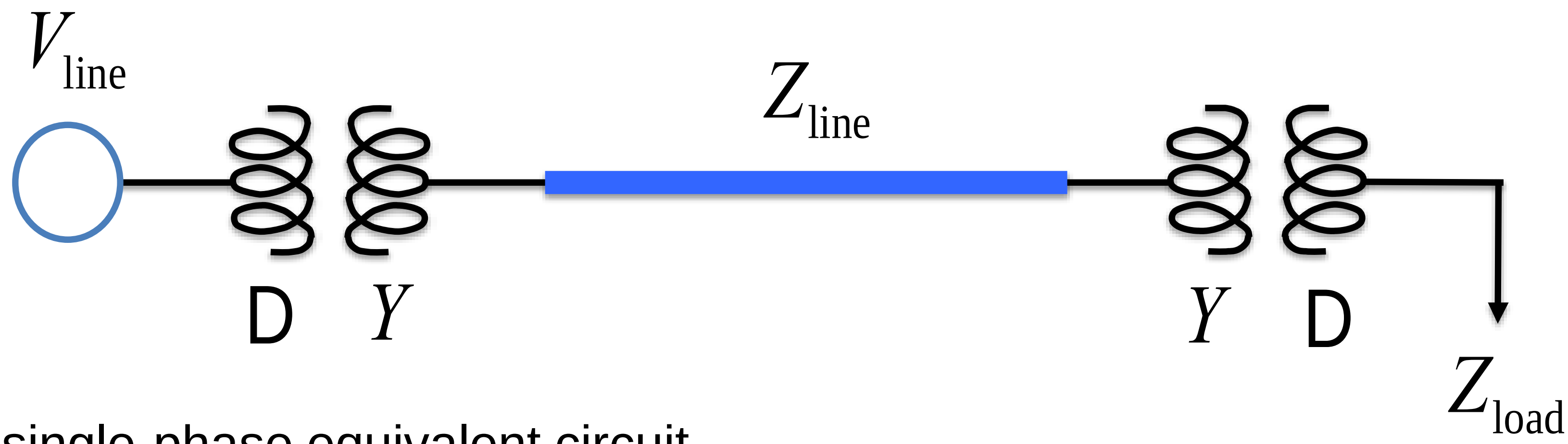
3p generator (terminal voltage)

3p transformer (stepup)

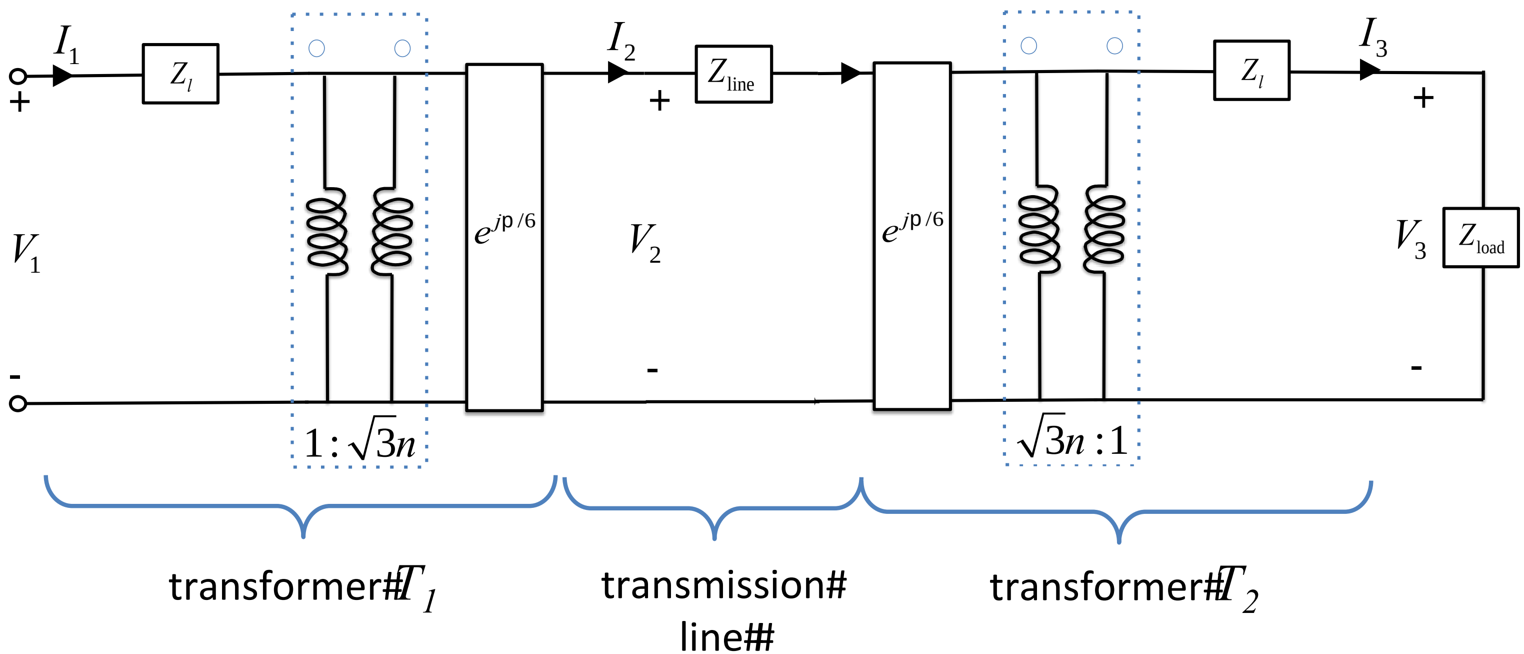
3p transmission line

3p transformer (stepdown)

Putting everything together



single-phase equivalent circuit



The flow of power II

Power flow and optimization

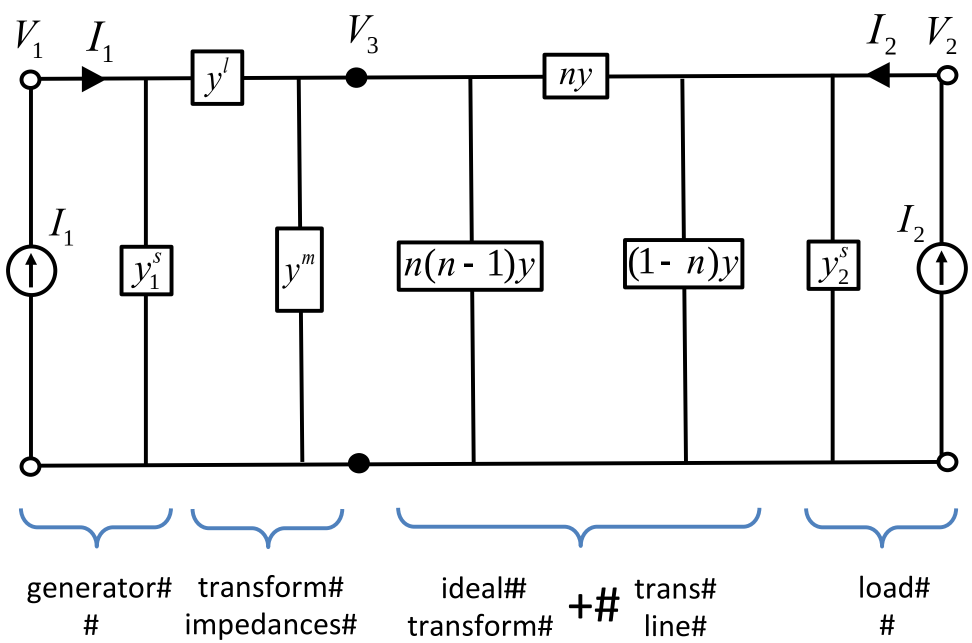
Network models (10mins)

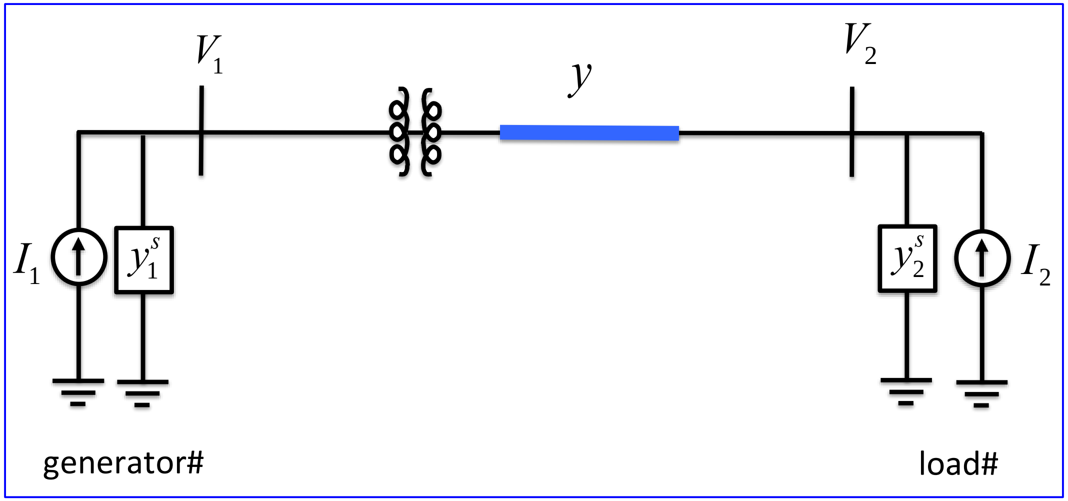
- Admittance matrix
- Power flow models

Optimal power flow problems (35mins)

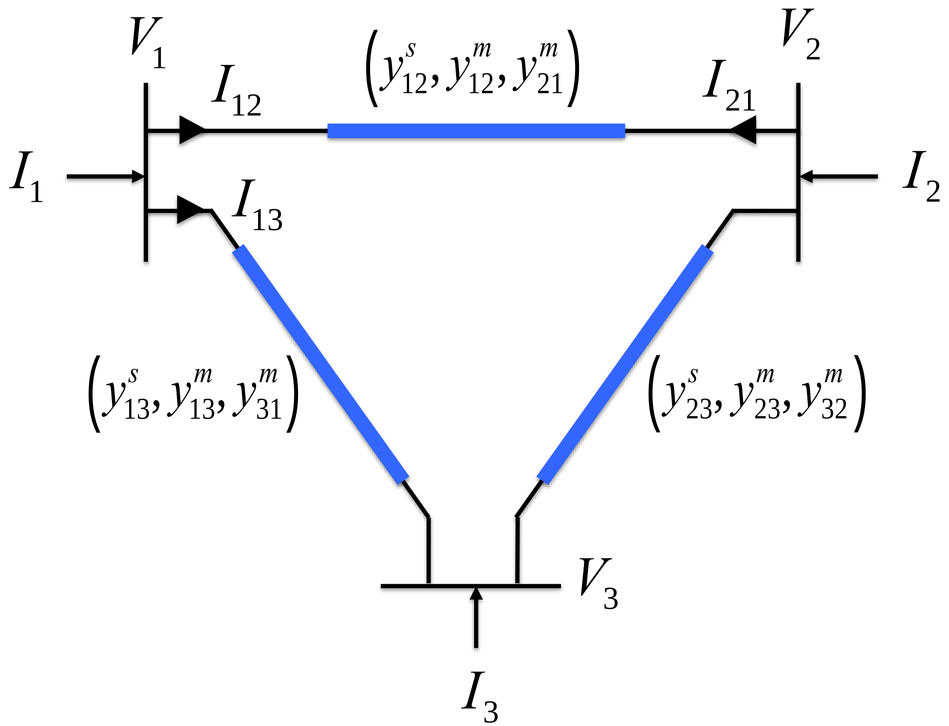
- Formulation and example
- Convex relaxations
- Real-time OPF

Example circuit model





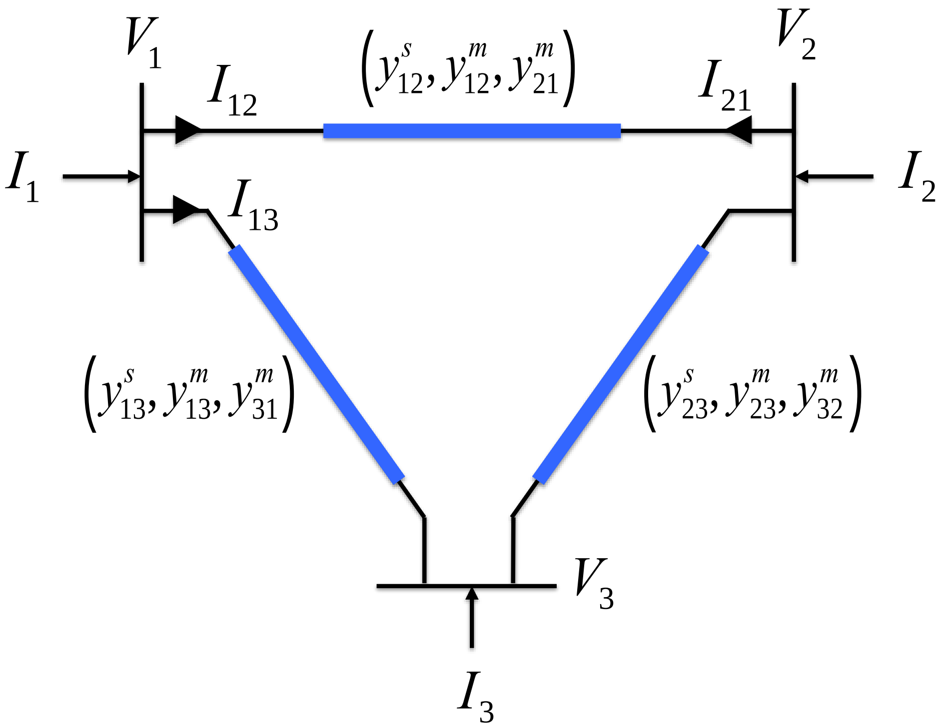
Network model



Each line modeled as P model

- Series impedance
- Shunt admittance at each end
- They may not be equal

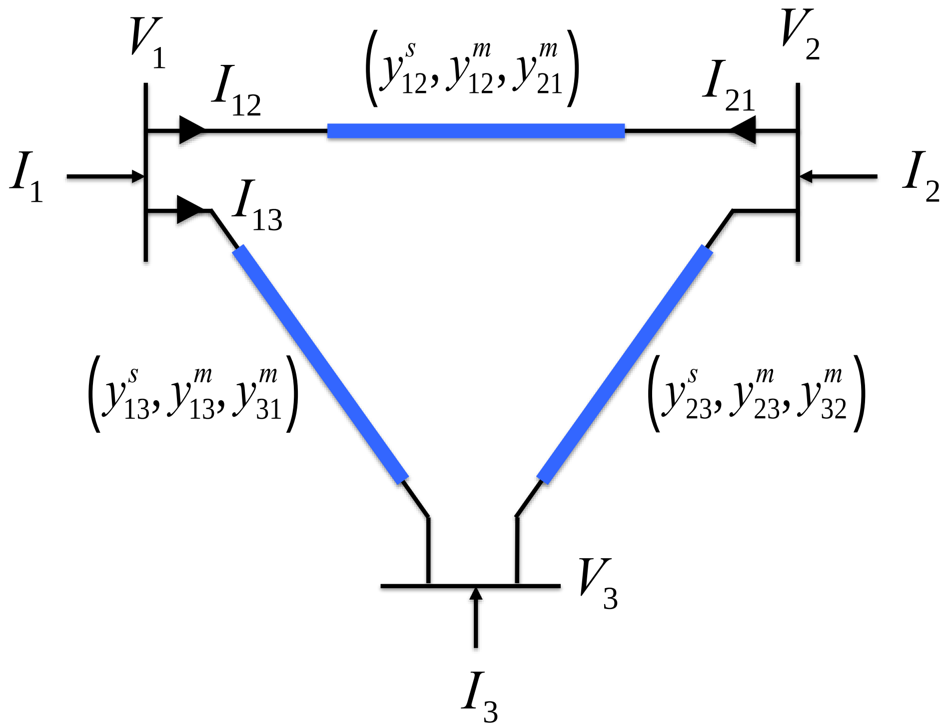
Network admittance matrix



$$I = YV$$

Y : network graph + admittances

Network admittance matrix



$$Y_{jk} = \begin{cases} -y^s_{jk}, & j \sim k \;\; (j \neq k) \\ \sum_{k:j\sim k} y^s_{jk} + y^m_{jj}, & j = k \\ 0 & \text{otherwise} \end{cases}$$

$$y^m_{jj} := \sum_{k:j\sim k} y^m_{jk}$$

The flow of power II

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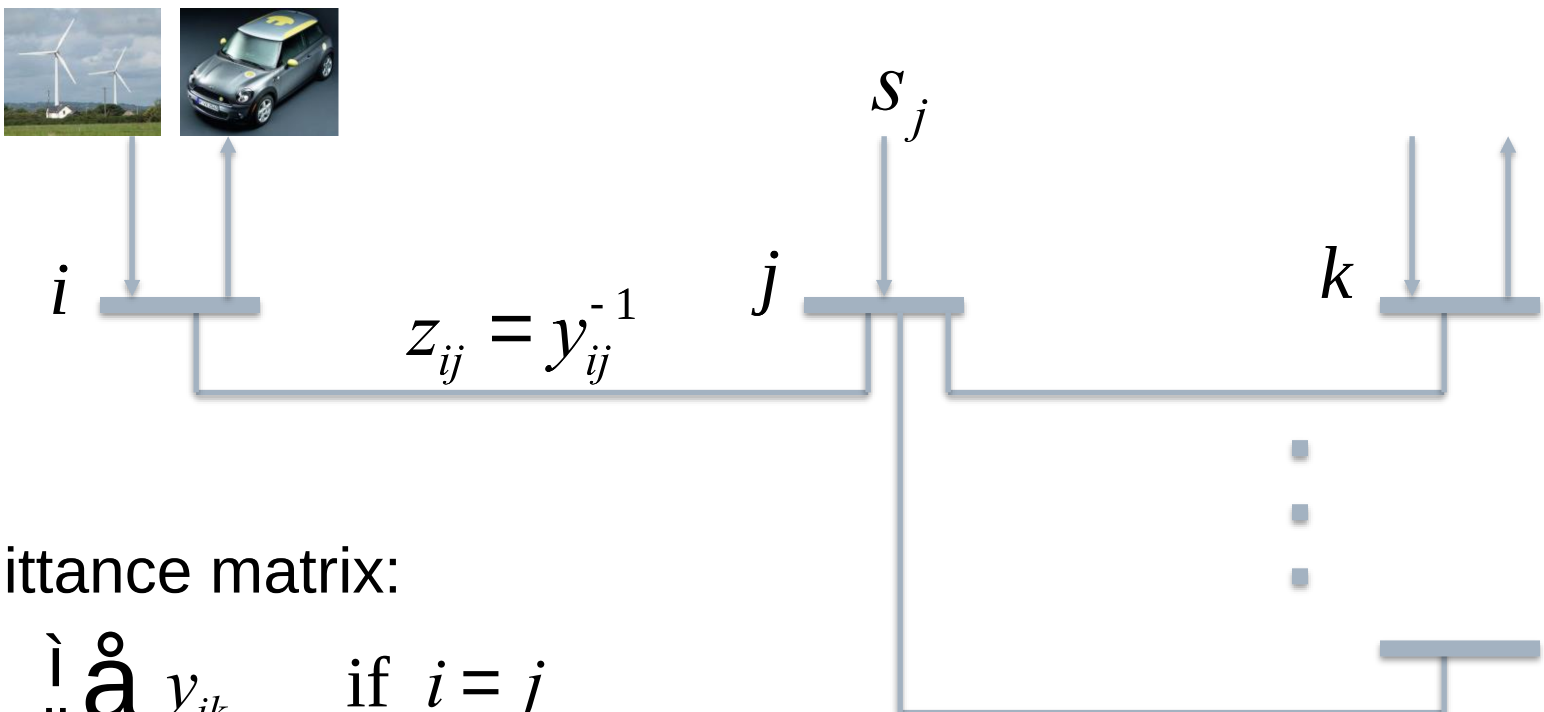
Bus injection model



admittance matrix:

$$Y_{ij} := \begin{cases} \sum_{k \sim i} y_{ik} & \text{if } i = j \\ -y_{ij} & \text{if } i \sim j \\ 0 & \text{else} \end{cases}$$

graph G: undirected

Y specifies topology of G and impedances z on lines

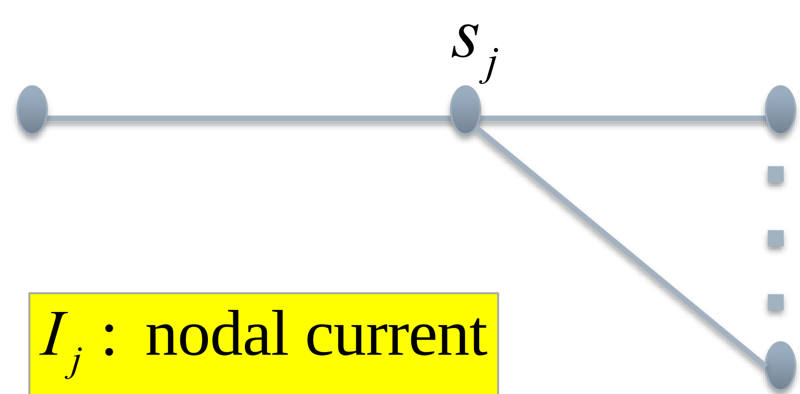
Bus injection model

$$I = YV$$ Kirchhoff law

$$s_j = V_j I_j^* \quad \text{for all } j$$ power balance

admittance matrix:

$$Y_{ij} := \begin{cases} \sum_{k \sim i} y_{ik} & \text{if } i = j \\ -y_{ij} & \text{if } i \sim j \\ 0 & \text{else} \end{cases}$$



I_j : nodal current

V_j : voltage

Bus injection model

$$I = YV$$ Kirchhoff law

$$s_j = V_j I_j^* \quad \text{for all } j$$ power balance

Eliminate I :

$$s_j = \sum_{k:k\sim j} y_{jk}^* \left(|V_j|^2 - V_j V_k^* \right) \quad \text{for all } j$$

Bus injection model

Complex form:

$$s_j = \sum_{k:k\sim j} y_{jk}^* \left(|V_j|^2 - V_j V_k^* \right) \qquad \text{for all } j$$

Polar form:

$$p_j = \left(\sum_{k=0}^{n} g_{jk} \right) |V_j|^2 - \sum_{k \neq j} |V_j||V_k| \left(g_{jk} \cos\theta_{jk} - b_{jk} \sin\theta_{jk} \right)$$

$$q_j = \left(\sum_{k=0}^{n} b_{jk} \right) |V_j|^2 - \sum_{k \neq j} |V_j||V_k| \left(b_{jk} \cos\theta_{jk} + g_{jk} \sin\theta_{jk} \right)$$

Cartesian form:

$$p_j = \sum_{k=0}^{n} \left(g_{jk} \left(e_j^2 + f_j^2 \right) - g_{jk}(e_j e_k + f_j f_k) + b_{jk}(f_j e_k - e_j f_k) \right)$$

$$q_j = \sum_{k=0}^{n} \left(b_{jk} \left(e_j^2 + f_j^2 \right) - b_{jk}(e_j e_k + f_j f_k) - g_{jk}(f_j e_k - e_j f_k) \right)$$

Bus injection model

DC power flow

$$p_j = \sum_{k=0}^{n} b_{jk} |V_j| |V_k| (\theta_j - \theta_k)$$

Assumptions:

- Lossless short line
- Small angle difference
- Fixed voltage magnitude
- Ignore reactive power

The flow of power II

Power flow and optimization

Network models (10mins)

- Admittance matrix
- Power flow models

Optimal power flow problems (35mins)

- Formulation and example
- Convex relaxations
- Real-time OPF

Optimal power flow (OPF)

OPF is solved routinely for

- network control & optimization decisions
- market operations & pricing
- at timescales of mins, hours, days, …

Non-convex and hard to solve

- Huge literature since 1962
- Common practice: DC power flow (LP)
- Also: Newton-Raphson, interior point, …

$$\min \; c(x) \quad \text{s. t.} \quad F(x) = 0, \;\; x \le \overline{x}$$

Optimal power flow

min	$\mathrm{tr}\left(CVV^H\right)$	gen cost, power loss
over	(V, s, l)	
subject to	$s_j = \mathrm{tr}\left(Y_j^H VV^H\right)$	power flow equation
	$l_{jk} = \mathrm{tr}\left(B_{jk}^H VV^H\right)$	line flow
	$\underline{s}_j$ £ s_j £ $\overline{s}_j$	injection limits
	$\underline{l}_{jk}$ £ l_{jk} £ $\overline{l}_{jk}$	line limits
	$\underline{V}_j$ £ $\lvert V_j \rvert$ £ $\overline{V}_j$	voltage limits

- Y_j^H describes network topology and impedances
- s_j is net power injection (generation) at node j

Optimal power flow

min	$\operatorname{tr}\left(CVV^H\right)$	gen cost, power loss
over	(V, s, l)	
subject to	$s_j = \operatorname{tr}\left(Y_j^H VV^H\right)$	power flow equation
	$l_{jk} = \operatorname{tr}\left(B_{jk}^H VV^H\right)$	line flow
	$\underline{s}_j \le s_j \le \bar{s}_j$	injection limits
	$\underline{l}_{jk} \le l_{jk} \le \bar{l}_{jk}$	line limits
	$\underline{V}_j \le \lvert V_j \rvert \le \overline{V}_j$	voltage limits

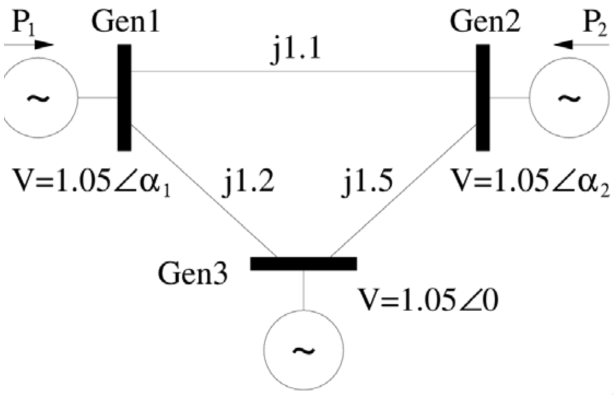
nonconvex feasible set (nonconvex QCQP)

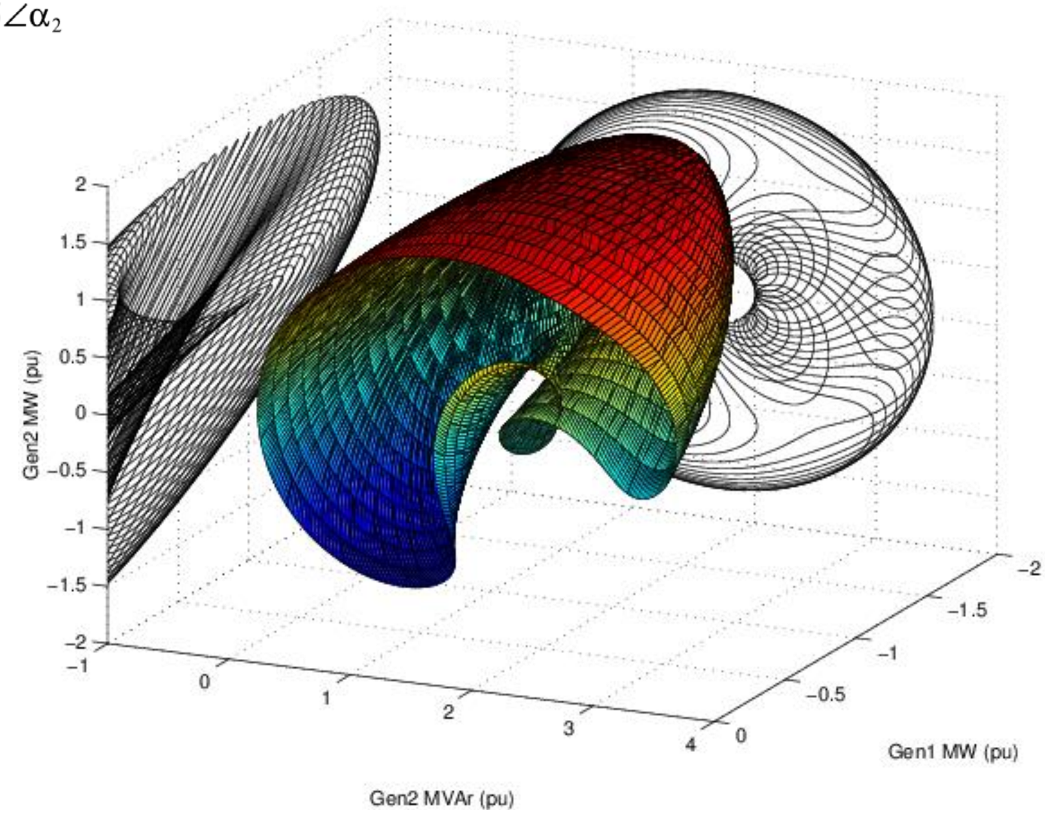
- Y_j^H not Hermitian (nor positive semidefinite)
- C is positive semidefinite (and Hermitian)

Optimal power flow

OPF problem underlies numerous applications

- nonlinearity of power flow equations ➔ nonconvexity





Ian Hiskens, Michigan

Dealing with nonconvexity

Linearization

- DC approximation

Convex relaxations

- Semidefinite relaxation (Lasserre hierarchy)
- QC relaxation (van Hentenryck)
- Strong SOCP (Sun)

Dealing with nonconvexity

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Realtime OPF

- Online algorithm, as opposed to offline
- Also tracks time-varying OPF

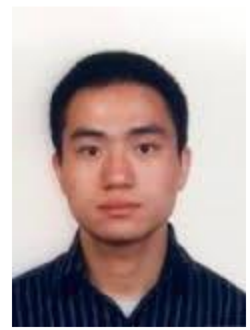
Relaxations of AC OPF

dealing with nonconvexity


Bose (UIUC)


Chandy


Farivar (Google)


Gan (FB)

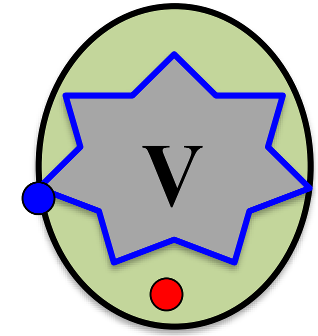

Lavaei (UCB)


Li (Harvard)

many others at & outside Caltech …

Low, Convex relaxation of OPF, 2014
http://netlab.caltech.edu

Equivalent feasible sets



$$\min \quad \operatorname{tr} CVV^H$$

$$\text{subject to} \quad \underline{s}_j \le \operatorname{tr}\left(Y_j^H VV^H\right) \le \bar{s}_j \qquad \underline{v}_j \le |V_j|^2 \le \bar{v}_j$$

quadratic in V
linear in W

Equivalent problem:

$$\min \quad \operatorname{tr} CW$$

$$\text{subject to} \quad \underline{s}_j \le \operatorname{tr}\left(Y_j^H W\right) \le \bar{s}_j \qquad \underline{v}_j \le W_{jj} \le \bar{v}_j$$

$$W \succeq 0, \quad \operatorname{rank} W = 1$$

convex in W
except this constraint

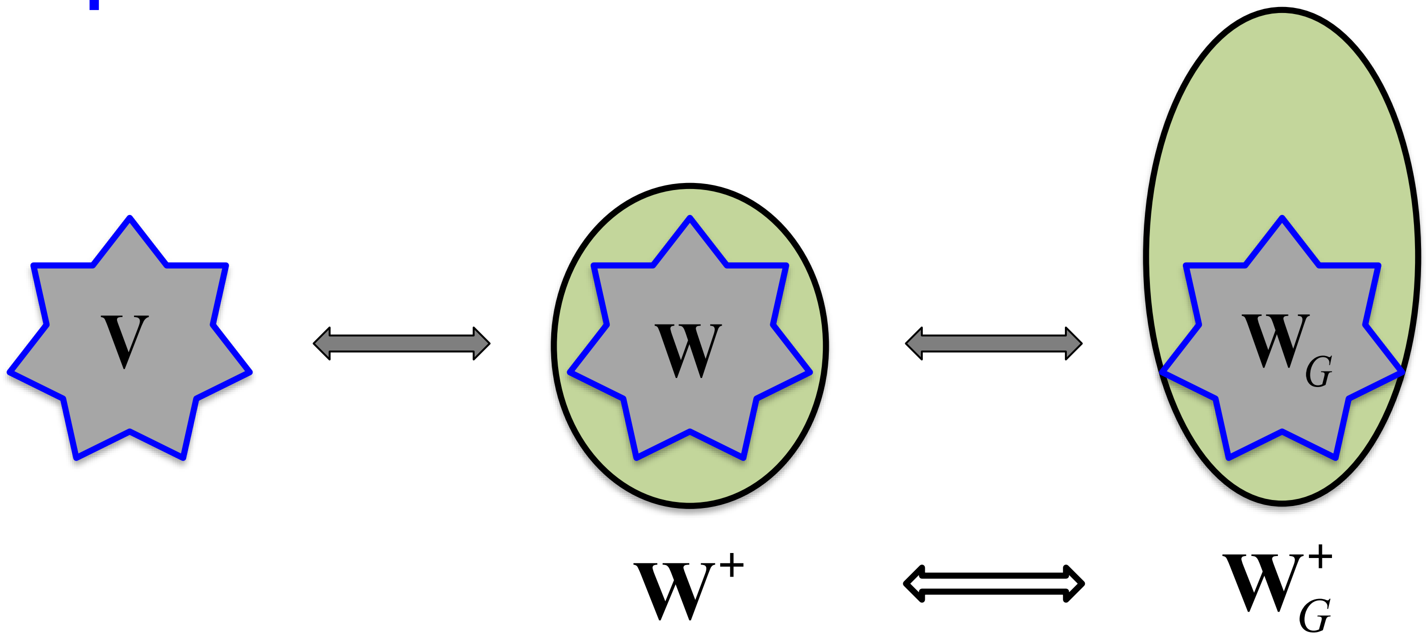
Solution strategy

OPF: $$\min_{x \in \mathbf{X}} f(x)$$

relaxation: $$\min_{\hat{x} \in \mathbf{X}^+} f(\hat{x})$$

If optimal solution $\hat{x}^*$ satisfies easily checkable conditions, then optimal solution x^* of OPF can be recovered

Equivalent relaxations



Theorem

- Radial G: SOCP is equivalent to SDP ($\mathbb{V} \subseteq \mathbb{W}^+ \equiv \mathbb{W}_G^+$)
- Mesh G: SOCP is strictly coarser than SDP

For radial networks: always solve SOCP !

Exact relaxation

For radial networks, sufficient conditions on

- power injections bounds, or
- voltage upper bounds, or
- phase angle bounds

Exact relaxation

For radial networks, sufficient conditions on

- power injections bounds, or
- voltage upper bounds, or
- phase angle bounds

Exact relaxation

QCQP (C, C_k)

$$
\begin{aligned}
\min \quad & \mathrm{tr}\left(C x x^H\right) \\
\text{over} \quad & x \in \mathbf{C}^n \\
\text{s.t.} \quad & \mathrm{tr}\left(C_k x x^H\right) \le b_k \qquad k \in K
\end{aligned}
$$

graph of QCQP

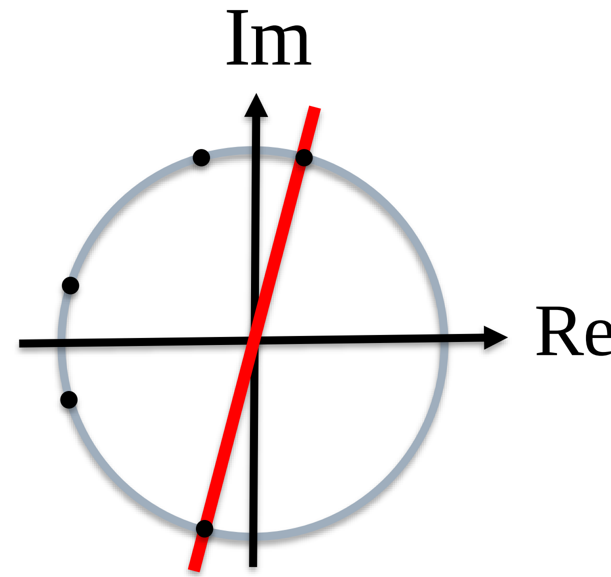
$G(C, C_k)$ has edge (i, j) $\Leftrightarrow$

$C_{ij} \neq 0$ or $[C_k]_{ij} \neq 0$ for some k

QCQP over tree

$G(C, C_k)$ is a tree

Exact relaxation



QCQP (C, C_k)

$$\begin{aligned} &\min && \operatorname{tr}\left(C x x^H\right) \\ &\text{over} && x \in \mathbf{C}^n \\ &\text{s.t.} && \operatorname{tr}\left(C_k x x^H\right) \le b_k \qquad k \in K \end{aligned}$$

Key condition

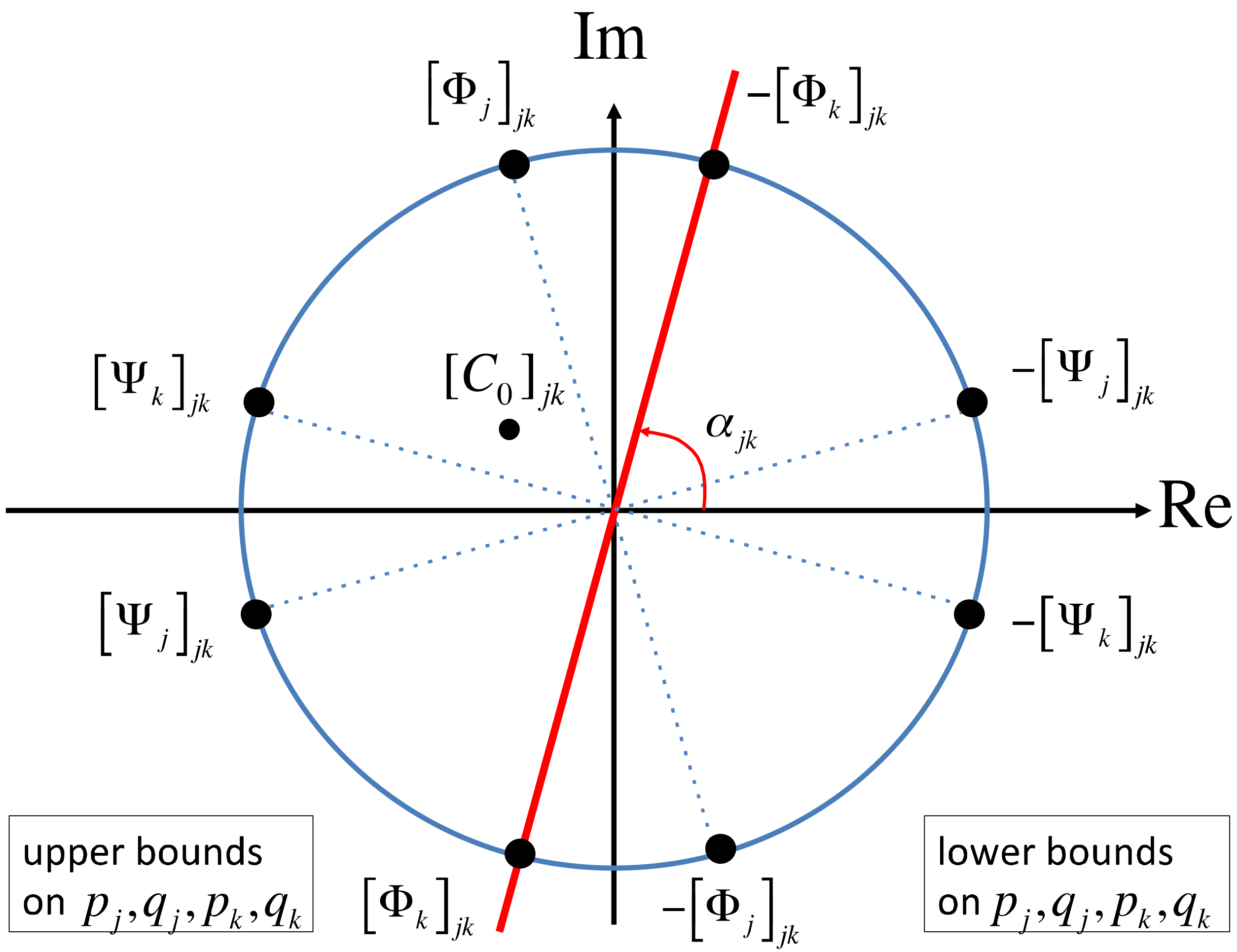
$i \sim j: \left(C_{ij}, [C_k]_{ij}, \ \forall k\right)$ lie on half-plane through 0

Theorem

SOCP relaxation is exact for QCQP over tree

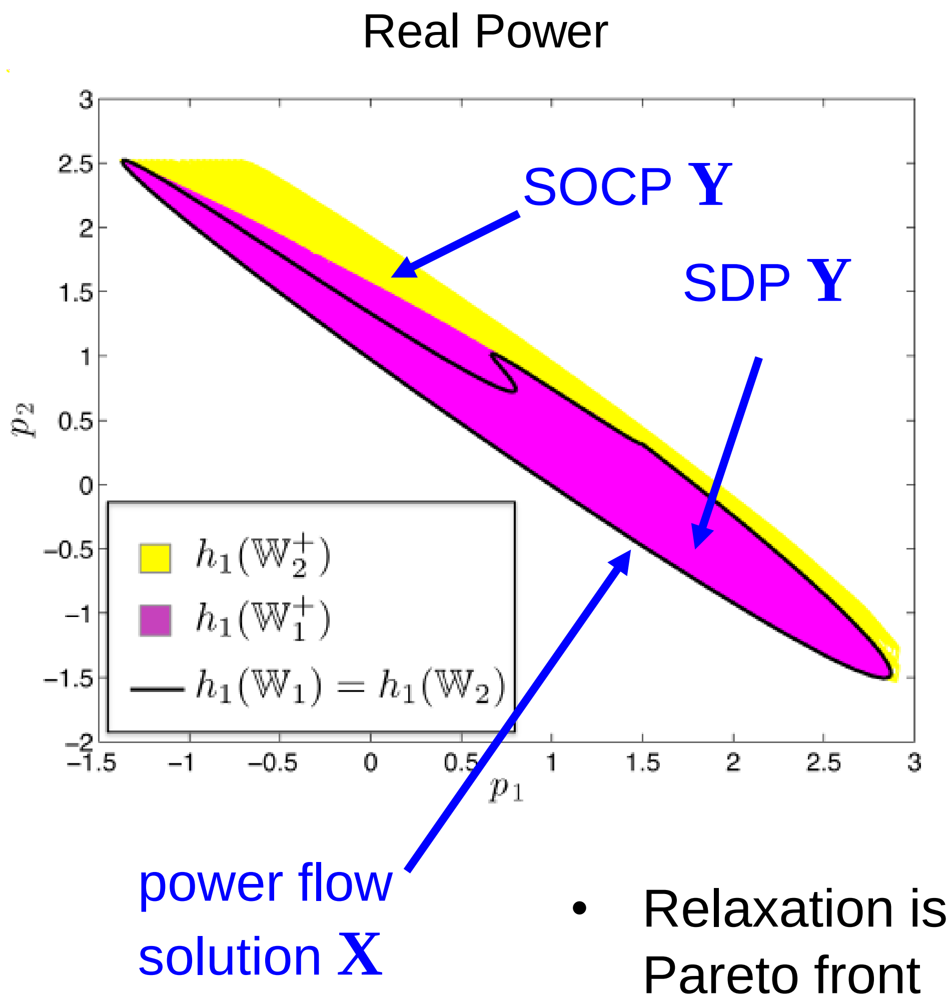
Bose et al 2012, 2014
Sojoudi, Lavaei 2013

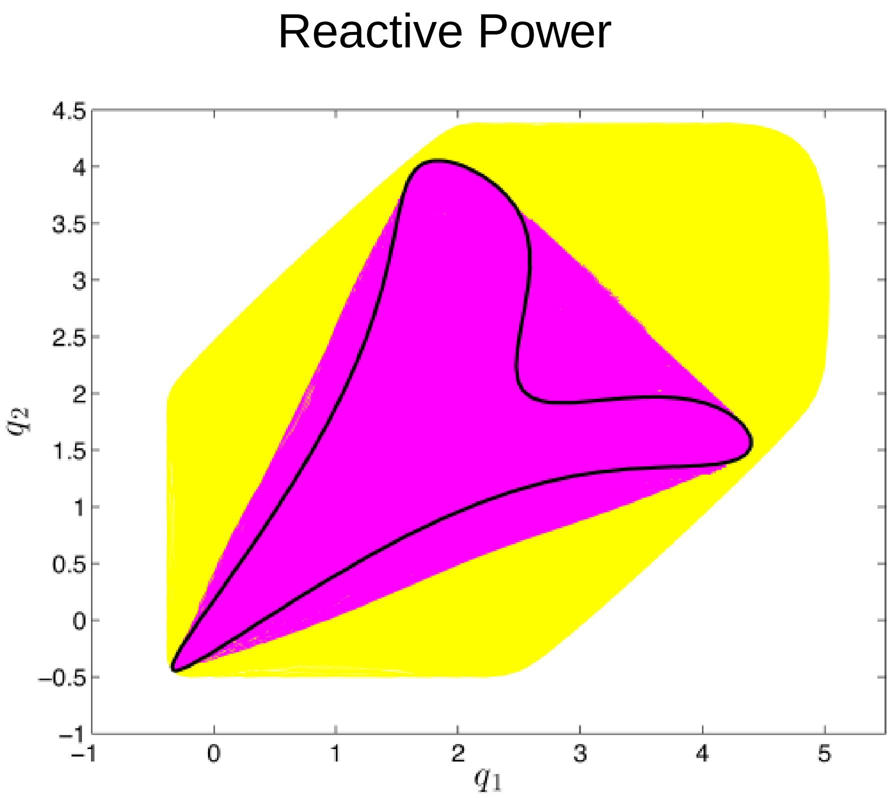
Implication on OPF



Not both lower & upper bounds on real & reactive powers at both ends of a line can be finite

Example





- Relaxation is exact if $\mathbf{X}$ and $\mathbf{Y}$ have same Pareto front
- SOCP is faster but coarser than SDP

Bose, Low, Teeraratkul, Hassibi TAC 2015

Potential benefits

IEEE test systems		SDP cost	MATPOWER cost
Syst.	$\text{rank}(\overline{X}_0)$	J°	$\overline{J}$
9	1	5296.7	5296.7
30	1	576.9	576.9
118	1	129661	129661
14A	1	8092.8	9093.8

12.4% lower cost than solution from nonlinear solver MATPOWER

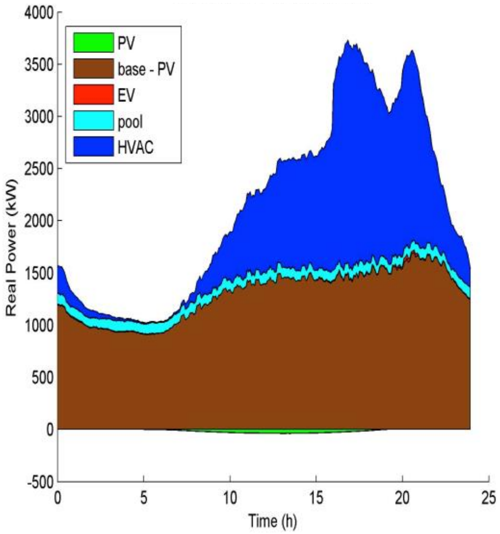
[Louca, Seiler, Bitar 2013]

Potential benefits

Case study on an SCE feeder

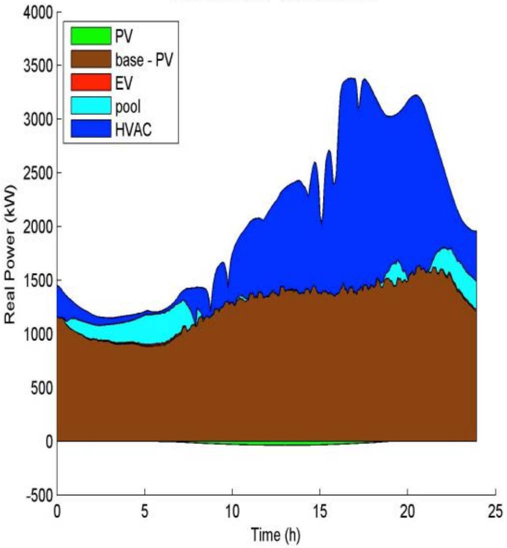
- Southern California
- 1,400 residential houses, ~200 commercial buildings
- Controllable loads: EV, pool pumps, HVAC, PV inverters
- Formulated as an OPF problem, multiphase unbalanced radial network

baseline



peak load reduction: 8%
energy cost reduction: 4%

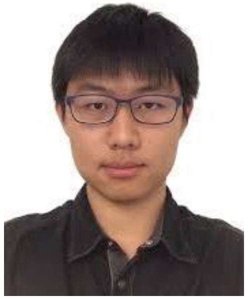
optimized



Realtime AC OPF

for tracking

Gan (FB)



Tang (Caltech)

Dvijotham (DeepMind)

See also: Dall'Anese et al, Bernstein et al, Hug & Dorfler et al, Callaway et al

Gan & L, JSAC 2016
Tang et al, TSG 2017

Motivations

Simplify OPF simulation/solution

- Solving static OPF with simulator in the loop
- Avoid modifying GridLab-D during ARPA-E GENI (2012-15)

Deal with nonconvexity

- Network computes power flow solutions in real time at scale for free
- Exploit it for our optimization/control

Track optimal solution of time-varying OPF

- Uncertainty will continue to increase
- Real-time measurements increasingly become available on seconds timescale
- Must, and can, close the loop in the future

Dealing with nonconvexity

Linearization

- DC approximation

Convex relaxations

- Semidefinite relaxation (Lasserre hierarchy)
- QC relaxation (van Hentenryck)
- Strong SOCP (Sun)

Realtime OPF

- Online algorithm, as opposed to offline
- Also tracks time-varying OPF

Literature

Static OPF:

- Gan and Low, JSAC 2016
- Dall'Anese, Dhople and Giannakis, TPS 2016
- Arnold et al, TPS 2016
- A. Hauswirth, et al, Allerton 2016

Time-varying OPF:

- Dall'Anese and Simonetto, TSG 2016
- Wang et al, TPS 2016
- Tang, Dvijotham and Low, TSG 2017
- Tang and Low, CDC 2017

Earlier relevant work on voltage control

- Survey: Molzahn et al, TSG 2017

OPF

$$\min \quad c_0(y) + c(x)$$

$$\text{over} \quad x, \; y$$

$$\text{s. t.}$$

controllable devices

uncontrollable state

OPF

$$\min \quad c_0(y) + c(x)$$

$$\text{over} \quad x,\ y$$

$$\text{s. t.} \quad F(x, y) = 0$$

power flow equations

OPF

$$\min \quad c_0(y) + c(x)$$

$$\text{over} \quad x,\ y$$

$$\text{s. t.} \quad F(x, y) = 0$$ power flow equations

$$y \le \overline{y}$$ operational constraints

$$x \in X := \{ \underline{x} \le x \le \overline{x} \}$$ capacity limits

$$\text{Assume: } \frac{\partial F}{\partial y} \ne 0 \quad \Rightarrow \quad y(x) \text{ over } X$$

OPF: eliminate y

$$\min_{x} \quad c_0(y(x)) + c(x)$$

$$\text{s. t.} \quad y(x) \le \overline{y}$$

$$x \in X := \{ \underline{x} \le x \le \overline{x} \}$$

Theorem [Huang, Wu, Wang, & Zhao. TPS 2016]
For DistFlow model, controllable (feasible) region

$$\{ x \mid y(x) \le \overline{y},\ x \in X \}$$

is convex (despite nonlinearity of $y(x)$)

OPF: add barrier or penalty

$$
\begin{aligned}
\min_{x} \quad & c_0(y(x)) + c(x) \\
\text{s. t.} \quad & y(x) \le \overline{y} \\
& x \in X := \{ \underline{x} \le x \le \overline{x} \}
\end{aligned}
$$

add barrier or penalty function
to remove operational constraints

$$
\begin{aligned}
\min \quad & f(x, y(x);\ \mu) \\
\text{over} \quad & x \in X
\end{aligned}
$$

f: nonconvex

Online (feedback) perspective

DER : gradient update

$$x(t+1) = G(x(t),\ y(t))$$

control $x(t)$

measurement, communication $y(t)$

Network: power flow solver

$$y(t) : F(x(t),\ y(t)) = 0$$

cyber network

physical network

- Explicitly exploits network as power flow solver
- Naturally tracks changing network conditions

Outline: realtime OPF

Motivation

Problem formulation

Static OPF

[Gan & Low, JSAC 2016]

- 1st order algorithm
- Optimality properties

Time-varying OPF

[Tang, Dj, & Low, TSG 2017]

- 2nd order algorithm
- Tracking performance
- Distributed implementation

[Tang & Low, CDC 2017]

Static OPF

$$\begin{array}{ll} \min & f(x, y(x);\ \mu) \\ \text{over} & x \in X \end{array}$$

gradient projection algorithm:

$$x(t+1) = \left[x(t) - \eta \frac{\partial f}{\partial x}(t) \right]_X \qquad \text{active control}$$

$$y(t) = y(x(t)) \qquad \text{law of physics}$$

[Gan & Low, JSAC 2016]

Local optimality

Under appropriate assumptions

- $x(t)$ converges to set of local optima
- if #local optima is finite, $x(t)$ converges

Global optimality

Assume: $p_0(x)$ convex over X

$v_k(x)$ concave over X

$$A := \{ x \in X : v(x) \le a\bar{v} + (1-a)\underline{v} \}$$

Theorem

If co{local optima} are in A then

- $x(t)$ converges to the set of global optima
- $x(t)$ itself converges a global optimum if #local optima is finite

Global optimality

Assume: $p_0(x)$ convex over X

$v_k(x)$ concave over X

$$A := \{ x \in X \ : \ v(x) \le \alpha \overline{v} + (1-\alpha)\underline{v} \}$$

Theorem

- Can choose a s.t.

  $A \rightarrow$ original feasible set

- If SOCP is exact over X, then assumption holds

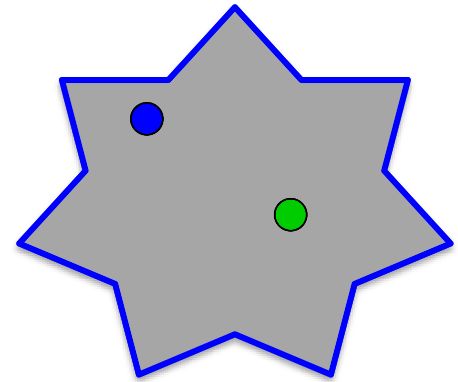
Incidentally, this turns out to be the convergence condition in Arnold,et al, “Model-Free Optimal Control of VAR Resources in Distribution Systems: An Extremum Seeking Approach,”

Suboptimality gap

any local optimum

any original feasible pt slightly away from X boundary

$f\left(x^*\right) - f\left(\hat{x}\right)$ £ r » 0



- Informally, a local minimum is almost as good as any strictly interior feasible point

Simulations

# bus	CVX		IPM		error	speedup
	obj	time(s)	obj	time(s)		
42	10.4585	6.5267	10.4585	0.2679	-0.0e-7	24.36
56	34.8989	7.1077	34.8989	0.3924	+0.2e-7	18.11
111	0.0751	11.3793	0.0751	0.8529	+5.4e-6	13.34
190	0.1394	20.2745	0.1394	1.9968	+3.3e-6	10.15
290	0.2817	23.8817	0.2817	4.3564	+1.1e-7	5.48
390	0.4292	29.8620	0.4292	2.9405	+5.4e-7	10.16
490	0.5526	36.3591	0.5526	3.0072	+2.9e-7	12.09
590	0.7035	43.6932	0.7035	4.4655	+2.4e-7	9.78
690	0.8546	51.9830	0.8546	3.2247	+0.7e-7	16.12
790	0.9975	62.3654	0.9975	2.6228	+0.7e-7	23.78
890	1.1685	67.7256	1.1685	2.0507	+0.8e-7	33.03
990	1.3930	74.8522	1.3930	2.7747	+1.0e-7	26.98
1091	1.5869	83.2236	1.5869	1.0869	+1.2e-7	76.57
1190	1.8123	92.4484	1.8123	1.2121	+1.4e-7	76.27
1290	2.0134	101.0380	2.0134	1.3525	+1.6e-7	74.70
1390	2.2007	111.0839	2.2007	1.4883	+1.7e-7	74.64
1490	2.4523	122.1819	2.4523	1.6372	+1.9e-7	74.83
1590	2.6477	157.8238	2.6477	1.8021	+2.0e-7	87.58
1690	2.8441	147.6862	2.8441	1.9166	+2.1e-7	77.06
1790	3.0495	152.6081	3.0495	2.0603	+2.1e-7	74.07
1890	3.8555	160.4689	3.8555	2.1963	+1.9e-7	73.06
1990	4.1424	171.8137	4.1424	2.3586	+1.9e-7	72.84

Outline: realtime OPF

Motivation

Problem formulation

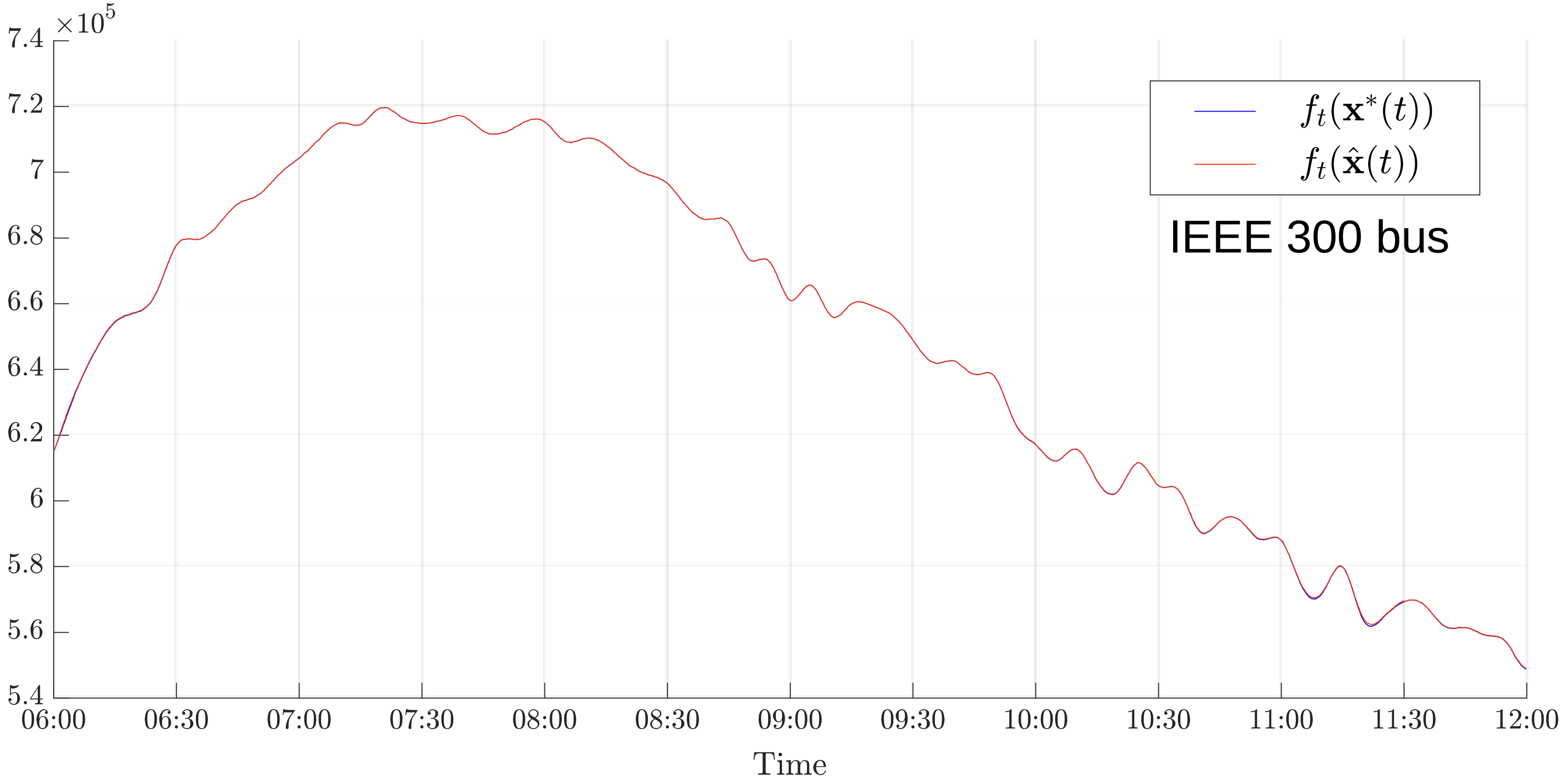
Static OPF [Gan & Low, JSAC 2016]

Dynamic OPF [Tang, Dj, & Low, TSG 2017]

- 2nd order algorithm
- Tracking performance
- Distributed implementation [Tang & Low, CDC 2017]

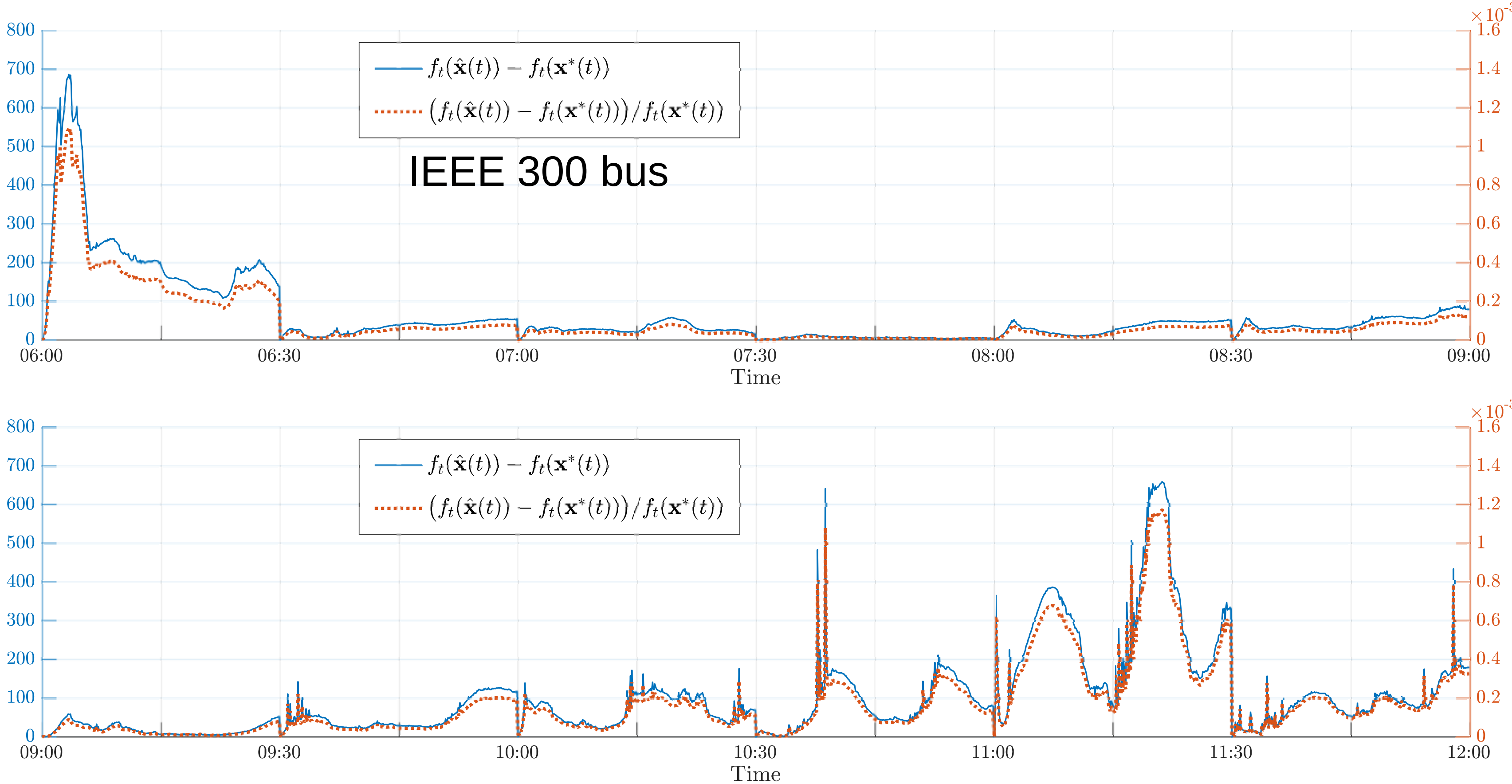
See also: Dall'Anese and Simonetto, TSG 2016
Wang et al, TPS 2016

Tracking performance



realtime OPF algorithms can track time-varying OPF well

Tracking performance



realtime OPF algorithms can track time-varying OPF well

Drifting OPF

$$\min_{x} \quad c_0(y(x)) + c(x)$$
$$\text{s. t.} \quad y(x) \le \overline{y}$$
$$x \in X$$

static OPF

$$\min_{x} \quad c_0(y(x), \gamma_t) + c(x, \gamma_t)$$
$$\text{s. t.} \quad y(x, \gamma_t) \le \overline{y}$$
$$x \in X$$

drifting OPF

Drifting OPF

$$\begin{aligned} \min \quad & f_t(x, y(x);\ \mu_t) \\ \text{over} \quad & x \in X_t \end{aligned}$$

Quasi-Newton algorithm:

$$x(t+1) = \left[x(t) - \eta \left(H(t) \right)^{-1} \frac{\partial f_t}{\partial x}(x(t)) \right]_{X_t} \qquad \text{active control}$$

$$y(t) = y(x(t)) \qquad \text{law of physics}$$

[Tang, Dj & Low, 2017]

Drifting OPF

$$\begin{array}{ll} \min & f_t(x, y(x);\ \eta_t) \\ \text{over} & x \in X_t \end{array}$$

Computing $x(t+1)$ by solving convex QP:

$$\min_x \ \left(\nabla f_t(x(t))\right)^T (x - x(t)) + \frac{1}{2}(x - x(t))^T B_t(x(t))(x - x(t))$$

$$\text{s. t.} \quad x \in X_t$$

e.g. approx Hessian

[Tang, Dj & Low, 2017]

Tracking performance

$$\text{error} := \frac{1}{T}\sum_{t=1}^{T}\left\| x^{\text{online}}(t) - x^{*}(t) \right\|$$

control error

$$(\text{assuming } x^{\text{online}}(0) = x^{*}(0))$$

Tracking performance

$$\text{error} := \frac{1}{T}\sum_{t=1}^{T} \left\| x^{\text{online}}(t) - x^{*}(t) \right\|$$

Theorem

$$\text{error} \le \frac{\epsilon}{\sqrt{\lambda_m/\lambda_M} - \epsilon} \cdot \frac{1}{T}\sum_{t=1}^{T}\left(\left\| x^{*}(t) - x^{*}(t-1) \right\| + \Delta_t \right)$$

avg rate of drifting

- of optimal solution
- of feasible set

[Tang, Dj, & Low, TSG 2017]

Tracking performance

$$\text{error} := \frac{1}{T}\sum_{t=1}^{T}\left\| x^{\text{online}}(t) - x^{*}(t) \right\|$$

Theorem

$$\text{error} \le \frac{\epsilon}{\sqrt{\lambda_m/\lambda_M} - \epsilon} \cdot \frac{1}{T}\sum_{t=1}^{T}\left(\left\| x^{*}(t) - x^{*}(t-1) \right\| + \Delta_t \right)$$

error in Hessian approx

[Tang, Dj, & Low, TSG 2017]

Tracking performance

$$\text{error} := \frac{1}{T}\sum_{t=1}^{T}\left\| x^{\text{online}}(t) - x^{*}(t)\right\|$$

Theorem

$$\text{error} \leq \frac{\epsilon}{\sqrt{\lambda_m/\lambda_M} - \epsilon} \cdot \frac{1}{T}\sum_{t=1}^{T}\left(\left\| x^{*}(t) - x^{*}(t-1)\right\| + \Delta_t\right)$$

"condition number"
of Hessian

[Tang, Dj, & Low, TSG 2017]

Implementation

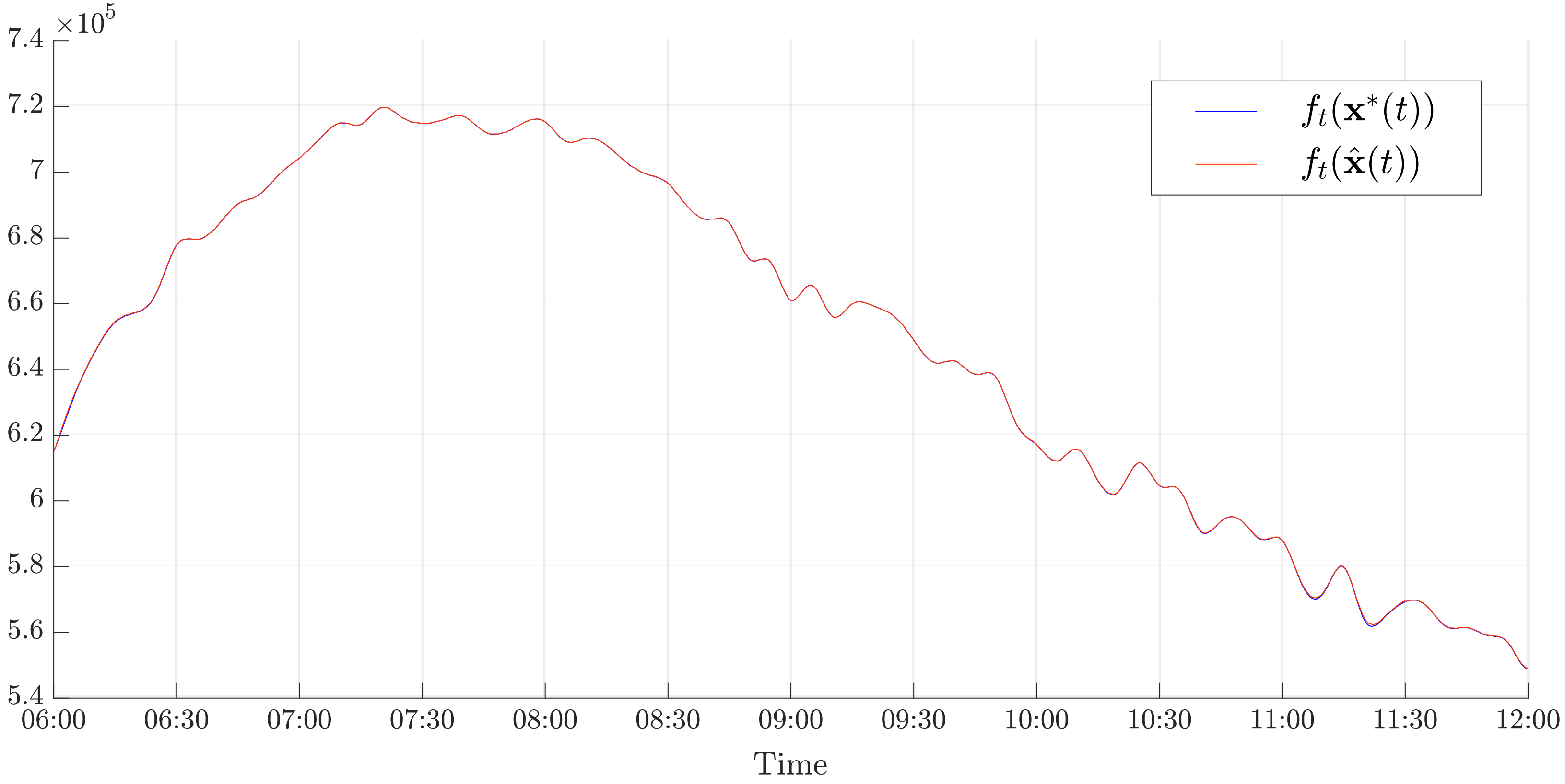
Implement L-BFGS-B

- More scalable
- Handles (box) constraints X

Simulations

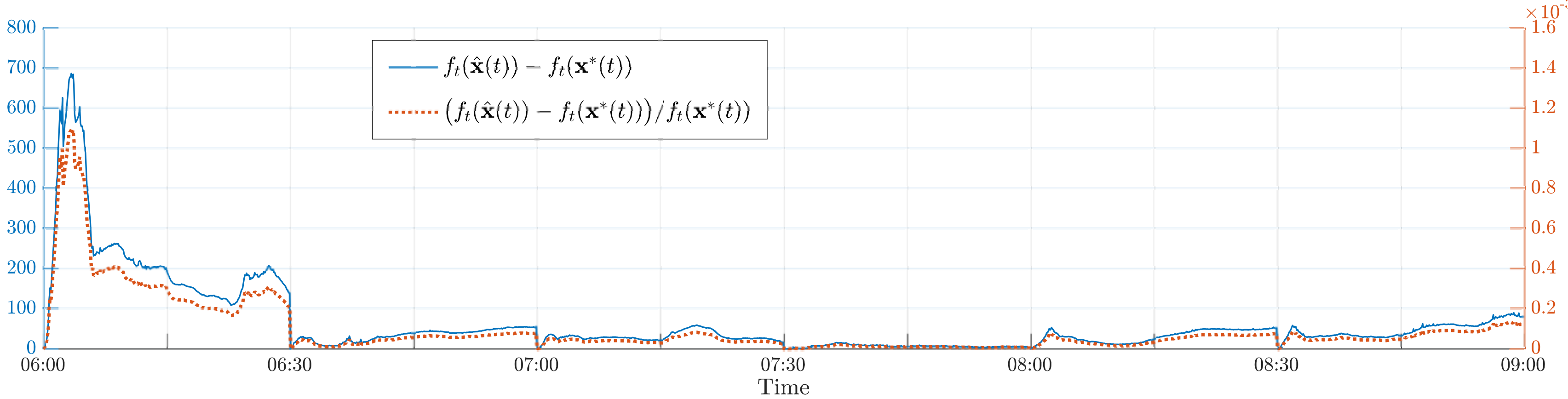
- IEEE 300 bus

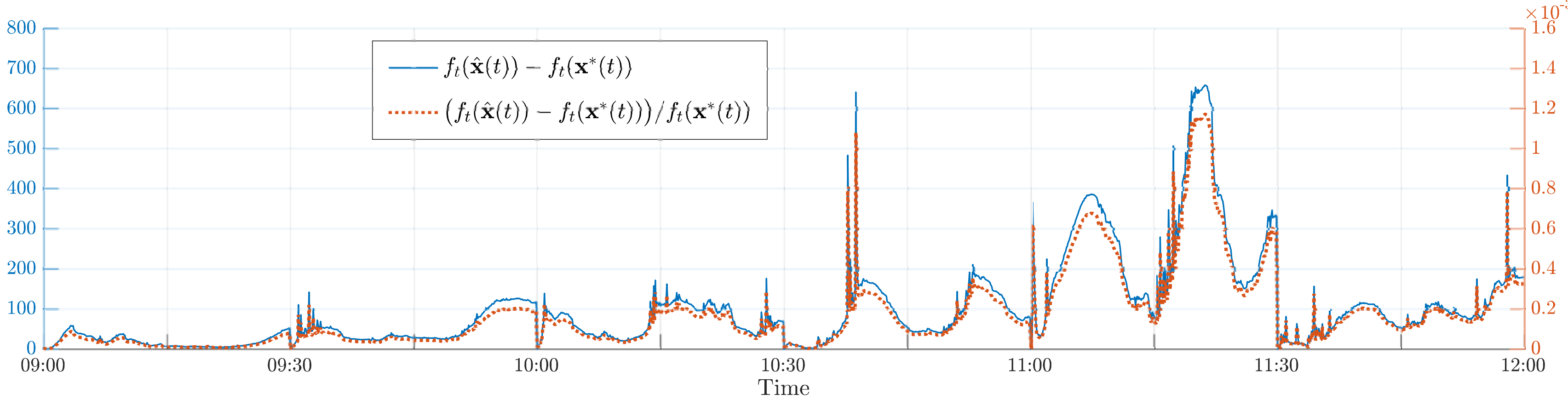
Tracking performance



IEEE 300 bus

Tracking performance





IEEE 300 bus

Key message

Large network of DERs

- Real-time optimization at scale
- Computational challenge: power flow solution

Online optimization [feedback control]

- Network computes power flow solutions in real time at scale for free
- Exploit it for our optimization/control
- Naturally adapts to evolving network conditions

Examples

- Slow timescale: OPF
- Fast timescale: frequency control